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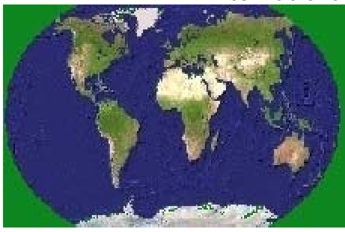

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Regular properties of lexicographic products

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Abstract
In this paper, the regular properties of the lexicographic products namely, lexicographic min-product and lexicographic max-product of two regular fuzzy graphs are studied. Also the conditions for the lexicographic products of two regular fuzzy graphs to be regular are provided.

Keywords: Regular Fuzzy Graph, Full Regular Fuzzy Graph, Lexicographic Min-product and Lexicographic Max-product.

1. introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Bhattacharya [1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson.J.N. and Peng.C.S. [3]. The conjunction of two fuzzy graphs was defined by Nagoor Gani.A and Radha.K. [4]. We defined the direct sum[6] and the strong product[8] of two fuzzy graphs and studied the effective, connected and regular properties of these operations. Also we defined the lexicographic min-product and lexicographic max-product of two fuzzy graphs and studied some of their properties[9]. In this paper, we have proved that lexicographic min-product and lexicographic max-product of two regular fuzzy graphs need not be regular and provided the conditions for the lexicographic products of two regular fuzzy graphs to be regular.

2. Preliminaries

First let us recall some preliminary definitions and results that can be found in [1]-[9].

A fuzzy graph G is a pair of functions (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ, μ) is denoted by G*:(V, E) where $E \subseteq V\times V$. The degree of a vertex u of a fuzzy graph G: (σ, μ) is defined as $d_G(u) = \sum_{u\neq v} \mu(uv) = \sum_{uv\in E} \mu(uv)$. The fuzzy graph G is called k-regular if $d_G(u) = k$ for all $u\in V$ and G is called full regular if G is both regular and partially regular.

Let G_1: (σ_1, μ_1) and G_2 :(σ_2,μ_2) denote two fuzzy graphs. Define G: (σ, μ) with underlying crisp graph G*: (V, E) where $V = V_1 \times V_2$, $E = \{(u_1, v_1)(u_2, v_2) / u_1 u_2\in E_1$ or $u_1= u_2$ and $v_1 v_2\in E_2\}$ by,

$\sigma(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$, for all $(u_1, v_1) \in V_1 \times V_2$ and

$$\mu((u_1, v_1)(u_2, v_2)) = \begin{cases} \mu_1(u_1u_2) & ,\text{if } u_1u_2 \in E_1 \\ \sigma_1(u_1)\wedge \mu_2(v_1v_2) & ,\text{if } u_1 = u_2, v_1v_2 \in E_2. \end{cases}$$

Then G :(σ, μ) is called the lexicographic min-product of G_1 with G_2 and is denoted by $G_1[G_2]_{min}$.

Define G:(τ, ν) by,

$\tau(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$, for all $(u_1, v_1) \in V_1 \times V_2$ and

$$\nu((u_1, v_1)(u_2, v_2)) = \begin{cases} \mu_1(u_1u_2) & ,\text{if } u_1u_2 \in E_1 \\ \sigma_1(u_1)\vee \mu_2(v_1v_2) & ,\text{if } u_1 = u_2, v_1v_2 \in E_2. \end{cases}$$

Then G :(τ, ν) is called the lexicographic max-product of G_1 with G_2 and is denoted by $G_1[G_2]_{max}$.

If G_1 :(σ_1,μ_1) and G_2:(σ_2,μ_2) are two fuzzy graphs such that $\sigma_1 \geq \mu_2$, then $d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$ & $d_{G_1[G_2]_{max}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i)$.

If $\sigma_1 \leq \mu_2$, then $d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i)$ & $d_{G_1[G_2]_{max}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$.

If $\sigma_1 \wedge \mu_2 = c$, a constant, then $d_{G_1[G_2]_{min}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$.

If $\sigma_1 \vee \mu_2 = c$, a constant, then $d_{G_1[G_2]_{max}}(u_i,v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$.

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Notation

The relation $\sigma_1 \le \sigma_2$ means that $\sigma_1(u) \le \sigma_2(v)$ for every $u \in V_1$ and for every $v \in V_2$ where σ_i is a fuzzy subset of V_i, i =1,2.

