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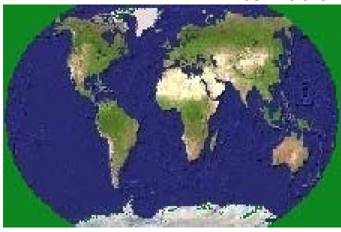

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Survival time to trace the threshold grade level in an organization

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Abstract
Manpower planning depends on the highly unpredictable human behaviour and the uncertain social environment in which the system functions. The study of stochastic models of Manpower systems have been proposed and studied extensively in the past by many researchers. In this paper we develop a model with Stochastic Process of manpower recruitment for an organization where the graded manpower system is observed. Existing numbers at each epoch's time and threshold level is been calculated. The simulation results were studied to illustrate the proposed model.

Keywords: Manpower planning, Survival, Stochastic, Threshold

1. Introduction

Manpower forecasts are important for assisting the Human resource agencies in the policy-making process. Forecasts based on an inaccurate analysis can cause either undersupply or oversupply of manpower. Management should not only be mindful of the outcome of the performance reward systems but also the process of how to implement those systems. The organization is exposed to a break down situation when the number of exits of personnel exceeds a "threshold level". The organization takes decisions on revising policies at random times, where the inter-decision times, which are called epochs, are i.i.d random variable. One can see for more details in [1, 4, 5].

The Cumulative density function (CDF) of the Modified Weibull Distribution (MWD)

$$(x; \alpha, \beta, \gamma)$$

$$F(x; \alpha, \beta, \gamma) = 1 - e^{-\alpha x - \beta x^{\gamma}} \;, \qquad x > 0$$

[2], [3]. It is observed that the MWD can have constant, increasing and decreasing hazard rate functions which are desirable for data analysis purposes. Application on set of real data showed that the MWD can be used rather than other known distribution.

2. Notations

X_i : continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i[th] occasion of policy announcement, $i =$ $1,2,3, \ldots k$ and $X'_i\,S$ are i.i.d and $X_i = X$ for all i.

Y_1, Y_2 : continuous random variable denoting the threshold levels for the two grades which follows Modified Weibull distribution.

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g^*(.)$: Laplace transform of $g(.)$; $g_k{}^*(.)$: Laplace transform of $g_k(.)$

$h(.)$: The probability density function of random threshold level which has Modified Weibull distribution and $H(.)$ is the corresponding Probability generating functions.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding Probability Density function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0,t]

$S(.)$: The survivor function i.e. $P(T > t)$; $1 - S(t) = L(t)$

3. Model Descriptions

The corresponding survival function of Modified Weibull distribution

$$\bar{H}(x) = e^{-(\alpha_2 x+\beta_2 x)} + e^{-(\alpha_1 x+\beta_1 x)} - e^{-(\alpha_1 x+\beta_1 x)(\alpha_2 x+\beta_2 x)} \quad (1)$$

$$P[Max(Y_1,Y_2)] = P[Y_1 < Y \cap Y_2 < Y] = P[Y_1 < Y]P[Y_2 < Y] \quad (2)$$

One is interested in an item for which there is a significant individual variation in ability to withstand shocks. There may be no practical way to inspect an individual item to determine its threshold y. In this case, the threshold must be a random variable. The shock survival probability are given by

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^\infty g_k(x)\,\bar{H}(x)dx = [g^*(\alpha_2+\beta_2)]^k + [g^*(\alpha_1+\beta_1)]^k + [g^*(\alpha_1+\beta_1)(\alpha_2+\beta_2)]^k \quad (3)$$

The survival function which gives the probability that the cumulative threshold will fail only after time t. $S(t) = P(T > t)$, Probability that the total damage survives beyond t.

It is also known from renewal process that

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)P(X_i < Y)$$

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\,[g^*(\alpha_2+\beta_2)]^k + [g^*(\alpha_1+\beta_1)]^k + [g^*(\alpha_1+\beta_1)(\alpha_2+\beta_2)]^k \quad (4)$$

$$= 1 - [1 - g^*(\alpha_2+\beta_2)]\sum_{k=1}^{\infty} F_k(t)\,[g^*(\alpha_2+\beta_2)]^{k-1} + [1 - g^*(\alpha_1+\beta_1)]\sum_{k=1}^{\infty} F_k(t)\,[g^*(\alpha_1+\beta_1)]^{k-1} + [1 - g^*(\alpha_1+\beta_1)(\alpha_2+\beta_2)]\sum_{k=1}^{\infty} F_k(t)\,[g^*(\alpha_1+\beta_1)(\alpha_2+\beta_2)]^{k-1} \quad (5)$$

Data that measure "the length of time" until the occurrence of an event are called lifetimes, failure times or survival data. $L(T) = 1 - S(t)$. Taking Laplace transform of $L(T)$, we get

$$L(T) = 1 - \left\{1 - [1 - g^*(\alpha_2 + \beta_2)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(\alpha_2 + \beta_2)]^{k-1} + [1 - g^*(\alpha_1 + \beta_1)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(\alpha_1 + \beta_1)]^{k-1} + [1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)]^{k-1}\right\} \qquad (6)$$

