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Comparison of multivariate GARCH models using volatility of EUR/ETB and USD/ETB exchange rates in Ethiopia

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Abstract
The price at which two distinct countries' currencies are traded varies in terms of volatility of exchange rate. Because exchange rate volatility is associated with risk and uncertainty, it is a major source of concern for macroeconomic policymakers. The main objective of this study was comparison of MGARCH models using volatility of daily Euro/Ethiopian birr and USD/ETB in Ethiopia. Secondary data were obtained from nation bank of Ethiopia and the sample data period runs from March 1, 2016 to February 24, 2020. The main variables considered under this study were USD/ETB and Euro/ETB daily exchange rates. There are 1,287 time series observations included and exclude Saturday and Sunday data from this study. Four types of multivariate GARCH models estimated in this study; namely: Constant Conditional Correlation (CCC), Dynamic Conditional Covariance (DCC), Dvech and Baba, Engle, Kraft and Kroner (BEKK) MGARCH models. The stationarity of sample data checked by Augmented Dickey Fuller (ADF) and Phillips-Perron **(**PP) test and the series were stationary after logarithm of first differenced. From estimated evidence that CCC-MGARCH (1, 2), DCC-MGARCH (1, 2), Dvech-MGARCH (1, 1) and BEKK-MGARCH (1, 1) models with Gaussian, student's-t and skew student's t-distribution are the best estimation models in terms of the volatility behavior of the series. Amongst these models, DCC-MGARCH (1, 2) with minimum value of information criteria and log-likelihood function was found to perform best in term of fit the volatility of Ethiopian Birr/USD and ETB/EUR. Hence, we recommend that future research works need to build on the current work and extend it a bit further currencies and models. For example, it would be interesting to consider fitting other MGARCH models of the major with other currencies the country uses in its international transactions.

Keywords: CCC, DCC, BEK, Dvech MGARCH models, volatility, exchange rate, Ethiopia

Introduction

Generalized autoregressive conditional Heteroscedasticity models, originally proposed by [7, 16], were a big advance in the statistical analysis of univariate financial returns and are widely fitted to describe and forecast their volatilities. Univariate GARCH models were extended to a multivariate framework by [10]. Since then, multivariate GARCH models have attracted a great deal of attention due to several applications that require estimates of conditional variances, covariances and correlations of multivariate time series. To recognize this feture through a multivariate model would generate a more reliable model than separate univariate GARCH mode. The broad number of representations of MGARCH models appear in the context of systems of financial time series returns; see, for example, [1-2], [6] [25-26] and [31-32], for a few selected asset pricing; volatility transmission between assets and markets futures hedging; influence of exchange rate volatility on trade and output and Value-at-Risk. From these, influence of exchange rate volatility is the most recognizable application of multivariate GARCH models. For instance: is the volatility of an asset transmitted to another directly (if the lagged conditional variance of the asset is significantly present in the conditional variance of the other asset) or indirectly (if the lagged conditional covariance between the asset and another enters in the other asset equation)? Does a shock on a market increase the volatility on another market, and by how much? Is the impact the same for negative and positive shocks of the same amplitude? Such like questions are raised in the impact of exchange rate volatility; see [1-2], and [27].

Volatility of exchange rate can be defined as the variation of price at which two different countries' currencies are traded [21]. Exchange rate volatility is a major source of concern for macroeconomic policy makers because it is synonymous with risk and uncertainty. It is well-documented that internationally-oriented countries are particularly sensitive to foreign exchange rate volatility [19]. Understanding the dynamics of volatility is therefore crucial for the design of a well-informed policy that seeks to minimize the deleterious influence of uncertainty on the national welfare. In addition, the volatility has shown to be auto-correlated; which means that today's volatility depends on the past volatility. Considering the fact that the volatility is not directly observable, the need of a good model to predict the future volatilities is essential. It is known that financial volatilities move together more or less closely over time across exchange rate markets. Hence, it is essential to take into account the dependence in the movements of exchange rates.

One method to estimate the covariance matrix between the assets is to extend the univariate GARCH into a multivariate GARCH model. Multivariate GARCH models are multivariate time-series models in which the conditional covariance matrix of the errors depends on its own past and its past shocks. It allows the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure and allow the conditional mean to follow a vector-autoregressive structure. Extending from univariate GARCH model to multivariate GARCH opens better opportunity to better econometric decision tools in macroeconomic markets. The main challenge in constructing multivariate GARCH models is to make them parsimonious enough, but still maintain the flexibility.

The first model for MGARCH models was Constant Conditional Correlation (CCC) model introduced by [8]. In this model, the conditional correlation is assumed to be constant over time, and only the conditional standard deviation is time varying. The conditional correlation parameters that weight the non-linear combinations of the conditional variance are constant in the CCC-MGARCH model. While the CCC model of [8] and [30] assumes time-invariant, but pairwise-specific, correlations, which can be estimated by a consistent estimator for the unconditional correlation.

In DCC-MGARCH model the conditional variances are modelled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariances are modelled as non-linear functions of the conditional variances. The conditional quasi-correlation parameters that weight the non-linear combinations of the conditional variances follow the GARCH-like process specified in [17] and [28]. In [14] introduced the DCC-MGARCH model, which is an extension of the CCC-MGARCH model, for which the conditional correlation matrix is designed to vary over the time.

Dvech-MGARCH estimates the parameters of diagonal vech (DVECH) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which each element of the conditional correlation matrix is parameterized as a linear function of its own past and past shocks.

In this study we were implemented four MGARCH models namely: CCC, DCC, Dvech and BEKK using volatility of daily USD/ETB and EUR/ETB exchange rate as a case study.

Methodology

Data Source

To achieve the objectives of the study, daily exchange rate data were collected from the National Bank of Ethiopia. In this study, the sample data period runs from March 1, 2016 to February 24, 2020. The main variables considered under this study were USD-ETB and Euro-ETB daily exchange rate data. There are 1,287 time series observations included and exclude Saturday and Sunday data from this study.