3. Regular property of the lexicographic min-product

If G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are two regular fuzzy graphs then the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of G_1: (σ_1, μ_1) with G_2: (σ_2, μ_2) need not be a regular fuzzy graph. It is illustrated through the following example.

3.1. Example

Consider the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$, $G_2:(\sigma_2, \mu_2)$ and their lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ given in the following Figure-1.

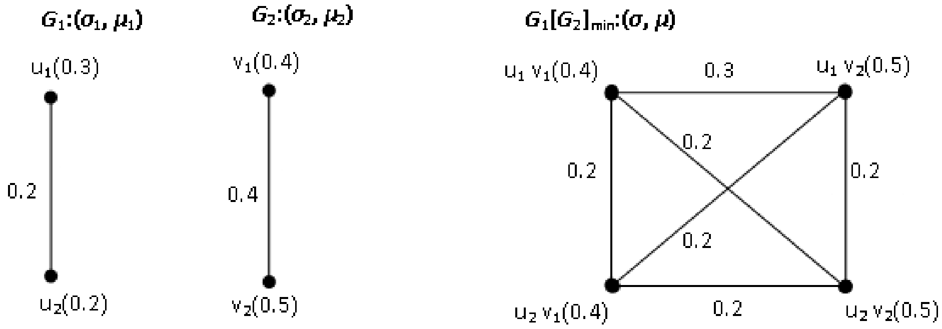

Fig: 1 Lexicographic min-product of G_1 with G_2

$d_{G_1}(u_i) = 0.2, \forall\, u_i \in V_1$ and $d_{G_2}(v_j) = 0.4, \forall\, v_j \in V_2$

But,

$$d_{G_1[G_2]_{min}}(u_1, v_1) = 2 \times 0.2 + 1 \times 0.3 = 0.7 \text{ and } d_{G_1[G_2]_{min}}(u_1, v_2) = 2 \times 0.2 + 1 \times 0.3 = 0.7$$
$$d_{G_1[G_2]_{min}}(u_2, v_1) = 2 \times 0.2 + 1 \times 0.2 = 0.6 \text{ and } d_{G_1[G_2]_{min}}(u_2, v_2) = 2 \times 0.2 + 1 \times 0.2 = 0.6$$

Thus the two fuzzy graphs G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are regular but their lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is not a regular fuzzy graph.

But with few restrictions it can be proved that the lexicographic min-product of two regular fuzzy graphs is regular. The following theorems explain the conditions for the lexicographic min-product of two regular fuzzy graphs to be regular.

3.2. Remark

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two partially regular fuzzy graphs then both the lexicographic min-products $G_1[G_2]_{min}:(\sigma, \mu)$ and $G_2[G_1]_{min}:(\sigma', \mu')$ are partially regular fuzzy graphs since $G_1^*:(V_1, E_1)$ and $G_2^*:(V_2, E_2)$ are regular graphs.

3.3. Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ then the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is a regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.

Proof:

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \ge \mu_2$. Then, the degree of any vertex in the lexicographic min-product of the two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$$

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs of degrees k_1 and k_2 respectively, then

$$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, k_1 + k_2$$

This is a constant since k_1, k_2 and $|V_2|$ are all constants and it is true for all $(u_i\, v_j) \in V_1 \times V_2$.

Thus we arrive at the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is regular.

Conversely assume that $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ and the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is a regular fuzzy graph.

To prove: $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.

Then for any two vertices (u_1, v_1) and (u_2, v_2) in $V_1 \times V_2$,

$$d_{G_1[G_2]_{min}}(u_1, v_1) = d_{G_1[G_2]_{min}}(u_2, v_2)$$
$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) + d_{G_2}(v_1) = |V_2|\, d_{G_1}(u_2) + d_{G_2}(v_2)$$

Fix $v \in V_2$ and consider (u_1, v) and (u_2, v) in $V_1 \times V_2$ where u_1, $u_2 \in V_1$ are arbitrary.

