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Some suzuki type fixed point results in partial metric spaces

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Abstract
In this paper, we prove a Suzuki type fixed point results for multivalued mapping using a partial Hausdorff metric in partial metric spaces. These results generalize and extend the several comparable results in the literature of metric and partial metric spaces.

Keywords: partial metric space, fixed point, multivalued mappings, partial hausdorff metric

Introduction

The Banach contraction principle plays a very important role in nonlinear analysis. Nadler [10] assured the multivalued version of Banach contraction principle by using Hausdorff metric. In 2008, Suzuki [15] introduced a new type of mapping which generalized Banach contraction principle. This result attracted several authors to work along these lines and it generalized to multivalued mappings by Kikkawa and Suzuki [7], Mot and Petrused [3], Dhompongsa and Yingtaweesittikul [13] and Singh and Mishra [14]. Also Singh *et al.* [12] presented a common fixed point theorem for a pair of multivalued maps in a complete metric space extending a recent theorem of Doric and Lazovic [2]. These are all Suzuki type theorems.

In 1994, Mathews [11] introduced the concept of a partial metric spaces as a part or the study of denotational semantics of dataflow networks. Aydi *et al.* [4] in 2012, introduced the definition of a partial Hausdorff metric and also proved the existence of the Banach contraction principle for multivalued mapping in complete partial metric spaces.

Ahmad *et al.* [6] generalized various known results proved by Kikkawa and Suzuki [7], Mot, R. K and Petrused [3], Dhompongsa and Yingtaweesittikul [13] in the case of partial metric spaces. In 2014. Bose [9] obtained some Suzuki type common fixed point theorems for multivalued mappings using a suitable continuous function. By motivated the above considerations, we investigate the possibility to extend the results in [8, 9] to the setting of partial Hausdorff metric spaces. Also, our theorem and corollaries generalize the well known results in the literature.

Let X and Y be non empty sets. T is said to be a multivalued mapping from X to Y if T is a function from X to the power set of Y. We denote a multivalued mapping by $X \to 2y$.

A point $x \in X$ is said to be a fixed point of multivalued mapping T if $x \in Tx$. We denote the set of fixed points of T by Fix (T).

Let (X, d) be a metric space and CB(X) denote the collection of non empty closed bounded subsets of X. For $A, B \in CB(X)$ and $x \in X$, define

$$d(x, A) = \inf_{a \in A} d(x, a)$$

And

$$H(A, B) = \max\left\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\right\}$$

Note that H is called the Hausdorff metric induced by the metric d.

Definition 1.1 [11]

Let X be a nonempty set. A function $p: X \times X \to R+$ is said to be a partial metric on X if for any $x, y, z \in X$, the following conditions hold:

(p1) $p(x, x) = p(y, y) = p(x, y)$ if and only if $x = y$;

(p2) $p(x, x) \leq p(x, y)$;

(p3) $p(x, y) = p(y, x)$;

(p4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

The pair (X, p) is then called a partial metric space.

If $p(x, y) = 0$, then (p1) and (p2) imply that $x = y$. But the converse does not hold always. A trivial example of a partial metric space is the pair (R+, p), where $p: R+ \times R+ \to R+$ is defined as $p(x, y) = \max\{x, y\}$.

Example 1.2 [11]

If $X = \{[a, b]: a, b \in R, a \leq b\}$,
Then
$p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$ defines a partial metric p on X.

Each partial metric p on X generates a T0 topology τp on X which has a base the family open p- balls $\{B_p(x, \varepsilon): x \in X, \varepsilon > 0\}$,
where
$B_p(x, \varepsilon) = \{y \in X: p(x, y) < p(x, x) + \varepsilon\}$, for all $x \in X$ and $\varepsilon > 0$.

Observe that a sequence {xn} in a partial metric space (X, p) converges to a point $x \in X$, with respect to τ_p, if and only if $p(x, x) = \lim_{n \to +\infty} p(x, x_n)$

If p is a partial metric on X, then the function $ps: X \times X \to R^+$ given by

$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y)$, defines a metric on X.

