International Journal of Multidisciplinary Research and Development
Online ISSN: 2349-4182, Print ISSN: 2349-5979; Impact Factor: RJIF 5.72
Received: 21-09-2020; Accepted: 06-10-2020; Published: 22-10-2020
www.allsubjectjournal.com
Volume 7; Issue 10; 2020; Page No. 75-79

Teaching of derivative through problems focused on oil and gas industry contexts

Maricela García Avalos[1], Manuel Sandoval Martínez[2], Gerardo Enrique Sepúlveda Palacios[3]
[1-3] Universidad Politécnica del Golfo de México, Carretera Federal Malpaso-Bellote, Tabasco, México

Abstract
Results of the application of problem solving focused on oil and gas industry contexts to freshman students of Petroleum Engineering at the Polytechnic University of the Gulf of Mexico (UPGM) are presented. The main objective was to improve the understanding of the application of the derivative concept and some logical-mathematical skills. For its development, it was worked on Differential and Integral Calculus where several individual and group activities aimed to solve problems in contexts were assigned and the Rudnick & Krulik (R&K) methodology was used. Such activities followed a didactic sequence that allowed them to effectively achieve the stated objectives. To measure the understanding of the topic, a performance evidence was applied, which consisted of a presentation of a problem in context. Deficiencies during student's presentation were found in such a way that only 26% approved. However, such deficiencies were overcome as students delved into the problems in context, which ended up in all (100%) the student's effectively performing presentations meaning that they acquired the proper logical-mathematical skills. This indicated that the use of problems in contexts as a complementary tool for learning derivatives is good compared to traditional methods.

Keywords: Mathematics teaching, contextualized problems, Rudnick & Krulik algorithm, teamwork, non-traditional teaching, and development of mathematical skills.

1. Introduction

Colleges usually consider Differential and Integral Calculus as a mandatory course, especially in engineering departments. This course become difficult for students because they do not have the basic background about it, they do not have the habit to study or simply because it is a math course; which at the end is reflected as a high failure rate.

The Polytechnic University of the Gulf of Mexico (UPGM) is not apart from this, since petroleum engineering students study Differential and Integral Calculus in the first semester and, for several years, it has been noted that students do not properly acquire the logical-mathematical skills they need and do not understand the concepts covered in the course, causing their academic performance to be low or to fail the course.

Most of the basic science professors at our university continue to teach in a traditional way, where students are treated only as recipients of information. For example, they solve purely mathematical exercises without including any application problem focused on the major they are studying. This causes students to be discouraged and makes them think that taking Differential and Integral Calculus is pointless, becoming a difficult subject to pass. This contribute to their negative attitude towards the class, decreasing the interest in learning. That is why it is necessary for teachers to implement new strategies to teach their classes.

On the other hand, one of the learning units of Differential and Integral Calculus is the application of the derivative, in this unit professors should emphasize on solving applied problems focused on the corresponding major, this could allow students to come up with some relevant questions, like: What is the derivative good for? How do I apply this tool in my workplace? – among others.

To improve the academic performance and attitude of students, the implementation of a methodology that involves the solution of contextualized problems focused on the oil and gas industry was proposed to observe how important the concept of the derivative is and how to apply it to solve problems in the workplace. Part of the methodology explains how to solve problems using the Rudnick & Krulik algorithm (Krulik & Rudnick, 1988) [17] which consists of 5 steps: Read, explore, plan, solve and extend. This algorithm helps students to find the solution in an orderly and clear way, since each of the steps allows them to better understand the solution to the problem.

All the problems analyzed in class were designed by the authors of this paper, and they involve derivatives of algebraic, trigonometric, and exponential functions.

2. Background

In the field of math didactics research, it is well known that the common teaching techniques of calculus are based on the transmission of knowledge with a very marked emphasis on the development of algebraic skills where intellectual discernment for the understanding of ideas, notions and concepts is neglected (Relime, 2007) [20]. Such situation has been addressed in various works that range from theoretical arguments to proposals that are intended to improve the quality of learning, which include both the prior knowledge a student would need to be successful in the study of calculus, as well as the elaboration of didactic materials (Farfán RM, 1991) [14] (Farfán RM, 1994) [15] (Artigue, 1995) [2] (Dolores, 1999) [13] (Salinas P., 2002) [22].

The differential and integral calculus is the study of functions, therefore its teaching aims to show important properties of functions (Cuevas, 2009). As functions are the mathematical model par excellence of almost any science, differential and integral calculus is a mandatory course in the curricula of engineering majors. However, reports of failure to learn calculus are frequent and, therefore, it becomes one of the problems that the educational community concerns the most (ANUIES, 2002) [1]. Traditionally, the pass rate in calculus course is very low, which is very common in engineering majors in Mexico, where there are failure rates of more than 70% for this course (Steen, 1987) [25] (Cuevas A ., 1996) [11] (Baker B., 2001) [3]. In this sense, (Tall, 1996) [26] mentions that even though students undergo a heavy regime of calculation exercises, the percentage of failures in this subject ranges between 30% and 50%.