Multivariate GARCH Models

The generalization from the univariate volatility model into a multivariate approach opens up a variety of modeling possibilities. This study aims to Comparison of Multivariate GARCH models using Volatility of EUR/ETB and USD/ETB Exchange Rates. Multivariate GARCH models are an extension of univariate GARCH models. It allows the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure and allow the conditional mean to follow a vector autoregressive structure, [20]. MGARCH implements five commonly used parameterizations: the diagonal vector MGARCH (DVECH) model, the constant conditional correlation (CCC) MGARCH model, the dynamic conditional correlation (DCC) MGARCH model, BEKK-MGARCH and the time-varying conditional correlation (VCC) MGARCH model. See for example [5], [10-11] and [34] provide general to MGARCH models. We give a formal definition of the general MGARCH model to establish notation that facilitates comparisons of the models.

Generally, MGARCH models can be classified into four types: (1) Models of conditional variances and correlations: At first the univariate conditional variances and correlations are computed and then used to get the conditional covariance matrix. For example: the Constant Conditional Correlations (CCC) model; the time-varying conditional correlation (VCC) model and the Dynamic Conditional Correlations (DCC) model. (2) Non-parametric and semi-parametric approaches: Models in this class form an alternative to parametric estimation of the conditional covariance structure. The advantages of these models are that they do not impose a particular structure (that can be misspecified) on the data. (3) Models of the conditional covariance matrix: The conditional covariance is computed in a direct way. For example the VECH and BEKK models. (4) Factor models: The return process is assumed to consist of a small number of unobservable Heteroscedasticity factors. This approach benefits from that the dimensionality of the problem reduces when the number of factors compared to the dimension of the return vector is small.

In this paper, we have used models of the conditional covariance matrix (BEKK and DVECH models) and models of conditional variances and correlations (CCC and DCC models). But before describing these models more profoundly, some definitions are required. The general MGARCH model is given as:

$$\left.\begin{array}{l} y_t = \mathbf{C}x_t + u_t \\ u_t = \mathbf{H}_t^{1/2}\nu_t \end{array}\right\} \quad (1)$$

Where y_t is an $m \times 1$ vector of dependent variables in (1); $\mathbf{C}$ is an $m \times k$ matrix of parameters; x_t is a $k \times 1$ vector of independent variables, which may contain lags of y_t; $\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$; and ν_t is an $m \times 1$ vector of zero-mean, unit-variance, u_t is error terms and independent and identically distributed innovations (iid).

Models of Conditional Variances and Correlations

The Constant Conditional Correlation (CCC) model

The CCC- MGARCH model assumes that negative and positive shocks of equal magnitude have identical influences on the conditional variance. [8] Proposed a CCC-MGARCH model in which the correlation matrix is time invariant. Conditional correlation model is used in non-linear combinations of univariate GARCH model to represent the conditional covariances. In each of the conditional correlation models, the conditional covariance matrix is positive definite by construction and has a simple structure, which facilitates parameter estimation. CCC model has a slower parameter growth rate than DVECH model as the number of time series increases. The CCC model was suggested by [8, 35], where the time varying covaiance matrix *H* at a time *t* expressed as.

$$H_t = D_t R_t D_t \quad (2)$$

Where, in equation (2) $D_t = diag(\delta_{1t}, \delta_{2t}, ...\delta_{Nt})$ the right hand side consists of the conditional correlation matrix R that is time invariant, meaning that R_t is a diagonal matrix of $(h_t, ..., h_k)$ in (2) such as:

$$D_t = \begin{bmatrix} \sqrt{h_{1,t}} & 0 & . & . & 0 \\ 0 & . & . & . & 0 \\ . & . & . & . & 0 \\ . & . & . & . & 0 \\ 0 & 0 & . & . & \sqrt{h_{k,t}} \end{bmatrix} \quad (3)$$

Where, each $h_{i,t}$ follows a univariate GARCH process. The conditional correlation matrix is given by $R_t = \lfloor \rho_{i,j} \rfloor$, and the non-diagonal elements of H_t are

$$|H_t|_{i,j} = \sqrt{h_{i,t}}\sqrt{h_{j,t}} * \rho_{i,j} \qquad \forall i \neq j \quad (4)$$

Since the return process $r_{i,t}$ is modeled with a univariate approach, the desired conditional variances can be expressed in vector form.

$$h_t = L + \sum_{i=1}^{q} \varpi_i u_{t-i}^{\ 2} + \sum_{j=1}^{p} \varphi_j h_{t-j}^{\ 2} \quad (5)$$

The first term L is a vector of the intercepts with a size of $n \times 1$ and the matrices of the coefficients are $n \times n$. Furthermore $u_{t-j}^{\ 2} = u_{t-j} * u'_{t-j.}$

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{12} & \Lambda & \rho_{1N} \\ \rho_{12} & 1 & \Lambda & \rho_{2N} \\ M & & O & \\ \rho_{1N} & \rho_{2N} & & 1 \end{bmatrix} \quad (6)$$

The basic idea is that every variance–covariance matrix can be decomposed in the above way. Therefore, we can characterize the dynamics in the following way.

$$\mathbf{H}_t = \begin{bmatrix} \delta^2{}_{1t} & \delta_{12,t} & \Lambda & \delta_{1Nt} \\ \delta_{12,t} & \delta^2{}_{21t} & \Lambda & \delta_{2N,t} \\ M & M & O & \\ \delta_{1N,t} & \delta_{2N,t} & & \delta^2{}_{Nt} \end{bmatrix} \quad (7)$$

$$\delta^2{}_{it} = \partial_i + \sum_{j=1}^{m} \theta_{i,j} u^2{}_{i,t-j} + \sum_{j=1}^{k} \psi_{i,j} \delta^2{}_{i,t-j}, i = 1,2,...,n \quad (8)$$

$$\delta_{ijt} = \rho_{ij}\delta_{it}\delta_{jt}, \ i.j = 1,2,...,n, \ i \neq j \quad (9)$$

The usual conditions to ensure the positivity of the variances and the stationarity hold in (9): $\partial_i > 0, \theta_{i,j} > 0, \psi_{i,j} > 0$ and $\sum_{j=1}^{m} \theta_{i,j} + \sum_{j=1}^{k} \psi_{i,j} < 1$

The advantage of the CCC model is that the computational procedure is more easily performed because the correlation matrix $|H_t|_{i,j}$ is constant. However, this means that the model may be too restrictive.