Definition 1.3 [11]
Let (X, p) be a partial metric space.
a. A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if $\lim_{n, m \to +\infty} p(x_n, x_m)$ exists and is finite.
b. (X, p) is said to be complete if every Cauchy sequence {xn} in X converges with respect to τ_p to a point $x \in X$ such that $\lim_{n \to +\infty} p(x_n, x) = p(x, x)$. In this case, we say that the partial metric p is complete.

Lemma 1.4 [5, 11]
Let (X, p) be a partial metric space. Then:
a. A sequence $\{x_n\}$ in X is said to be a Cauchy sequence (X, p) if and only if it is a Cauchy sequence in metric space (X, ps).
b. A partial metric space (X, p) is complete if and only if the metric space (X, p^s) is complete. Moreover,

$$\lim_{n \to +\infty} p^s(x_n, x) = 0 \text{ iff } \lim_{n, m \to +\infty} p(x_n, x_m) = \lim_{n \to +\infty} p(x_n, x) = p(x, x)$$

In 2012, Aydi *et al*. [4] defined a partial Hausdorff metric as follows:

Let (X, p) be a partial metric space. Let $CB^p(X)$ be the family of all nonempty, closed and bounded subsets of the partial metric space (X, p), induced by the partial metric p. Note that closedness is take from (X, τ_p), τ_p is the topology induced by p and boundedness is given as follows:
A is a bounded subset in (X, p) if there exist $x_0 \in X$ and $M \geq 0$ such that for all $a \in A$, we have $a \in B_p(x_o, M)$, that is, $p(x_0, a) < p(a, a) + M$. For $A, B \in CB_p(X)$ and $x \in X$, define

$$p(x, A) = \inf\{p(x, a), a \in A\},$$

$$\delta_p(A, B) = \sup\{ p(a, B): a \in A\}$$

and

$$\delta_p(B, A) = \sup\{ p(b, A): b \in B\}.$$

The mapping $H_p: CB^p(X) \times CB^p(X) \to [0, +\infty)$ define by $H_p(A, B) = \max\{\delta_p(A, B), \delta_p(B, A)\}$ is called a partial Hausdorff metric induced by p.
It is immediate to check that $p(x, A) = 0 \Rightarrow p^s(x, A) = 0,$ where $p^s(x, A) = \inf\{p^s(x, a), a \in A\}$.

Lemma 1.5 [5]
Let (X, p) be a partial metric space and A any nonempty set in (X, p), then $a \in \overline{A}$ if and only if $p(a, A) = p(a, a)$, where $\overline{A}$ denotes the closure of A with respect to the partial metric p. Note that A is closed in (X, p) if and only if $A = \overline{A}$.
Now, we shall study some properties of mapping

$$\delta_p : CB^p(X) \times CB^p(X) \to [0, +\infty).$$

Proposition 1.6 [4]
Let (X, p) be a partial metric space. Forany A, B, C $\in CB_p(X)$, we have the following:

(i) $\delta_p(A, A) = \sup\{ p(a, a): a \in A\}$;

(ii) $\delta_p(A, A) \leq \delta_p(A, B)$;

(iii) $\delta_p(A, B) = 0$ implies that $A \subseteq B$;

(iv) $\delta_p(A, B) \leq \delta_p(A, C) + \delta_p(C, B) - \inf_{c \in C} p(c, c)$.

Proposition 1.7 [4]
Let (X, p) be a partial metric space. For any A, B$\in$CBP (X), we have the following:

(i) $H_p(A, A) \leq H_p(A, B)$;

(ii) $H_p(A, B) = H_p(B, A)$;

(iii) $H_p(A, B) \leq H_p(A, C) + H_p(C, B) - \inf_{c \in C} p(c, c)$.