(Artigue, 1995) [2] points out that many investigations show that even though students are taught to mechanically perform derivative calculations and solve standard problems, it was found that this is not enough for them to reach a satisfactory understanding of the concepts and methods of thinking in the field of calculus. On the other hand, other research indicate that the traditional method focuses on evaluating how capable the student is of finding the result of the problem and little emphasis on the procedure to reach it.

For example, (Moreno, 2005) [18] indicates that: "Teaching the principles of calculus is quite problematic, and although we are able to teach students to solve more or less mechanically some standard problems, or to perform some derivatives or integrals, such actions are far from what a true understanding of the concepts and thinking methods of this part of math would entail. " A major problem linked to this situation is that knowledge is generally dealt with outside of appropriate contexts. Thus, when it is intended to show students the usefulness of the contents that are studied, the most that is reached in a common calculus course is to solve the so-called application problems that are proposed in the texts, which almost never correspond to reality (Relime, 2007) [20].

In practice, it often happens that the professor's didactic strategy consists of arriving with their textbook or with their notes, their marker, and an eraser, they ask their group: what was the last topic we went over? And they give a lecture — sometimes masterful — or make an excellent presentation of the topics that follows the agenda; at other times, the strategy is to copy or transcribe your course notes on the board. The low quality of teaching strategies is not the only aspect of teaching practice that we need to worry about; the evaluation and assignment of grades is also a fundamental point. What do we really evaluate with our evaluation procedures? Perhaps, the mental agility of the students? Their knowledge? Their abilities to answer exams? Are they good at transcribing? How self-taught are they? Or maybe the learning goals that we want them to achieve in classes? And what about self-evaluation: do we give feedback on our work to ourselves? Do we validate our evaluation instruments? How fair, but above all, how significant are the grades that we assign to each student? (Rubí Vázquez Gloria Elena, 2010) [21].

This has negative consequences when those who learn are students whose majors require knowledge and skills to allow them to solve real problems. Such is the case of those pursuing and engineer major. (Camarena P., 1990) [8] mentions "part of the problem in engineering is that math is totally unlinked to engineering courses, and the real world of an engineer demands this link, which is educationally in no man's land".

This problem has impact on the classroom environment, the students' willingness to learn and their attitude to face new knowledge. Knowing mathematics means, for students, having some skill in solving equations, developing procedures, applying formulas and methods. Rarely does a student conceive mathematics as something that can be useful beyond what was stated before, and when it does happen, it is not entirely clear for them. What can be done? How to link the mathematical contents to areas that may interest the student? In this regard, (Camarena P., 2000) [9] mentions: "Mathematics in context: it helps the student to build their own knowledge of meaningful mathematics, with firm and non-volatile ties; it reinforces the development of mathematical skills, through the process of solve problems related to the interests of the student".

Now, regarding the design of learning activities, three types of contexts used to encourage the participation of students in essential activities of the discipline can be distinguished: contexts about real situations, hypothetical or realistic, and formal or purely mathematical (Barrera, 2002) [25]. Problem solving is identified as an important activity for learning calculus. Learning calculus goes beyond memorizing a set of definitions, algorithms, and techniques for solving routine activities. Additionally, an environment should be fostered in the classroom where students can communicate their ideas, ask questions, use multiple representations, make conjectures, formulate counterexamples, make predictions, and build mathematical models (Benitíz Mojica, 2009) [6].

Some works (Moreno, 2005) [18] (Hermon, McCartan, & Cunningham, 2009) [16] agree that traditional, mechanistic, decontextualized, and technical teaching hinders the understanding of the mathematical study object meanings and their links with other sciences. Indeed, it is often expected math teaching to be based on contextualized problems; that is, directly applicable to the objective or subjective needs of the student (Cruz, 2006) [10].

Some research on the problem-solving process has concluded that students should be taught to solve problems, focusing on the solution process and not so much on the answer to the problem. Also, it was found that procedures for solving problems could be taught as habits and transformed through appropriate training and practice (Bloom & Broder, 1950) [7], thus these investigations made it clear that the process is more important than the product as such, and that for the student to solve successfully a problem, adequate information about the solution process is required, in addition to specific information in the field to which the problem belongs.

Although problem solving plays an important role in the math teaching, accumulated knowledge is scattered. However, solving problems is a distinctive sign of mathematical activity and even more so in engineering field. According to (Schoenfeld, 2015) [24], success in solving a problem depends, among other factors, on the strategies that each individual has to solve a problem; (Reif, 1981) [19] points out that understanding a problem begins by building a description of the problem, which helps in the search for an appropriate solution. The Rudnick & Krulik algorithm (Krulik & Rudnick, 1988) [17] has shown to be very useful for students, since among other things, it allows them to solve problems in an orderly manner, with more clarity and confidence (Sandoval, García, & Velázquez, 2014) [23]. Although this tool was used in the teaching of algebra, now there is an opportunity to apply it to calculus.

