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A mathematical approach to investigate the temperature distribution on skin surface with sinusoidal heat flux condition

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Abstract
This work is a mathematical approach to study the temperature distribution of semi-infinite living biological tissues subject to sinusoidal heat flux on the skin with the thermal model of bioheat transfer which are often confront in cancer hyperthermia, laser surgery, thermal comfort analysis and tissue thermal parameter estimation etc.. There is equation which is known as Pennes equation that provide a relation for heat transfer between the tissue and blood has been solved with the help of Laplace transform with surface sinusoidal heating condition. This obtained solution defines the temperature distribution reactions on the surface of the skin. The results show that the temperature distribution due to the sinusoidal heating on the skin surface, it is linear initially and later on the temperature profile shows relation of temperature with respect to time when a constant heat flux is applied at the surface of the skin and it takes some finite time for skin to reach to a constant temperature.

Keywords: mathematical, sinusoidal, semi-infinite, temperature

Introduction

Skin plays a significant role in thermoregulation and provides the protection from the surroundings. Purpose of thermoregulation is to permit our body to maintain its core internal temperature, it deals with absorption, heat generation, radiation, vaporization and transmission. A basic process which takes place between the skin and the environment is thermal energy exchange. The function of skin depend on the location of the skin and its function as well as structure and thickness [6, 7, 9]. There are two basic layers in the skin are dermis and epidermis [13, 21, 28]. Dermis and epidermis permit heat let go by radiation, convection and perspiration. Regarding temperature distribution the temperature of the skin is 31°C and the temperature of the bone is 33°C in the forearm of human. For this reason, there is a need of imposing higher temperature on the outer surface of skin to get an increase in the deep tissues. Micro-circulatory bed made of artery, capillaries, vein, arterioles and venules and are responsible for thermoregulation and it regulate by changing blood flow in blood vessels [12, 14, 17]. The blood circulation increases if there is an increase in the skin temperature to 42°C in order to dissolve a quantity of the thermal energy and to allow increase in the heat conduction mechanism through the deep tissue beneath [10, 15, 18]. In the resting condition, fat of the body also affects the deep tissue and skin temperature [15, 19]. Several investigator have studied the blood perfusion and temperature response while heating of the surface [7-11]. Heat transfer study seldom requires to simultaneously deal with spatial heating and transient both on the deep biological bodies and on skin surface**.** The temperature distribution for thermal therapy to exchange magnitude of heat transfer between tissue and blood, is known as Pennes bioheat transfer equation and is widely used now a days [20].

And there are some assumptions that there is only heat transfer in capillary between which take place in the blood and the tissue. The assumption has been made by continuum method to neglect the local effects "thermally significant" so the blood vessels disappear in the field of temperature. Nevertheless, Nyborg [22] has done a study in details where cooling effect of thermally significant blood vessels has been considerd**.** Kengne and Liu, [12] conducted a study for transient bioheat transfer equation for porous medium and explained the encompassing directional effect of blood flow**.** It has also demonstrated by several researcher [10, 22, 27], for localized hyperthermia the analytical solution for both steady state and transient for three tumor models with the help of Pennes bioheat transfer equation has good results. Arkin *et al*., [2] have also perfomed a study for sinusoidal heating on the skin surface in closed-form by using Pennes bioheat transfer equation. This theoretical and analytical study is important for the periodic temperature oscillation for long run steady condition. Although, that solution would not be able to perform a satisfactory solution to the starting transient temperature. Liu and Xu [5] have also performed a study for analysis for the skin surface with simple various boundary conditions and also investigated the phase shift between the surface temperature and the Shih *et al*., [29] also presented accurate correlations between human pathophysiology and skin thermal image information for possible diagnostics with the help of Monte Carlo algorithm. By using Green's fun ction method, Schürmann and Serov [25] has obtained many solution to bioheat transfer equation with transient heating or space or inside bodies or space. Pennes bioheat transfer equation using has been used by Kou *et al*., [14], he has applied finitedifference method to silve one dimensional equation and the finite-element method to solve two dimensional equation to analyse the results and to predict the state of skin burns in human skin. Ocheltree and, Frizzel [23] has developed a model for four-noded tetrahedrons and a mesh less-sensitive numerical approximation to model the tissue temperature and surface temperature of a breast with a tumor and normal breast in order to obtained an new system for cancer in breast and its early detection. Their team have also investigated the temperature distribution on surface and changes with depth of the tumor and the size of the tumor. Furthermore, many studies have done on the heating kinetics and predicted the solution during millimetre wave exposures of human skin for a short period of time [24, 26]. Thus, temperature should be transient for estimation of blood perfusion in thermal medical condition. Therefore, the objective of this paper is to obtained the solution of the Pennes bioheat transfer equation associated with the sinusoidal heating boundary condition.