Corollary 1.8 [4]
Let (X, p) be a partial metric space. For any A, B $\to$CBP(X), the following holds

$H_p(A, B) = 0$ implies that $A = B$.

Remark 1.9 [4]
The converse of Corollary 1.8 is not true in general as it is clear from the following example.

Example 1.10 [4]
Let X = [0,1] be endowed with the partial metric p: X × X $\to$R+ defined by p(x, y) = max{x, y}. From (i) of proposition 1.6, we have

$$H_p(X, X) = \delta_p(X, X) = \sup\{x : 0 \leq x \leq 1\} = 1 \neq 0.$$

Remark 1.11 [4]
It is easy to show that any Hausdorff metric is a partial metric, but the converse is not true (see example 1.10).

Definition 1.12 [1]
An element $x \in X$ is common fixed point of $T, S : X \to CB^p(X)$ and $f : X \to X$ if $x = fx \in Tx \cap Sx$.

Definition 1.13 [9]
Let $\Phi: [0,\infty)^5 \to [0,\infty)$ be continuous (or semi continuous) and increasing in each coordinate variable and $\Phi(t, t, t, at, bt) \leq t$, for every $t \in [0, \infty)$, where a + b = 2 and a, b $\in$ {0, 1, 2}.

Main Results
Lemma 2.1 [4]
Let (X, p) be a partial metric space, A, B $\in$ CBp(X) and h > 1, then, for any $a \in A$ there exist $b \in B$ such that $p(a, b) \leq h H_p(A, B)$.

Theorem 2.2
Let (X,p) be a complete partial metric space and let M, N: X $\to CB^p(X)$ be two multivalued mappings and φ: [0, 1) $\to$ (0, 1] be a non increasing function defined by

$$\varphi(r) = \begin{cases} 1 & if\ 0 \le r < \frac{1}{2}, \\ 1-r & if\ \frac{1}{2} \le r < 1. \end{cases}$$

If there exist $r\in [0, 1)$ such that for every $x, y\in X$, $\varphi(r) \min\{p(x, Mx), p(y, Ny)\} \le p(x, y)$,
implies
$HP(Mx, Ny) \le r \in \{p(x, y), p(x, Mx), p(y, Ny), p(x, Ny) - p(x, x), p(y, Mx) - p(y, y)\}$. (1)

Then there exists a $u\in X$ such that $u\in Mu \cap Nu$. (Here Φ is as specified in Definition 1.13).

Proof
Let $u_0\in X$ be arbitrarily chosen. For all $u_1\in Nu0$, we have by Lemma2.1 there exist $u_2\in Mu_1$ such that

$p(u_2, u_1) \le h\ HP(Mu_1, Nu_0)$.

Similarly, there exist $u_3\in Nu_2$ such that $p(u_3, u_2) \le h\ HP(Nu2, Mu1)$.

Continuing in this manner, we find a sequence $\{u_n\}$ in X such that $u_{2n+1}\in Nu_{2n}$ and $u_{2n+2}\in Mu_{2n+1}$ And $p(u_{2n+1}, u_{2n}) \le h\ HP(Nu_{2n}, Mu_{2n-1})$.

Then in either case, we have
$\varphi(r) \min \{p(u_{2n-1}, Mu_{2n-1}),p(u_{2n}, Nu_{2n})\} \le p(u_{2n-1}, u_{2n})$

This implies
$H_P(Mu_{2n-1}, Nu_{2n}) \le r\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, Mu_{2n-1}), p(u_{2n}, Nu_{2n}),$
$p(u_{2n-1}, Nu_{2n}) - p(u_{2n-1}, u_{2n-1}), p(u_{2n}, Mu_{2n-1}) - p(u_{2n}, u_{2n})\}$