$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) + d_{G_2}(v) = |V_2|\, d_{G_1}(u_2) + d_{G_2}(v)$$
$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) = |V_2|\, d_{G_1}(u_2)$$
$$\Rightarrow \quad d_{G_1}(u_1) = d_{G_1}(u_2)$$

This is true for all $u_1, u_2 \in V_1$. Thus G_1: (σ_1, μ_1) is a regular fuzzy graph.

Now fix $u \in V_1$ and consider (u, v_1) and (u, v_2) in $V_1 \times V_2$ where $v_1, v_2 \in V_2$ are arbitrary.

$$\Rightarrow \quad |V_2|\, d_{G_1}(u) + d_{G_2}(v_1) = |V_2|\, d_{G_1}(u) + d_{G_2}(v_2)$$
$$\Rightarrow \quad d_{G_2}(v_1) = d_{G_2}(v_2)$$

This is true for all $v_1, v_2 \in V_2$. Thus G_2: (σ_2, μ_2) is also a regular fuzzy graph.

3.4. Remark

(i) Similarly it can be proved that if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_2 \ge \mu_1$ then the lexicographic min-product $G_2[G_1]_{min}:(\sigma', \mu')$ is a regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.

(ii) If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ then the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ is a full regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are full regular fuzzy graphs.

(iii) If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ then the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$is a complete regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are complete regular fuzzy graphs.

(iv) In the above two remarks (ii) and (iii), if the condition "$\sigma_1 \ge \mu_2$" is replaced with "$\sigma_2 \ge \mu_1$" then the lexicographic min-product "$G_1[G_2]_{min}:(\sigma, \mu)$" will be replaced with "$G_2[G_1]_{min}:(\sigma', \mu')$".

3.5. Example

Consider the two complete regular fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ and the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of them which is also a complete regular fuzzy graph in the following Figure-2. In fact $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are full regular fuzzy graphs and the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of these two full regular fuzzy graphs is also a full regular fuzzy graph.

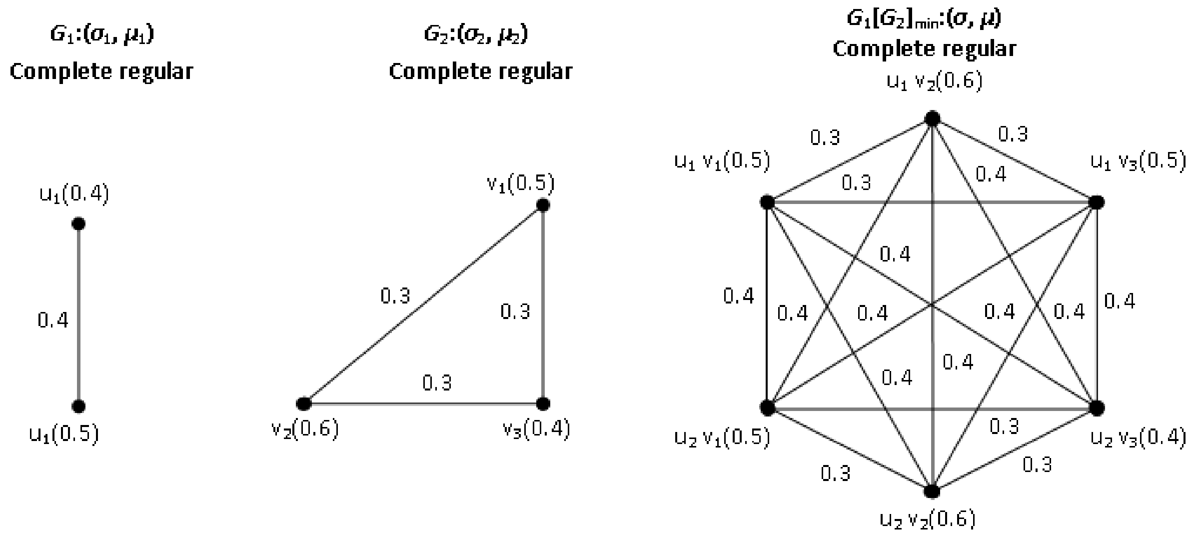

Fig 2: Lexicographic min-product of two complete regular fuzzy graphs

3.6. Theorem

If $G_1:(\sigma_1,\mu_1)$ is a regular fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a partially regular fuzzy graph such that $\sigma_1 \wedge \mu_2$ is a constant then the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is a regular fuzzy graph.