The Dynamic Conditional Correlation (DCC) model

DCC model is an extension of Constant Conditional Correlation (CCC) model. It is flexible like univariate GARCH model and parsimonious parametric models for the correlations. They are not linear but can often be very simply estimated using univariate or two step likelihood function based methods. DCC model perform well in a variety of situations [17]. The models of [17] and [37] are frankly multivariate and are useful for modelling high dimensional data sets [5]. The Dynamic Conditional Correlation model is in equation (2): where, H_t is the covariance matrix and R_t is an $n \times n$ matrix of the conditional correlation of the returns. The diagonal matrix D_t is expressed as

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & . & 0 \\ 0 & \sqrt{h_{2t}} & . & . \\ . & . & . & . \\ 0 & . & . & \sqrt{h_{nt}} \end{bmatrix} \quad (10)$$

This matrix consists of the univariate GARCH model. Furthermore, H_t has to be positive definitive, which is automatically obtained while R_t in equation (2) is a correlation matrix that is symmetric by definition. When this matrix is defined, two requirements are needed. Firstly, H_t needs to be positive definite since it is a covariance matrix. Secondly, the parts that belong to R_t need to be less than one. These requirements are met throug h a decomposition $R_t = diag(q_{ii_t})^{-1} Q_t diag(q_{ii_t})^{-1}$, for I = 1, 2,…,n w here, $Q_t = (1-\psi-\varphi)\overline{Q} + \psi u'_{t-1} + \varphi Q_{t-1}$, $\overline{Q} = Cov(u_t u'_t) = E(u_t u'_t)$. Additionally, the parameters ψ and φ are scalars and $diag(Q_t)$ is used to rescale the parts of Q_t in order to fulfill that $|\rho_{ij}| = \left|\frac{q_{int}}{q_{iit} q_{ijt}}\right| \leq 1$ where q_{int} is the content of the matrix $diag(q_{iit})$.The estimate of $\overline{Q}$ is $\frac{1}{T}\sum_{t=1}^{T} u_t u_t'$ Moreover, the scalars ψ and φ must be larger than zero, but the sum has to be less than one. One may note that these are conditions of the univariate GARCH to be stationary, but which is applied in the DCC model [14-15] and [18].

Models of the Conditional Covariance Matrix

The VECH model

The VEC model is a generalization of a univariate GARCH model, and developed by [10]. MGARCH models are dynamic multivariate regression models in which the conditional variances and covariances of the errors follow an autoregressive-moving-average structure [24] and [33]. The DVECH-MGARCH model parameterizes each element of the current conditional covariance matrix as a linear function of its own past and past shocks. In a DVECH-MGARCH model with one ARCH term and one GARCH term, the $(i, j)^{th}$ element of conditional covariance matrix is modelled by

$$h_{ij,t} = \eta_{ij} + \varpi_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \psi_{ij}h_{ij,t-1} \quad (11)$$

Where η_{ij}, ϖ_{ij} and ψ_{ij} are parameters and ε_{t-1} is the vector of errors from the previous period. This expression shows the linear form in which each element of the current conditional covariance matrix is a function of its own past and past shocks.
The general VEC model is defined as

$$vech(H_t) = L + \sum_{i=1}^{p}\varpi_i.vech(u_{t-i}u_{t-i}') + \sum_{j=1}^{q}\varphi_j.vech(H_{t-j}) \quad (12)$$

And the case when $p = q = 1$ is defined as

$$vech(H_t) = L + \varpi.vech(u_{t-1}u_{t-1}') + \varphi.vech(H_{t-1}) \quad (13)$$

Where, the left term is the lower diagonal matrix that is transformed into a $N(N+1)/2\times1$ vector.

The DVECH model

The diagonal VECH is a simplified VEC, where ϖ_i and φ_j are diagonal matrices in objective to obtain a positive H_t. This new version was also introduced by [10].
MGARCH dvech estimates the parameters of diagonal vector GARCH (DVECH) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which each element of the conditional correlation matrix is parameterized as a linear function of its own past and past shocks.
The Dvech (p, q) in MGARCH model is defined as

$$y_t = Cx_t + u_t \quad (14)$$

$$u_t = \sqrt{\mathbf{H}_t}\upsilon \quad (15)$$

$$\mathbf{H}_t = \mathbf{L} + \sum_{i=1}^{p}\varpi_i o\left(\varepsilon_{t-i}\varepsilon_{t-i}'\right) + \sum_{j=1}^{q}\varphi_j oH_{t-j} \quad (16)$$

Where, ϖ_i and φ_j are diagonal matrices and o is the Hadamard product of two matrices.
y_t is an $m\times1$ vector of dependent variables (13); L is an $m\times k$ matrix of parameters; x_t is a $k\times1$ vector of independent variables, which may contain lags of y_t; $\sqrt{H_t}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$; v_t is an $m\times1$ vector of normal, independent, and identically distributed innovations;
L, ϖ *and* φ *are* $N\times N$ symmetric parameter *matrices*
The $DVEC(1,1)$ is

$$\mathbf{H}_t = \mathbf{L} + \varpi_i o\left(\varepsilon_{t-i}\varepsilon_{t-i}'\right) + \varphi_j oH_{t-1} \quad (17)$$

The model above can be decomposed into univariate GARCH models of variances and covariance's.