Let the random variable U denoting inter arrival time which follows exponential with parameter. Now, $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the below equation (7) we get,

$$L^*(s) = \frac{[1 - g^*(\alpha_2 + \beta_2)]\, f^*(s)}{[1 - g^*(\alpha_2 + \beta_2)\, f^*(s)]} + \frac{[1 - g^*(\alpha_1 + \beta_1)]\, f^*(s)}{[1 - g^*(\alpha_1 + \beta_1)\, f^*(s)]} - \frac{[1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)]\, f^*(s)}{[1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2) f^*(s)]} \qquad (7)$$

$$= \frac{[1 - g^*(\alpha_2 + \beta_2)]\, \frac{c}{c+s}}{\left[1 - g^*(\alpha_2 + \beta_2)\, \frac{c}{c+s}\right]} + \frac{[1 - g^*(\alpha_1 + \beta_1)]\, \frac{c}{c+s}}{\left[1 - g^*(\alpha_1 + \beta_1)\, \frac{c}{c+s}\right]} - \frac{[1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)]\, \frac{c}{c+s}}{\left[1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2) \frac{c}{c+s}\right]}$$

On simplifications we get,

$$= \frac{c\, [1 - g^*(\alpha_2 + \beta_2)]}{[c + s - g^*(\alpha_2 + \beta_2)\, c]} + \frac{c\, [1 - g^*(\alpha_1 + \beta_1)]}{[c + s - g^*(\alpha_1 + \beta_1)\, c]} - \frac{c\, [1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)]}{[c + s - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)\, c]} \qquad (8)$$

$$E(T) = -\frac{d}{ds}\; l^*(s)\;\; given\; s = 0$$

$$= \frac{1}{c\, [1 - g^*(\alpha_2 + \beta_2)]} + \frac{1}{c\, [1 - g^*(\alpha_1 + \beta_1)]} - \frac{1}{c\, [1 - g^*(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)]} \qquad on\ simplification \qquad (9)$$

The inter-arrival time of the threshold follows exponential distribution. The Laplace transformation of the exponential is given by $\frac{\mu}{\mu+\lambda}$

$$g^*(.) \sim \exp(\mu), \qquad g^*(\lambda) \sim \exp\left(\frac{\mu}{\mu + (\alpha_1 + \beta_1)}\right) \quad g^*(\lambda) \exp\left(\frac{\mu}{\mu + (\alpha_2 + \beta_2)}\right)$$

$$= \frac{1}{c\left[1 - \frac{\mu}{\mu + (\alpha_2 + \beta_2)}\right]} + \frac{1}{c\left[1 - \frac{\mu}{\mu + (\alpha_1 + \beta_1)}\right]} - \frac{1}{c\left[1 - \frac{\mu}{\mu + (\alpha_2 + \beta_2)(\alpha_1 + \beta_1)}\right]}$$

$$E(T) = \left[\frac{\mu + (\alpha_2 + \beta_2)}{c(\alpha_2 + \beta_2)} + \frac{\mu + (\alpha_1 + \beta_1)}{c(\alpha_1 + \beta_1)} + \frac{\mu + (\alpha_2 + \beta_2)(\alpha_1 + \beta_1)}{c[(\alpha_2 + \beta_2)(\alpha_1 + \beta_1)]}\right] \qquad (10)$$

4. Numerical Illustration

On the basis of the expressions derived for the expected time, the behaviour of the same due to the change in different parameters is shown in the below Figure

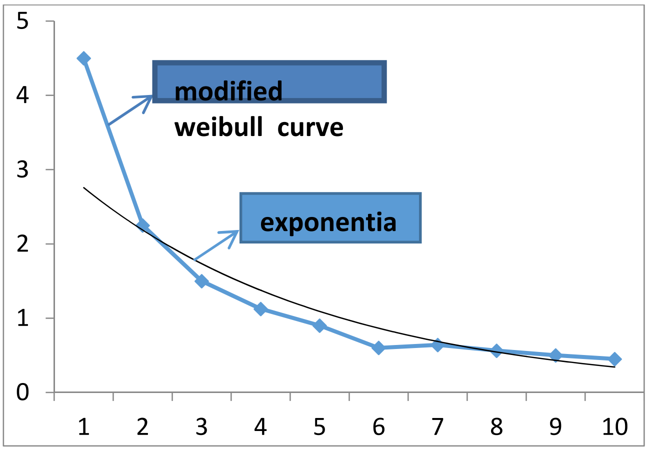

Fig 1: Expected Time

5. Conclusion

From Figure-1 it is observed that with fixed parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \mu$ and for increase change in inter-arrival time $'c'$ the Expected time to attain the threshold level decreases. The practical implication of the result is that the spread of damage through unknown factors is faster as the intensity of the organization is lower. Pronouncement making arrangements should be taken in regular interval for the development of the organization. Comparing with the exponential curve it is observed that as risk increase the threshold level of an organization shrinks.

6. Reference

1. Esary JD, Marshall AW, Proschan F. Shock models and their processes. Ann Probability 1973; 627-649.
2. Ammar M. Sarhan and Mazen Zaindin., (2009). Modified Weibull distribution, *Applied Sciences*, Vol.11, pp. 123-136.
3. Lawless JF. Statistical Models and Methods for Lifetime Data, John Wiley and Sons, New York, 2003.
4. Kannadasan K, Pandiyan P, Vinoth R, Saminathan R. Time to Recruitment in an Organization through Three Parameter Generalized Exponential Model. Journal of Reliability and Statistical Studies 2013; 6:21-28.
5. Pandiyan P, Kannadasan K, Vinoth R. Prospective Control in an Organization through Two Grade Systems. 4D International Journal of Management and Science 2013; 3(1):1-9.