Then
$p(u_{2n}, u_{2n+1}) \le h\ H_P(Mu_{2n-1}, Nu_{2n}),\quad h > 1$
$\le k\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, Mu_{2n-1}), p(u_{2n}, Nu_{2n}), p(u_{2n-1}, Nu_{2n}) - p(u_{2n-1}, u_{2n-1}), p(u_{2n}, Mu_{2n-1}) - p(u_{2n}, u_{2n})\}\quad k = rh < 1$
$\le k\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, u_{2n}), p(u_{2n}, u_{2n+1}), p(u_{2n-1}, u_{2n+1}) - p(u_{2n-1}, u_{2n-1}), p(u_{2n}, u_{2n}) - p(u_{2n}, u_{2n})\}$
$\le k\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, u_{2n}), p(u_{2n}, u_{2n+1}), p(u_{2n-1}, u_{2n}) +p(u_{2n}, u_{2n+1}) - p(u_{2n}, u_{2n}) - p(u_{2n-1}, u_{2n-1}), 0\}$
$\le k\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, u_{2n}), p(u_{2n}, u_{2n+1}), p(u_{2n-1}, u_{2n}) + p(u_{2n}, u_{2n+1}), 0\}$

If $p(u_{2n}, u_{2n+1}) > p(u_{2n-1}, u_{2n})$, we arrive at a contradiction.
Hence $p(u_{2n}, u_{2n+1}) \le p(u_{2n-1}, u_{2n})$
and this leads to
$p(u_{2n}, u_{2n+1}) \le k\ \Phi\{p(u_{2n-1}, u_{2n}), p(u_{2n-1}, u_{2n}), p(u_{2n-1}, u_{2n}), 2p(u_{2n-1}, u_{2n}), 0\}$
$\le kp(u_{2n-1}, u_{2n})$.[since $\Phi(t, t, t, at, bt) \le t, t\in [0,\infty)$]

That is
$p(u_{2n}, u_{2n+1}) \le k\{p(u_{2n-1}, u_{2n})$. In similar manner, we have $p(u_{2n+2}, u_{2n+1})\le kp(u_{2n+1}, u_{2n})$.
From this, we have $p(u_{n+1}, u_n)\le kp(u_n, u_{n-1})$ for any $n \in \mathbb{N}$.
Thus we have
$p(u_{n+1}, u_n) \le k^n\{p(u_0, u_1)$ for all $n \in \mathbb{N}$. (2)

Using (2) and the triangle inequality for partial metrics (p4), for all $n, m\in \mathbb{N}$, we have

$p(u_n, u_{n+m}) \le p(un, un+1) + p(un+1, un+m) - p(un+1, un+1)$

$\le p(u_n, u_{n+1}) + p(u_{n+1}, u_{n+m})$

$\le p(u_n, u_{n+1}) + p(u_{n+1}, u_{n+2}) + p(u_{n+2}, u_{n+m}) - p(u_{n+2}, u_{n+2})$

$\le p(u_n, u_{n+1}) + p(u_{n+1}, u_{n+2}) + p(u_{n+2}, u_{n+m})$.

Inductively, we have

$p(u_n, u_{n+m}) \le p(u_n, u_{n+1}) + p(u_{n+1}, u_{n+2}) + \ldots + p(u_{n+m-1}, u_{n+m})$

$\le k^n\{p(u_0, u^1) +k^{n+1}\{p(u_0, u_1) + \ldots + k_{n+m-1}\{p(u_0, u_1)$

$\le k^n + k^{n+1} + \ldots + k^{n+m-1})p(u_0, u_1)$

$\le \frac{k^n}{1-k} p(u_0, u_1) \to 0$

as $n \to +\infty$ since $0 \le k < 1$. By the definition of ps, we get

$p^s(u_n, u_{n+m}) \le 2\ p(u_n, u_{n+m}) \to 0$

As $n \to +\infty$, which implies that {un} is a Cauchy sequence in (X, ps). Since (X, p) is complete, by Lemma 1.4, the corresponding metric space (X, ps) is also complete. Therefore, the sequence {un} converges to some $u \in X$ with respect to the metric p^s, which is $p^s(u_n, u) = 0$. by Lemma 1.4, we have

$\lim_{n\to\infty}$

$p(u, u) = \lim_{n\to\infty} p(u_n, u) = \lim_{n\to\infty} p(u_n, u_m) = 0.$ (3)

Next we prove that for any $y \in X - \{u\}$,

$p(u, Ny) \le r.\max\{p(u, y), p(y, Ny)\}$,

$p(u, My) \le r.\max\{p(u, y), p(y, My)\}$.