Formulation of the Problem

The study for in the blood perfused tissue for transient temperature field associated with oscillated heating conditions has been done in this presented work. One dimensional heat exchange equation has been considered for the ease of skin geometrical feature. The study also contains an oscillated boundary condition $q(0,t) = q_0 e^{i\omega t}$ to discover the temperature variation in the tissues.

$$\rho_t c_t \frac{\partial T}{\partial t} + W_b c_b (T - T_a) = k \frac{\partial^2 T}{\partial x^2} \quad (1)$$

where *c*t is the specific heat for the tissue, *ρ*t is the density of the tissue, *T* is the temperature of the tissue, *t is* time, *c*b is the specific heat of blood, *W*b is the blood perfusion rate and *T*a is the supplying arterial blood temperature, *x* is the distance from the skin surface and *k* the thermal conductivity of tissue.

The boundary condition for heat flux is described as below:

$$-k \frac{\partial T}{\partial x}\Big|_{x=0} = q_0 e^{iwt} \quad (2)$$

where, ω is heating frequency and q_0 is heat flux.

Solution of the Problem

The dimensionless scheme defined as,

$$z \equiv \omega t, \quad \alpha \equiv \frac{k}{\rho_t c_t}, \quad X \equiv \sqrt{\frac{\omega}{\alpha}}\, x, \quad c_1 \equiv \frac{W_b c_b}{\rho_t c_t \omega}, \quad \theta \equiv \frac{k(T - T_a)}{q_0}\sqrt{\frac{\omega}{\alpha}} \quad (3)$$

Writing equation (1) after using dimensionless scheme,

$$\frac{\partial \theta}{\partial z} + c_1 \theta = \frac{\partial^2}{\partial x^2} \quad (4)$$

After applying dimensionless scheme, the boundary conditions are as below,

$$\frac{\partial T}{\partial x}\Big|_{x=0} = -e^{iz} \quad (5)$$

Applying Laplace transform on Eq. (4), it reduces as below,

$$\frac{\partial^2 \Phi}{\partial x^2} - (s + c_1)\Phi = 0 \quad (6)$$

Solution for the Eq. (6) is as below,

$$\Phi = c_2 e^{-\sqrt{s + c_1} X} \quad (7)$$

Using Eq. (7), the dimensionless boundary condition are as,

$$\frac{d\Phi}{dX}\Big|_{X=0} = -\sqrt{s + c_1}\, c_2 \quad (8)$$

Applying Laplace transform on Eq. (5), it is written as below,

$$\frac{d\Phi}{dX}\Big|_{X=0} = -\frac{1}{s - i} \quad (9)$$

where, $c_2 = \frac{1}{(s-i)\sqrt{s+c_1}}$ (10)

Putting the value from Eq. *(*10*)* to Eq. *(7)*, we get,

$$\Phi = \frac{e^{-\sqrt{s+c_1}X}}{(s-i)\sqrt{s+c_1}} \quad (11)$$

Now taking the Inverse Laplace transform of Eq. *(*11*)*,

$$\theta = \frac{e^{iz}}{2\sqrt{c_1+i}} e^{-\sqrt{c_1+i}} erfc[\frac{X}{2\sqrt{z}-sqrtc_1+i}] - e^{sqrtc_1+i} erfc[\frac{X}{2\sqrt{z}+sqrtc_1+i}] \quad (12)$$