The BEKK model

Where BEKK is an abbreviation for the creators of the model: - Baba, Engle, Kraft and Kroner [4]. As the matrix expects to attend real values, for the same reasons as when assuming a real number to be positive when taking the square root, an assumption has to be done, namely that the matrix H_t needs to be positive definite so it is possible to take the power of one half. The BEKK model is a further development of the DVEC model. The parameters of this model can be configured in different ways, allowing the BEKK model to have a different degree of restrictions. A diagonal BEKK has diagonal matrices as parameters and the full BEKK uses $n\times n$ parameter matrices. The general full and diagonal BEKK model is

$$\mathbf{H}_t = \mathbf{L}*\mathbf{L}' + \sum_{i=1}^{p}\sum_{k=1}^{k}\varpi_{ik}\left(u_{t-i}u_{t-i}'\right)*\varpi_{ik}' + \sum_{j=1}^{q}\sum_{k=1}^{k}\varphi_{jk}H_{t-j}*\varphi_{jk}' \quad (18)$$

Where ϖ_{ik} and φ_{jk} are parameter matrices and L is a lower triangular matrices in equation (18). According to [33] when $p = q = 1$ the model becomes a $BEKK(1,1)$

$$\mathbf{H}_t = L*L' + \varpi*\left(u_{t-i}u_{t-i}'\right)*\varpi' + \varphi*H_{t-j}*\varphi' \quad (19)$$

The advantage of the BEKK model is that H_t by definition is positive. The matrices of parameters are multiplied with an arbitrary symmetrical matrix and the transpose of the parameter matrix. For example: LIL' where I, is the identity matrix in equation (25). This certifies that each term in the model becomes positive semi-definite. One condition has to be fulfilled in order ensure covariance stationarity, that the absolute Eigen values of the expression, $\sum_{i=1}^{p}\sum_{j=1}^{k}\varpi_{ij}\otimes\varpi_{ij}\sum_{j=1}^{q}\sum_{k=1}^{k}\varphi_{jk}\otimes\varphi_{jk}$: have to be less than one [33].

Diagnostics of MGARCH models

Graphical diagnostics for MGARCH models can be fulfilled by examining plots of the sample autocorrelation (ACF) and the sample cross correlation functions (XCF). To ensure the inference from the estimated parameters in the MGARCH model is enough valid, the residuals should be exhibited as a set of white noise with features like expected zero mean vector, no autocorrelations, constant variance, and normal distribution of the residuals. The autocorrelation and cross correlation functions for the squared process are shown to be useful in identifying and checking time series behavior in the conditional variance equation of the GARCH form.
In the literature, several tests have been developed to test the autocorrelation no matter in univariate or multivariate form. Box and Pierce derived a goodness-of-fit test, called the portmanteau test. It may be the most popular one among all the diagnostics for conditional heteroscedasticity models. The test statistic may be expressed as a function of the covariances between the residuals of the fitted model [8, 17].
A multivariate version is given by

$$HM(k) = T^2\sum_{j=1}^{k}(T-j)^{-1}tr\{W_Y^{-1}(0)W_Y(j)W_Y^{-1}(0)W'_Y(j)\} \quad (20)$$

Where T is the number of observations, $W_Y(j)$ is the sample auto-covariance matrix of order j and, $Y_t = vech(y_t y_t')$. The distribution of $HM(k)$ is the asymptotical $\chi^2(k^2p)$ under the null hypothesis that there is no MGARCH effects.

Lagrange Multiplier Tests

Lagrange multiplier tests usually have a higher power than portmanteau tests when the alternative is correct (although they can be asymptotically equivalent in certain cases), but they may have low power against other alternatives. [18, 15], among others, have developed LM tests for MGARCH models [14]. Have developed a simple preliminary test for ARCH effects in common factor models. To reduce the number of parameters in the estimation of MGARCH models, it is usual to introduce restrictions. For instance, the CCC model of [8, 36] assumes that the conditional correlation matrix is constant over time.

Distributional Assumptions

The unconditional distribution of many financial time series seems to have fatter tails than allowed by the Gaussian family. Some of this can be explained by the presence of ARCH; that is, even if error term has a Gaussian distribution, the unconditional distribution of u_t is non-Gaussian with heavier tails than the Gaussian distribution. Even so, there is a fair amount of evidence that the conditional distribution of u_t is often non Gaussian as well. The same basic approach can be used with non-Gaussian distributions. See for example, [7], proposed that u_t might be drawn from a t- distribution with v degree of freedom, where v is regarded as a parameter to be estimated by maximum likelihood method. And also there exist many candidates for the multivariate skew Student's *t*-distribution [3]. If u_t has a t-distribution with v degree of freedom and a scale parameter s_t, then its probability density function is given by for Multivariate student t-distribution

$$f(u_t/v)=\prod_{t=1}^{T}\frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})[\prod v-2]^{n/2}|\psi_t|^{1/2}}\left[1+\frac{u_t^T u_t}{v-2}\right]^{-(\frac{v+1}{2})} \quad (21)$$

Where $\Gamma(.)$ is the Gamma function. This density can be used in place of Gaussian specification along with the same specification of the conditional mean and conditional variance [14]. If v >2, then u_t has mean zero and variance constant:

The log-likelihood function written as

$$\ln(L(\theta))=\sum_{t=1}^{T}\left(\begin{array}{l}\ln\left[\Gamma\left(\frac{v+n}{2}\right)\right]-\ln\left[\Gamma\left(\frac{v}{n}\right)\right]-n/2\ln[\Pi(v-2)]\\ -1/2\ln[|D_tR_tD_t|]-\frac{v+n}{2}\left[1+\frac{\alpha_t^T D_t^{-1}R_t^{-1}D_t^{-1}\alpha_t}{}\right]\end{array}\right) \quad (22)$$

as for Gaussian standardized error is divided into two groups: $(\psi,\eta)=(\psi_1,\psi_2,...\psi_n,\eta)$, where $\psi_i=\alpha_{0i},\alpha_{1i},....,\alpha_{q_i},\beta_{1i},\beta_{2i},...,\beta_{pi}$ are the parameters of the univariate GARCH model for the i^{th} asset series, i=1,…and $\eta=(\alpha,\beta,v)$.