Since the sequence $\{u_n\}$ converges to u, there exists $n_0\in N$ such that $p(u, u_n) \le \frac{1}{3} p(u, y)$ for $y \ne u$ and for all $n \ge n_0$.
We have

$\varphi(r)p(u_{2n-1}, Mu_{2n-1}) \le p(u_{2n-1}, Mu_{2n-1}) \le p(u_{2n-1}, u_{2n})$

$\le p(u_{2n-1}, u) + p(u, u_{2n}) - p(u, u)$

$\le p(u_{2n-1}, u) + p(u, u_{2n})$

$\le \frac{2}{3} p(y, u) = p(y, u) - \frac{1}{3} p(y, u)$

$\le p(y, u) - p(u_{2n-1}, u)$

$\le p(u_{2n-1}, y)$.

Then $p(u_{2n-1}, Mu_{2n-1}) \le p(u_{2n-1}, y)$ (4)

Now either $p(u_{2n-1}, Mu_{2n-1}) \le p(y, Ny)$ or $p(y, Ny) \le p(u_{2n-1}, Mu_{2n-1})$. In either case, by (4) and the assumption,

$\varphi(r) \min \{p(u_{2n-1}, Mu_{2n-1}), p(y, Ny)\} \le p(u_{2n-1}, y)$.

Also we have

$p(u_{2n}, Ny) \le H_P(Mu_{2n-1}, Ny)$

$\le r\Phi\{p(u_{2n-1}, y), p(u_{2n-1}, Mu_{2n-1}), p(y, Ny),$

$p(u_{2n-1}, Ny) - p(u_{2n-1}, u_{2n-1}), p(y, Nu_{2n-1}) - p(y, y)\}$

$\le r\Phi\{p(u_{2n-1}, y), p(u_{2n-1}, u_{2n}), p(y, Ny), p(u_{2n-1}, Ny) -$

$p(u_{2n-1}, u_{2n-1}), p(y, u_{2n}) - p(y, y)\}$.

Taking limit as $n \to \infty$, we have

$p(u, Ny) \le r\Phi\{p(u, y), p(u, u), p(y, Ny), p(u, Ny) - p(u, u), p(y, u) - p(y, y)\}$

$p(u, Ny) \le r\Phi\{p(u, y), 0, p(y, Ny), p(u, Ny), p(y, u)\}$.

This implies that

$p(u, Ny) \le r\, p(u, y)$ or $p(u, Ny) \le r\, p(y, Ny)$

that is $p(u, Ny) \le r.\max\{p(u, y), p(y, Ny)\}$

Similarly it can be shown that

$p(u, My) \le r.\max\{p(u, y), p(y, My)\}$ (5)

Now we prove that $u \in Mu \cap Nu$.

Let $0 \le r < \frac{1}{2}$. Then following three cases are to be considered.