Hence the temperature profile on the skin surface is as below,

$$\theta|_{X=0} = \frac{e^{iz}}{\sqrt{c_1+i}} erf[\sqrt{(c_1+i)z}] \quad (13)$$

writting Eq. *(*5*)* and Eq. *(*13*)*, the surface temperature profile and surface heat flux oscillation are as below,

$$\theta|_{X=0} = -\frac{erf[\sqrt{((c_1+i)z)}]}{\sqrt{c_1+i}} \frac{\partial \theta}{\partial X}|_{X=0} \quad (14)$$

Hence, as a result transient bioheat transfer one-dimensional equation is written,

$$T = T_a + \frac{q_0}{2k}(\sqrt{\frac{\alpha}{(c_1+i)\omega}} e^{i\omega t})\{e^{-\sqrt{\frac{(c_1+i)\omega}{\alpha}}} erfc \times [\frac{x}{\sqrt{4\alpha t}} - \sqrt{(c_1+i)\omega t}] - e^{\sqrt{\frac{(c_1+i)\omega t}{\alpha}}x}$$

$$erfc \times [\frac{x}{\sqrt{4\alpha t}} + \sqrt{(c_1+i)\omega t}]\} \quad (15)$$

where, $\beta = \sqrt{(\frac{W_b c_b}{k}) + \frac{i\omega}{\alpha}}$ equ.(15)can be rewritten as:

$$T = T_a + \frac{q_0}{2k\beta} e^{i\omega t}[e^{-\beta x} erfc \times (\frac{x}{\sqrt{4\alpha t}} - \beta\sqrt{\alpha t}) - e^{\beta x} erfc \times (\frac{x}{\sqrt{4\alpha t}} + \beta\sqrt{\alpha t})] \quad (16)$$

Equ. (16), can be written as:

$$T|_{t\to 0} = T_a \quad (17)$$

$$T|_{t\to larger\ time} = T_a + \frac{q_0}{k\beta} e^{i\omega t} e^{-\beta x} \quad (18)$$

The two different heating cases are shown below**.**

Case I*:* $W_b c_b = 0$ *(*keeping blood perfusion neglected*)*

By assuming (c_1 =0) in Eq. *(*15*), **we get,***

$$T = T_a + \frac{q_0}{2k}(\sqrt{\frac{\alpha}{i\omega}} e^{i\omega t})[e^{-\sqrt{\frac{i\omega}{\alpha}}x} erfc \times (\frac{x}{\sqrt{4\alpha t}} - \sqrt{i\omega t}) - e^{-\sqrt{\frac{i\omega}{\alpha}}x} erfc \times (\frac{x}{\sqrt{4\alpha t}} + \sqrt{i\omega t})] \quad (19)$$

Case II*:* $q(0, t) = q_0$ *(keeping* Heat flux constant*)*

The boundary condition is written as***:***

$$-k\frac{\partial T}{\partial x}|_{x=0} = q_0 \quad (20)$$

The dimensionless scheme is as below,

$$X' \equiv x\sqrt{\frac{W_b c_b}{k}}, \alpha \equiv \frac{k}{\rho_t c_t}, z' \equiv \frac{W_b c_b}{\rho_t c_t}, \theta' \equiv \frac{k(T-T_a)}{q_0}\sqrt{kW_b c_b} \quad (21)$$

Now the Eq. (1) is as below,

$$\frac{\partial \theta'}{\partial z'} + \theta' = \frac{\partial^2 \theta'}{\partial X'^2} \quad (22)$$