Unit Root Test of the Variables

There are many tests for determining whether a time series is stationary or non-stationary. Those are the Phillips-perron test and Augmented Dickey- fuller (ADF) test.

Augmented Dickey-Fuller test

To identify autocorrelation problems, [13] have developed a test called Augmented Dickey Fuller test. This test constructs a parametric correction for higher order correction by assuming that the series y_t follows an autoregressive or AR (P) process and adding P lagged difference terms of the dependent variable to the right hand side of the test regression. The ADF test equation is specified as:

$$\Delta y_t=\alpha+\beta_t+\gamma y_{t-1}+\delta_1\Delta y_{t-1}+...+\delta_{t-1}\Delta y_{t-p+1}+\varepsilon_t \quad (23)$$

Where ε_t is the error term, α the intercept and, β_t time trend. The test statistic is given below:

$$DF_t=\frac{\hat{\gamma}}{SE(\hat{\gamma})} \quad (24)$$

Where in equation (2) $SE(\hat{\gamma})$ standard error of $\hat{\gamma}$. The test is carried out by testing the joint effect hypothesis: $Ho:\beta=\hat{\gamma}=0$ where $\hat{\gamma}=\gamma-1$ by using the correctional F-test but comparing the test statistic with the interval F- values developed by Dickey and Fuller.

Phillips-Perron Test

Phillips in [29] developed a number of unit root tests that have become popular in the analysis of financial time series. The Pillips-Perron (PP) unit root tests differ from ADF test mainly in how they deal with serial correlation and Heteroscedasticity in the errors. The test regression for the PP test is:

$$\Delta y_t=\alpha_o+\vartheta_t+\zeta y_{t-1}+\varepsilon_t \quad (25)$$

Where, in equation (3) α_o is intercept, ϑ is the time trend and u_t is the disturbance term. The tests correct for any serial correlation and Heteroscedasticity in the error term u_t of the test regression by directly modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$. This modified statistics, denoted Z_t and Z_π, are given by:

$$Z_t=\left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{1/2}t_{\pi=0}-\frac{1}{2}\left(\frac{\hat{\lambda}^2-\hat{\sigma}^2}{\hat{\lambda}^2}\right)\left(\frac{T.SE(\hat{\pi})}{\hat{\sigma}^2}\right) \quad (26)$$

$$Z_\pi=T_{\hat{\pi}}-\frac{1}{2}\left(\frac{T^2.SE(\hat{\pi})}{\hat{\sigma}^2}\right)(\hat{\lambda}^2-\hat{\sigma}^2) \quad (27)$$

Where, the terms $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are consistent estimates of the variance parameters.

$$\sigma^2=\lim_{T\to\infty}T^{-3}\sum_{t=1}^{T}E(u_t^2)$$

And

$$\lambda^2=\lim_{\lambda\to\infty}\sum_{t=1}^{T}E\left(T^{-1}*S^2{}_t\right)$$

Where $S_T=\sum_{t=1}^{T}u_t$. The sample variance of the least squares residual $\hat{u}_t$ is a consistent estimate of σ^2, and the Newey-West long run variance estimate of u_t using $\hat{u}_t$ is consistent estimate of λ^2. Under the null hypothesis that $\pi=0$ the PP Z_t and Z_π statistics is correctional F-test but comparing the test statistic with the interval F- values developed by Dickey and Fuller.

Empirical Results

Unit Root Test

In econometric practice in the analysis of financial time series data begins with an examination of unit roots test. The Augmented Dickey-Fuller and Phillips-Perron tests are used to test for EUR/ETB and USD/ETB returns under the null hypothesis of a unit root against the alternative hypothesis of stationarity.

The results of unit root tests are presented in (Table-1), it revealed that large negative values in a both cases for levels, such that the individual returns series rejected the null hypothesis at the 1% significance level, so that all returns series are stationary at logarithm of first difference.

Table 1: ADF and PP unit root test logarithm of first differenced daily EUR/ETB and USD/ETB Exchange rate

Euro/ETB	**Test statistics**	**Critical values**			**p- values**
	Z(t)	**1%**	**5%**	**10%**	
ADF	-49.295	-2.329	-1.646	-1.282	0.000
Pillips-Perron	-49.295	-3.430	-2.860	-2.570	0.000
USD/ETB					
ADF	-45.419	-2.329	-1.646	-1.282	0.000
Pillips-Perron	-45.419	-3.430	-2.860	-2.570	0.000

Source: author’s computation, 2020

Descriptive Statistics

Table 2 presents the descriptive statistics for the returns series of EUR/ETB and USD/ETB. These EUR/ETB returns series have high Kurtosis, which indicates the presence of fat tails.

The negative skewness statistics signify the series has a longer left tail than right tail. The Jarque-Bera Lagrange multiplier statistics of EUR/ETB and USD/ETB returns are statistically significant, thereby implying that the distribution of these prices is not normal. These suggest that it may be worth exploring some non-normal distributions such as the student-t distribution.