Case 1. Suppose $u \notin Mu$ and $u \notin Nu$. Let $a \in Nu$ be such that $2rp(a, u) < p(u, Nu)$.
Then from (5) with $y = a$ we have

$p(u, Na) \le r.\max\{p(u, a), p(a, Na)\}$

$p(u, Ma) \le r.\max\{p(u, a), p(a, Ma)\}$ (6)

Since $\varphi(r)p(u, Nu) \le p(u, Nu) \le p(u, a)$, we have

$\varphi(r)\{p(a, Ma), p(u, Nu)\} \le p(u, a)$.
This implies
$p(Ma, a) \le HP(Ma, Nu)$

$\le r\Phi\{p(a, u), p(a, Ma), p(u, Nu), p(a, Nu) - p(a, a), p(u, Ma) - p(u,u)\}$

$\le r\Phi\{p(a, u), p(a, Ma), p(u, a), p(a, a) - p(a, a), p(u, Ma) - p(u,u)\}$

$\le r\Phi\{p(a, u), p(a, Ma), p(u, a), p(a, a) - p(a, a), p(u, a) + p(a, Ma) -$

$p(a, a) - p(u,u)\}$

$\le r\Phi\{p(a, u), p(a, u), p(u, a), 0, p(u, a) + p(a, u)\}$

$\le r\Phi\{p(a, u), p(a, u), p(u, a), 0, 2p(u, a)\}$.

That is

$p(Ma, a) \le HP(Ma, Nu) \le rp(a, u) < p(a, u) \Rightarrow p(u, Ma) \le r\, p(a, u)$. (7)

Starting from (p4) using the fact that

$p(Ma, Nu) = \inf\{p(x, y): x \in Ma, y \in Nu\}$

$\le \inf\{p(x, y): x \in Ma\} = p(Ma, a)$ since $a \in Nu$ and by (6)

and (7) we have

$p(u, Nu) \le p(u, Ma) + p(Ma, Nu)$

$\le p(u, Ma) + HP(Ma, Nu)$

$\le r.\max\{p(u, a), p(a, Ma)\} + r\, p(a, u)$

$= r\, p(u, a) + r\, p(a, u)$

$= 2r\, p(a, u)$

$< p(u, Nu)$

That is a contradiction and hence $u \in Nu$. In similar fashion, we can show that $u \in Mu$.

Case 2. $u \in Mu$ and $u \notin Nu$ and Case 3. $u \notin Mu$ and $u \in Nu$ can be similarly disposed of.

Case 4. Let $\frac{1}{2} \le r < 1$. First we proved that
$H_P(Mx, Nu) \le r\,\Phi\{p(x, u), p(x, Mx), p(u, Nu), p(x, Nu) - p(x, x), p(u, Mx) - p(u, u)\}$. (8)

Assume that $x \ne u$. Then for every $n \in N$, there exists $z_n \in Mx$ such that

$$p(u, z_n) \le p(u, Mx) + \frac{1}{n} p(x, u).$$

Therefore we have

$p(x, Mx) \le p(x, zn)$

$\le p(x, u) + p(u, zn) - p(u, u)$

$\le p(x, u) + p(u, Mx) + \frac{1}{n} p(x, u)$

$\le p(x, u) + r.\max\{p(u, x), p(x, Mx)\} + \frac{1}{n} p(x, u)$. (9)

If $\max\{p(u, x), p(x, Mx)\} = p(u, x)$, then from (9) we have

$p(x, Mx) \le p(x, u) + r\, p(u, x) + \frac{1}{n} p(x, u) \le (1 + r + \frac{1}{n})\, p(x, u)$.

This implies that $\frac{1}{1+r}\, p(x, Mx) \le [1 + \frac{1}{(1+r)n}]\, p(x, u)$ and since $\varphi(r) = 1-r$, it follow that

$$\varphi(r)\, p(x, Mx) = (1-r)\, p(x, Mx) \leq \frac{1}{1+r}\, p(x, Mx) \leq [1 + \frac{1}{(1+r)n}]\, p(x,u).$$

Taking limit as $n \to \infty$, we have $\varphi(r)\, p(x, Mx) \leq p(x, u)$ and

$$\varphi(r) \min \{p(x, Mx), p(u, Nu)\} \leq p(x, u).$$

Then from (1) with taking y = u we get (8).