The dimensionless Eq. (20) is written as,

$$\frac{\partial \theta'}{\partial X'}|_{X'=0} = -1 \quad (23)$$

Applying Laplace transform the Eqs. (22) and (25)، written as,

$$\frac{d^2\Theta}{dX^2} - (s+1)\Theta = 0 \quad (24)$$

hence, the equation can be written as,

$$\frac{d\Theta}{dX'}|_{X'=0} = -\frac{1}{s} \quad (25)$$

As result the Eqs. (24) and (25) is written as,

$$\Theta = \frac{e^{-\sqrt{s+1}X'}}{s\sqrt{s+1}} \quad (26)$$

Applying Inverse Laplace transform, the dimensionless equation can be written as below,

$$\theta' = \frac{1}{2}[e^{-X'}erfc(\frac{X'}{2\sqrt{z}} - \sqrt{z'}) - e^{X'}erfc(\frac{X'}{2\sqrt{z'}})] \quad (27)$$

$$T = T_a + \frac{q_0}{\sqrt{4kW_b c_b}}[e^{-\sqrt{\frac{W_b c_b}{k}}x} erfc \times (\frac{x}{\sqrt{4\alpha t}} - \sqrt{\frac{W_b c_b}{\rho_t c_t t}}) - e^{-\sqrt{\frac{W_b c_b}{k}}x} erfc \times (\frac{x}{\sqrt{4\alpha t}} + \sqrt{\frac{W_b c_b}{\rho_t c_t t}})] \quad (28)$$

The Eq. (27) with help of Eq. (27) can be written as below,

where, $\lambda = \sqrt{\frac{W_b c_b}{k}}$

and the equation can be written as below,

$$T = T_a + \frac{q_0}{2k\lambda}[e^{-\lambda x} erfc \times (\frac{x}{\sqrt{4\alpha t}} - \lambda\sqrt{\alpha t}) - e^{-\lambda x} erfc \times (\frac{x}{\sqrt{4\alpha t}} + \lambda\sqrt{\alpha t})] \quad (29)$$

Results and Discussion

This work has predicted the transient temperature profiles in living tissue subjected by a sinusoidal heating at the skin surface. All the parameters which are used in the study has been taken from the existing research and are shown in table (1) below. In Figure 1, when the time is zero, the graph shows a constant temperature. It implies that skin temperature is maintained constant which matches the initial conditions. In Figure 2, when distance is zero i.e, at the surface of skin, the temperature profile shows relation of temperature with respect to time when a constant heat flux is applied at the surface of the skin. The curve implies that it takes some finite time for skin to reach to a constant temperature.

Table 1

Parameters	Values
Density of tissue (ρ_t)_	1050 kg/m3

Specific heat of tissue(c_t)	3770 J/kg/C
Specific heat of blood(c_b)	3770 J/kg/C
Thermal conductivity of tissue(k)	0.5 W/m/C
Temperature of artery(T_a)	310 K
Heat flux(q_0)	5000 W/m2
Blood perfusion rate(w_b)	0.5 kg/m3
Heat frequency(w)	0.05