Table 2: Descriptive Statistics

	N	**Mean**	**SD**	**Variance**	**Skewness**	**Kurtosis**	**Jarque Bera**	**p-value**
EUR/ETB	1287	26.844547	4.2487874	18.052	-0.791	6.213	1365.55	0.000
USD/ETB	1287	24.645289	3.2575081	10.611	-0.770	5.676	976.64	0.000
Correlation	0.93							

Note: The Jarque-Bera test is a goodness–of–fit measure of departure from normality, based on the sample kurtosis and skewness. Under the null hypothesis of normality, the statistic JB has an asymptotic chi-square distribution with two degrees of freedom. Source: author’s computation, 2020

The MGARCH model results

We have estimated four multivariate GARCH models (CCC, DCC, BEKK and Dvech) for each error distribution, currency exchange rate from (table 3-6) respectively. Table-3 shows that, the estimates for the CCC-MGARCH model. The ARCH and GARCH estimates of the conditional variance are statistically significant. The ARCH estimates are generally close one and the GARCH effects are generally close to one. The mean or variance parameters used to model each dependent variable. Subsequently, the output table represents the results for the conditional correlation parameters. The conditional correlation between the standardized residuals for EUR/ETB and USD/ETB is estimated to be 0.6226. It indicates that these returns tend to move in the same directions; in other words, an increase in the return for the EUR/ETB tends to be associated with an increase in the return for the USD/ETB, and vice versa. It indicates that we may not need all the vector autoregressive parameters, but that each of the univariate ARCH, univariate GARCH, and conditional correlation parameters are statistically significant. The estimated conditional correlation parameters are statistically significant. It indicates that the returns in this case rise time to time.

The DCC- MGARCH model nests the CCC-MGARCH model. We have tested the time-invariance assumption with Wald tests on the parameters of these models we reject the null hypothesis that these conditional correlations are time invariant. To confirm that the DCC-MGARCH model is preferred over the CCC-MGARCH model, a test of dynamic correlation is made in order to see which one of the models that is most suitable for the data. A *p*-value of less than 0.05 is received, pointing out the absence of constant correlation and the data is thus suitable for using a DCC-MGARCH model compared to the CCC-MGARCH model where the correlation is assumed to be constant. Consequently, an estimation of the DCC-MGARCH model will be done. λ_{12} and λ_{21} in (table-4) of the estimated DCC-MGARCH model indicates that the model adjustment. The magnitudes of the lambda parameters indicate that the evolution of the conditional covariances depends more on their past values than on lagged residuals’ innovations. As can be seen all the coefficients are significantly not equal to zero, which implies that they all have a significant influence. A noticeable fact is that the degree of significance seems to follow a particular pattern, where the *t*-values of the intercepts and the ARCH parameter are relatively lower compared to the GARCH parameters which are highly significant. As stated it the theoretical description of the DCC (1, 2) model the scalars λ_{12} and λ_{21} have to individually larger than zero but at the same time the sum of them has to be strictly less than one. The results in Table 4, show that $\lambda_{12} + \lambda_{21} = 0.0003 + 0.9904 = 0.9907$ and in addition, the parameters of the inherent univariate processes are $a11 + b11$ = 0.4994+0.2428= 0.7422 and $a22 + b22$ = -0.4591+0 = -04591. To interpret this, a high *b* means, that the conditional variance is persistent. A high *a* means, that the volatility is spiky. Since our values of *b* are high, the conditional variances seem to be more persistent. As can be seen $a11 > a22$, indicating a more spiky time series graph in appendix, in (Figure-3), this is also true. In (table-5) BEKK (1, 1) model the results indicated the transmission of volatility among the series. In this model lies in the fact that all the coefficients were statistically significant at 1% or 5% level of significance. According to BEKK estimation results, the general impression is that the correlation seems to be trending upwards during this five years period. A few times the correlations have been negative, especially in the beginning of the estimated period. Furthermore, another impression is that the graph shows tendencies of correlational drops during periods of economic crises in figure 7. The model estimation employed here is the Gaussian quasi MLE and log-likelihood method. One of BEKK model assumptions is that the residuals have a Gaussian distribution. Hence, to test whether the estimations of the model parameters are robust, we can check whether the residuals of the estimated process are white noise (see figure-6 in Appendix).

In table 6 Dvech (1, 1) the elements of η are reported in the $\eta's$ equation. The estimate of η[1; 1] is 0.2208104, and the estimate of η[2; 1] is 0.04599. The ARCH term results are reported in ϖ equation. In the ARCH equation, ϖ(1, 1) is the coefficient on the ARCH term for the conditional variance of the first dependent variable, ϖ(2, 1) is the coefficient on the ARCH term for the conditional covariance between the first and second dependent variables, and ϖ (2, 2) is the coefficient on the ARCH term for the conditional variance of the second dependent variable. The GARCH term results are reported in ψ equation.

Table 3: Estimated result for CCC-MGARCH

Parameter	Coefficients	Std. Error	*t*-value	p-value
$c11$	0.0223	0.0061	3.62	0.000
$a11$	0.5005	0.0793	6.31	0.000
$a12$	-0.3732	0.0720	-5.18	0.000
$b11$	0.8864	0.0133	66.27	0.000
C21	0.0022	0.0004	5.46	0.000
C22	0.0181	0.0103	1.77	0.076
$a21$	0.5556	0.0892	6.23	0.000
$a22$	-0.4593	0.0907	-5.06	0.000
$b22$	0.9106	0.0145	62.91	0.000
C31	0.0050	0.0011	4.60	0.000
CCC	0.6226	0.1583	3.93	0.000

Source: author computation, 2020

Table 4: Estimated results for DCC model with student-t distribution

Parameter	Coefficients	Std. Error	*t*-value	p-value
$c11$	0.0223	0.0062	3.61	0.0000
$a11$	0.4994	0.0793	6.30	0.0000
$a12$	-0.3724	0.0719	-5.18	0.0000
$b11$	0.2428	0.0001	52.0495	0.0000
C12	0.0022	0.0004	5.46	0.0000
C21	0.0181	0.0103	1.77	0.0090
$a21$	0.5553	0.0892	6.23	0.0000
$a22$	-0.4591	0.0907	-5.06	0.0000
$b12$	0.9107	0.0145	62.96	0.0000
C22	0.0050	0.0011	4.60	0.0000
λ_{12}	0.0003	0.0022	5.13	0.0000
λ_{21}	0.9904	0.0138	71.69	0.0000
Wald chi2(2)= 49495.34				
P-value = 0.0000				