Proof:

Let $G_1:(\sigma_1,\mu_1)$ be a regular fuzzy graph of degree k_1 and $G_2:(\sigma_2,\mu_2)$ be a partially regular fuzzy graph such that the underlying crisp graph $G_2^*:(V_2,E_2)$ is a regular graph (say r_1-regular). Also take $\sigma_1 \wedge \mu_2 = c$, a constant. Then, the degree of any vertex in the lexicographic min-product of the two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1[G_2]_{min}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)(\sigma_1 \wedge \mu_2)$$
$$= |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$$
$$= |V_2|\, k_1 + r_1 c.$$

This is a constant since $|V_2|$, k_1, r_1 and c are all constants and it is true for all $(u_i\, v_j) \in V_1 \times V_2$. Thus the lexicographic min-product $G_1[G_2]_{min}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is a regular fuzzy graph.

4. Regular property of the lexicographic max-product

Consider the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$, $G_2:(\sigma_2, \mu_2)$ and their lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ given in the following Figure-3.

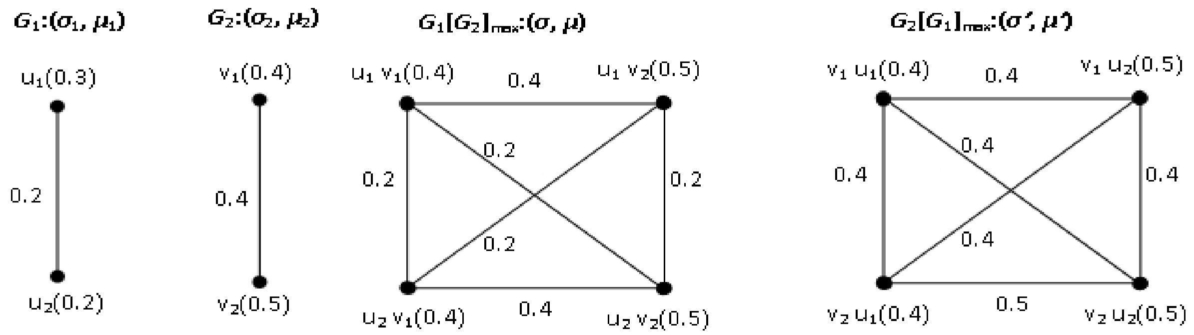

Fig 3: Lexicographic max-product of G_1 with G_2

$$d_{G_1}(u_i) = 0.2, \forall\, u_i \in V_1 \text{ and } d_{G_2}(v_j) = 0.4, \forall\, v_j \in V_2$$

But,

$$d_{G_1[G_2]_{max}}(u_1, v_1) = 2\times 0.2 + 1\times 0.3 = 0.7 \text{ and } d_{G_1[G_2]_{max}}(u_1, v_2) = 2\times 0.2 + 1\times 0.3 = 0.7$$
$$d_{G_1[G_2]_{max}}(u_2, v_1) = 2\times 0.2 + 1\times 0.2 = 0.6 \text{ and } d_{G_1[G_2]_{max}}(u_2, v_2) = 2\times 0.2 + 1\times 0.2 = 0.6$$

Thus the two fuzzy graphs $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ are regular fuzzy graphs but their lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is not a regular fuzzy graph. Also $G_2[G_1]_{max}:(\sigma', \mu')$ is not a regular fuzzy graph. Hence the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of two regular fuzzy graphs $G_1: (\sigma_1, \mu_1)$ with $G_2: (\sigma_2, \mu_2)$ need not be a regular fuzzy graph.

4.2. Remark

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two partially regular fuzzy graphs then both the lexicographic max-products $G_1[G_2]_{max}:(\sigma, \mu)$ and $G_2[G_1]_{max}:(\sigma', \mu')$ are partially regular fuzzy graphs since $G_1^*:(V_1, E_1)$ and $G_2^*:(V_2, E_2)$ are regular graphs.

4.3. Theorem
If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ then the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is a regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.