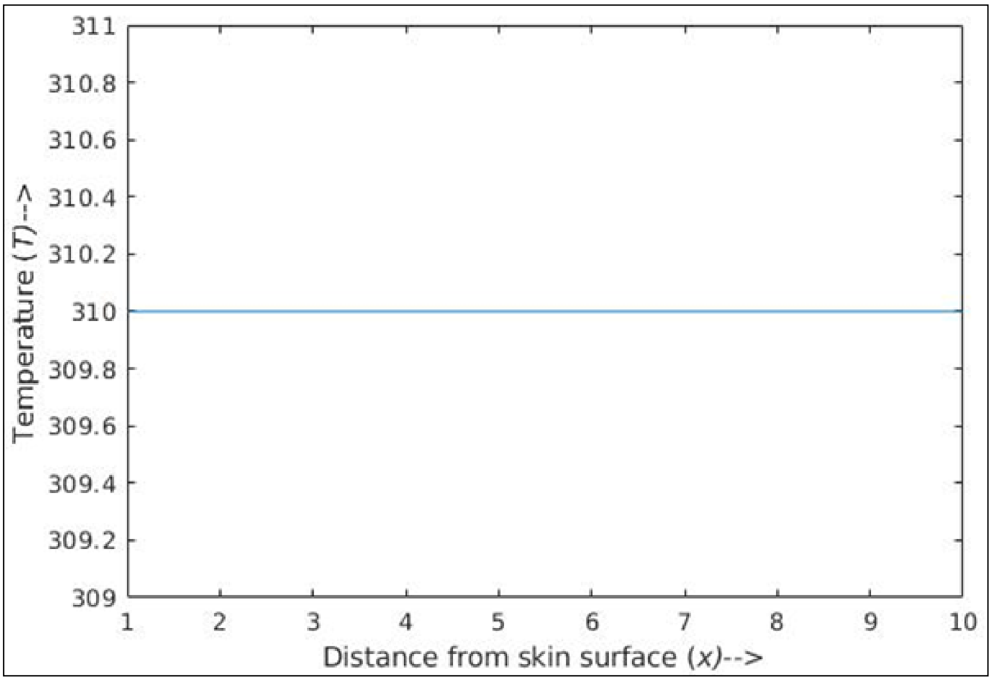

Fig 1: Variation of temperature v/s distance from skin surface when t=0

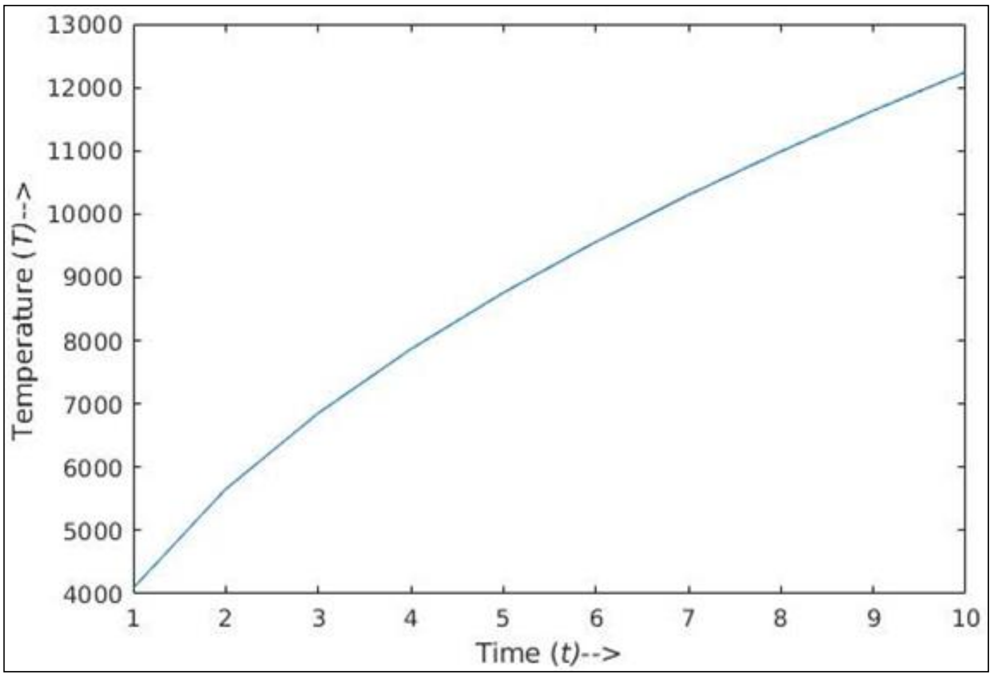

Fig 2: Variation of temperature v/s time when x=0

Conclusions

This study looks into effects of sinusoidal heat flux at the skin surface in a one-dimensional biological tissue. Laplace transform method is used to solve the Pennes bioheat Equation. The study concludes that at initial time the biological system is unstable and reaches to stability after passage of time. The instability is also dependent on frequency of heat flux. A low frequency system takes more time to stabilize while a system with high frequency of heat flux reaches stability faster in comparison to the former. This study was limited to special cases, but the model can be used to find the behavior of heat is skin at any general conditions.

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