Source: author's computation, 2020

Table 5: Estimated results for Diagonal BEKK Model

Parameter	Coefficients	Std. Error	*t*-value	p-value
$c11$	0.500	0.0803	6.30	0.0000
$c21$	0.6053	0.0902	6.20	0.0000
$c22$	0.6755	0.1259	2.6829	0.0000
$a11$	0.9852	0.017142	57.4800	0.0000
$a12$	-0.000	0.0006	-0.0010	0.4993
$a21$	0.000	0.0024	0.0121	0.4904
$a22$	0.1126	0.0825	6.8349	0.0000
$b11$	0.2428	0.0001	52.0495	0.0000
$b12$	0.0898	0.3885	0.3885	0.8172
$b21$	0.000	0,0085	-0.1527	0.3800
$b22$	0.4856	0.0021	119.7158	0.0000

Source: author's computation, 2020

Table 6: Estimated result for Dvech (1, 1) model

	Coef.	Std. Err.	z	P>|z|
C1	-0.3112	0.1731	-1.80	0.002
C2	-0.0826	0.0492	-1.68	0.003
η_{11}	0.2208	0.0465	4.74	0.000
η_{21}	0.4599	0.1140	4.04	0.000
η_{22}	0.00497	0.0021	2.40	0.001
ϖ_{11}	4.3063	1.0159	4.24	0.000
ϖ_{21}	0.1115	0.0491	2.27	0.023
ϖ_{22}	0.1819	0.0335	5.42	0.000
ψ_{11}	0.6401	0.0445	14.37	0.000
ψ_{21}	-0.4787	0.2324	-2.06	0.009
ψ_{22}	0.5145	0.0238	21.63	0.000
Model adequacy				
Distribution: Gaussian				
Log likelihood = 9273.104				
Wald chi2(2) = 272.47				
Prob > chi2 = 0.0000				