$H_P(Mx, Nu) \leq r\, \Phi\{p(x, u), p(x, Mx), p(u, Nu), p(x, Nu) - p(x, x), p(u, Mx) - p(u, u)\}$

If $p(u, x) < p(x, Mx)$, similarly proceeding we have

$H_P(Mx, Nu) \leq r\, \Phi\{p(x, u), p(x, Mx), p(u, Nu), p(x, Nu) - p(x, x), p(u, Mx) - p(u, u)\}$

Let $x = u_{2n+1}$. Then we obtain $p(u_{2n+2}, Nu) \leq H_p(Mu_{2n+1}, Nu)$

$\leq r\, \Phi\{p(u_{2n+1}, u), p(u_{2n+1}, Mu_{2n+1}), p(u, Nu), p(u_{2n+1}, Nu) -$

$p(u_{2n+1}, u_{2n+1}), p(u, Mu_{2n+1}) - p(u, u)\}$.

Taking limit $n \to \infty$, we have

$p(u, Nu) \leq r\, \Phi\{p(u, u), p(u_{2n+1}, u_{2n+2}), p(u, Nu), p(u_{2n+1}, Nu) - p(u_{2n+1}, u2_{n+1}),$

$p(u, u_{2n+2}) - p(u, u)\}$

$\leq r\, \Phi\{p(u, u), p(u, u), p(u, Nu), p(u, Nu) - p(u, u), p(u, u) - p(u, u)\}$

$\leq r\, \Phi\{0, 0, p(u, Nu), p(u, Nu), 0)\}$

$\leq r\, p(u, Nu)$.

Since $r < 1$, therefore $p(u, Nu) = 0 = p(u, u)$ and hence by Lemma 1.5, we have $u \in Nu$. Similarly we can show that $u \in Mu$.

Corollary 2.3

Let (X, p) be a complete partial metric space and let N: X $\in CBp(X)$ be a multivalued mapping. If there exist $r \in [0, 1)$ such that N satisfy the condition $\varphi(r)\, p(x, Nx) \leq p(x, y)$ implies
$H_P(Nx, Ny) \leq r \in \{p(x, y), p(x, Nx), p(y, Ny), p(x, Ny) - p(x, x), p(y, Nx) - p(y, y)\}$. for every $x, y \in X$, where the function φ is defined as in Theorem 2.2. Then N has a fixed point, that is, there exist $u \in X$ such that $u \in Nu$.

Corollary 2.4

Let (X, p) be a complete partial metric space and let N: X $\to CB^p(X)$ be a multivalued mapping. If there exist $r \in [0, 1)$ such that N satisfy the condition $\varphi(r)\, p(x, Nx) \leq p(x,y) \Rightarrow HP(Nx, Ny) \leq r \max\{p(x, y), p(x, Nx), p(y, Ny)\}$ for every $x, y \in X$, where the function φ is defined as in Theorem 2.2. Then N has a fixed point, that is, there exist $u \in X$ such that $u \in Nu$.

Proof

If $\Phi\{x1, x2, x3, x4, x5\} = \max\{x1, x2, x3\}$, then by using Corollary 2.3, we conclude that N has a fixed point.

Corollary 2.5

Let (X, p) be a complete partial metric space and let N: X $\to CB^p(X)$ be a multivalued mapping. If there exist $r \in [0, 1)$ such that N satisfy the condition $\varphi(r)\, p(x, Nx) \leq p(x,y) \Rightarrow HP(Nx, Ny) \leq a\, p(x, y) + b\, p(x, Nx) + c\, p(y, Ny)$ for every $x, y \in X$, where the function φ is defined as in Theorem 2.2. Then N has a fixed point, that is, there exist $u \in X$ such that $u \in Nu$.

Proof

If $\Phi\{x1, x2, x3, x4, x5\} = ax1 + bx2 + cx3$, where $a + b + c < 1$. Put $r = a + b + c < 1$, then by using Corollary 2.3, we conclude that N has a fixed point.

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