Proof:
Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \le \mu_2$. Then, the degree of any vertex in the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2}(v_j)$$

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs of degrees k_1 and k_2 respectively, then

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, k_1 + k_2$$

This is a constant since k_1, k_2 and $|V_2|$ are all constants and it is true for all $(u_i\, v_j) \in V_1 \times V_2$.
Thus we arrive at the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is regular.
Conversely assume that $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ and the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is a regular fuzzy graph.
To prove: $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.
Then for any two vertices (u_1, v_1) and (u_2, v_2) in $V_1 \times V_2$,

$$d_{G_1[G_2]_{max}}(u_1, v_1) = d_{G_1[G_2]_{max}}(u_2, v_2)$$

$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) + d_{G_2}(v_1) = |V_2|\, d_{G_1}(u_2) + d_{G_2}(v_2)$$

Fix $v \in V_2$ and consider (u_1, v) and (u_2, v) in $V_1 \times V_2$ where $u_1, u_2 \in V_1$ are arbitrary.

$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) + d_{G_2}(v) = |V_2|\, d_{G_1}(u_2) + d_{G_2}(v)$$

$$\Rightarrow \quad |V_2|\, d_{G_1}(u_1) = |V_2|\, d_{G_1}(u_2)$$

$$\Rightarrow \quad d_{G_1}(u_1) = d_{G_1}(u_2)$$

This is true for all $u_1, u_2 \in V_1$. Thus $G_1: (\sigma_1, \mu_1)$ is a regular fuzzy graph.
Now fix $u \in V_1$ and consider (u, v_1) and (u, v_2) in $V_1 \times V_2$ where $v_1, v_2 \in V_2$ are arbitrary.

$$\Rightarrow \quad |V_2|\, d_{G_1}(u) + d_{G_2}(v_1) = |V_2|\, d_{G_1}(u) + d_{G_2}(v_2)$$

$$\Rightarrow \quad d_{G_2}(v_1) = d_{G_2}(v_2)$$

This is true for all $v_1, v_2 \in V_2$. Thus $G_2: (\sigma_2, \mu_2)$ is also a regular fuzzy graph.

4.4. Remark
(i) Similarly it can be proved that if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_2 \le \mu_1$ then the lexicographic max-product $G_2[G_1]_{max}:(\sigma', \mu')$ is a regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are regular fuzzy graphs.
(ii) If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ then the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is a full regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are full regular fuzzy graphs.
(iii) If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ then the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ is a complete regular fuzzy graph if and only if $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are complete regular fuzzy graphs.
(iv) In the above two remarks (ii) and (iii), if the condition "$\sigma_1 \le \mu_2$" is replaced with "$\sigma_2 \le \mu_1$" then the lexicographic max-product "$G_1[G_2]_{max}:(\sigma, \mu)$" will be replaced with "$G_2[G_1]_{max}:(\sigma', \mu')$".

4.5. Theorem
If $G_1:(\sigma_1,\mu_1)$ is a regular fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a partially regular fuzzy graph such that $\sigma_1 \vee \mu_2$ is a constant then the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is a regular fuzzy graph.

Proof:
Let $G_1:(\sigma_1,\mu_1)$ be a regular fuzzy graph of degree k_1 and $G_2:(\sigma_2,\mu_2)$ be a partially regular fuzzy graph such that the underlying crisp graph $G_2^*:(V_2,E_2)$ is a regular graph (say r_1-regular). Also take $\sigma_1 \vee \mu_2 = c$, a constant. Then, the degree of any vertex in the lexicographic max-product of the two fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1[G_2]_{max}}(u_i, v_j) = |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)(\sigma_1 \vee \mu_2)$$
$$= |V_2|\, d_{G_1}(u_i) + d_{G_2^*}(v_j)\, c$$
$$= |V_2|\, k_1 + r_1 c.$$

This is a constant since $|V_2|$, k_1, r_1 and c are all constants and it is true for all $(u_i\, v_j) \in V_1 \times V_2$. Thus the lexicographic max-product $G_1[G_2]_{max}:(\sigma, \mu)$ of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is a regular fuzzy graph.

5. Conclusion
In this paper, we have studied the regular properties of the lexicographic min-product and the lexicographic max-product. Also we have provided the conditions for the lexicographic products of two regular fuzzy graphs to be regular. These properties will be helpful to study large fuzzy graph as a combination of small fuzzy graphs and to derive its properties from those of the small ones.

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