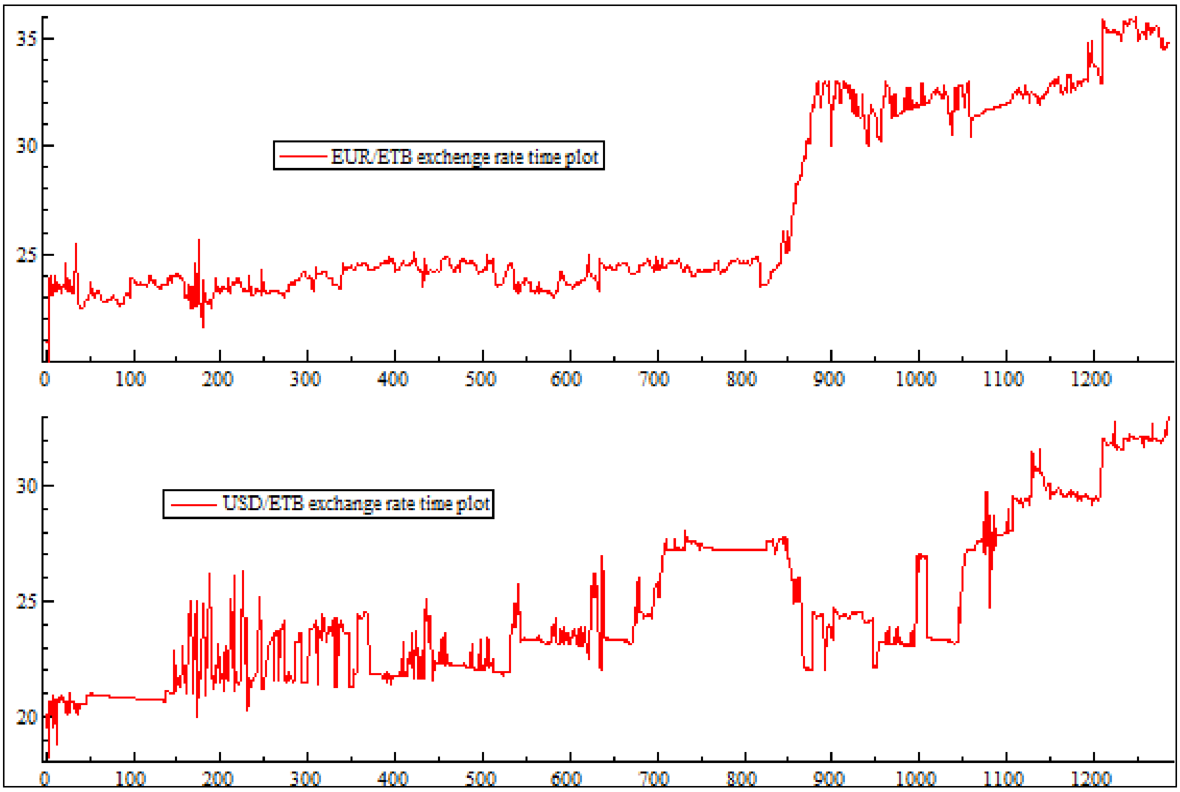

Fig 1: The daily EUR/ETB and USD/ETB exchange rate at level

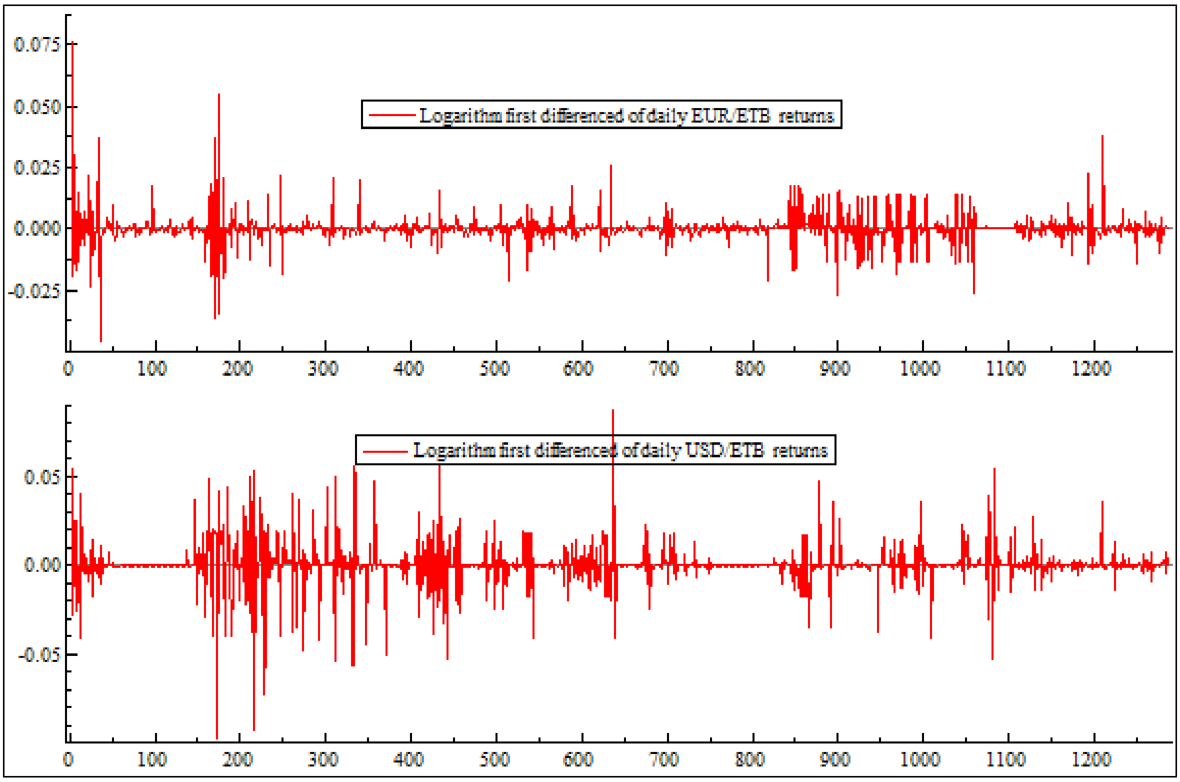

Fig 2: Volatility of daily EUR/ETB and USD/ETB returns

Conclusion

This paper estimated three multivariate volatility models, namely CCC, DCC and BEKK, for the volatility of daily EUR/ETB and USD/ETB exchange rate in Ethiopia. The empirical results for daily USD-ETB and Euro-ETB exchange rates considered in this study. There are 1,287 time series observations included and exclude Saturday and Sunday data from this study. The data was collected from national bank. The visual inspection of these series (Figure 1) clearly indicated the presence of volatility at time series plot. In addition, the skewness and kurtosis coefficients suggested the asymmetry and fat-tailed distribution of the series. This motivated us to use the MGARCH model with multivariate Student-t distribution instead of the usual multivariate normal distribution. The MGARCH – Lagrange multiplier (LM) test was carried out on the square of the residuals obtained the test whether the residuals exhibit heteroscedasticity. The volatility clustering phenomenon is observed in the exchange rate series and justifies the implementation of MGARCH models. We also explore the Stationarity of the series by implementing the augmented Dickey– Fuller (ADF) and Phillips-Perron **(**PP) test and we find that the series are stationary after logarithm of first differencing (figure 2). The EUR/ETB seems in figure-2, compared to the USD/ETB, to be more volatile in the beginning of the sample period, especially for the year of 2017. Besides that, the last period is highly volatile for EUR/ETB series. The series is satisfies the assumption of normality (see figure-12).

The estimated conditional correlations are presented in Figure 1, whereas Table-7 in appendix-a shows the sample correlation matrix of the estimated time-varying correlations. The correlations from the diagonal DCC model and the CCC–GARCH model are very strongly positively correlated (0.9201), which is also obvious from Figure 1. The second-highest correlation of correlations is the one between the BEKK–GARCH and the Dvech–GARCH model which is negatively correlated.

The BEKK (1, 1) model the results indicated the transmission of volatility among the series. In this model lies in the fact that all the coefficients were statistically significant at 1% or 5% level of significance. The estimates of the CCC (1, 2) model were found to be significant and the series transmitted volatility among them with a correlation of 0.6226 (Table 3). After achieving a positive insight of the conditional correlation, we tested for the presence of dynamic conditional correlation using [36] Lagrange Multiplier (LM) test. The χ^2 statistic (3975.4) was rejected at 1% level of significance, confirming the presence of dynamic properties of the series. Further, we modelled the series using DCC (1, 2) model with multivariate normal distribution. According to the model selection criteria DCC-MGARCH model is more appropriate to fit this study. The results of the DCC model were encouraging as all the parameters were found to be significant at 1% level of significance (Table 4). Further, to incorporate the leptokurtic effect DCC model with multivariate Student-t distribution was fitted to the series. The results obtained here were also encouraging and in line with that of DCC model with multivariate normal distribution. Along with it the shape parameter (λ) of the Student-t distribution was also found to be significant at 1% level of significance (Table-4). The high achievement of DCC-MARCH model with Student-t distribution to model leptokurtic exchange return goes with the findings in literature [22]. Hence, to test whether the estimations of the model parameters are robust, we can check whether the residuals of the estimated process are white noise see figures (5, 6 and 10) in Appendix.

Recommendation

Any Academia and researchers seeking to model the volatility of ETB against other foreign currencies; this study should prove useful in that it highlights the importance of using high frequency data, and basing one's analysis on a range of comparison of MGARCH models techniques, and for employing alternative proxy variables that capture realized volatility. This study was focused only on selected MGARCH models. Namely: CCC, DCC, BEKK and Dvech models. Therefore, future researchers should be to build on the current work and extend it a bit further. For example, it would be interesting to consider fitting GO-MGARCH-ML and DCC-DECO-MGARCH etc.--models of the major currencies the country uses in its international transactions.

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Availability of data and materials

The data that support the findings of this study can be obtained from the authors based on request
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Competing interests

The authors declare that they have no competing interests.

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