To address the problem of math teaching and learning in professional engineering majors, it is necessary to build proposals on alternative didactic structures that allow better learning having a practical meaning in engineering processes, close to the interest of students and where the study of calculation makes sense. In addition, there is a great social demand aimed to researchers in educational mathematics, where they not only analyze the problem of teaching mathematics, but also contribute to the production of material at the service of this teaching (Cuevas & Francois, 2013) [12]. Hence, the importance of elaborating a series of problems in context focused on the oil industry.

On the other hand, the educational model used in polytechnic universities considers professional training based on competencies, which characteristics manifested in the curricular design are different from the traditional one, in the way of conducting the teaching-learning process through the use of diverse didactic strategies and techniques, and in the evaluation of learning (Barcelata & Rodríguez, 2009) [4]. Competency-based education (CBE) aims for students to acquire generic and specific competencies, during their academic training, useful for life and work. The generic competence seeks to fulfill a personal and professional profile upon completion of their studies, and the specific competence focuses on an area of knowledge. In this sense, the Rudnick & Krulik strategy (useful for problem solving) allows them to develop many specific theoretical and procedural skills in one or more fields of mathematical knowledge.

3. Methodology

To develop this research, a didactic sequence that includes application of the derivative focused on petroleum engineering problems was proposed, which had to be solved using the Rudnick and Krulik algorithm. The topics involved refer to functions: algebraic, trigonometric, and exponential.

The didactic sequence was used as a complementary tool in the Differential and Integral Calculus (CDI) course of the first semester of the Petroleum Engineering major at the Polytechnic University of the Gulf of Mexico, with a 23 students' group. During the development of the learning unit: application of the derivative (4 weeks), teams of 3 people were formed to work on the activities assigned for an approximate time of 15 min for each activity, at the end the results were discussed in a group supported by the instructor, who served as a moderator. To evaluate the logical mathematical skills acquired by the students in the learning unit, they were assigned to a problem in context for which they should prepare a presentation (performance evidence) following an observation guide (see Annex 1) with the intention of letting them know which items were to be evaluated. According to what is indicated by the competence-based model, students had two more opportunities to present the evidence in case they were considered not competent in the first time. Students who presented their evidence in the first or second opportunity received counseling using the problems in contexts, with the intention of reinforcing their academic performance. It is worth mentioning that a student passes the evidence only if they obtained a score greater than or equal to 70%.

4. Results

Here is a problem in context, in which the application of R & K methodology is shown.

Problem 1: In a laboratory, a PVT test will be performed on a sample from a reservoir. Such test involves subjecting the sample to temperature variations T (°C) so its behavior can be interpreted. Temperature is modeled by:

$$\mathrm{T(t)} = 60 + 12\mathrm{sen}\left(\frac{\pi t}{10} - \frac{5}{6}\right);\ 0 < t < 10 \text{ minutes}$$

The lab manager wants to do a 75 °C test. The manual indicates that if the maximum temperature is exceeded, the sample will suffer irreversible damage. Will the sample be damaged by the proposal of the laboratory manager?

Solution with RK

1. **Read:** Laboratory, PVT test, reservoir, sample, and temperatures.
2. **Explore:** The temperature is modeled with; $\mathrm{T(t)} = 60 + 12\mathrm{sen}\left(\frac{\pi t}{10} - \frac{5}{6}\right);\ 0 < t < 10$ minutes. A test will be performed at 75 °C. See Figure 1.

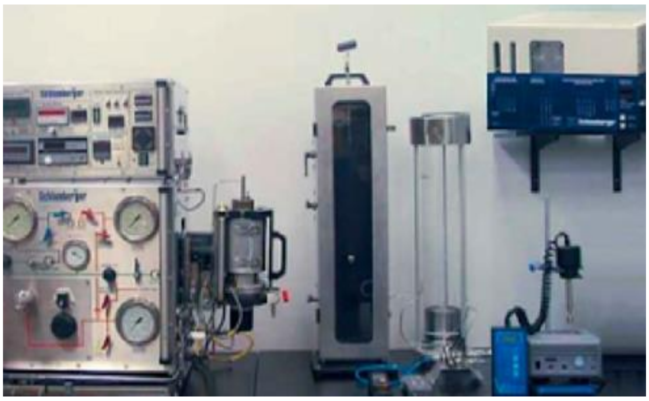

Fig 1: PVT Test Lab

3. **Strategy:** To calculate in what time the maximum temperature is reached, the critical point of the function T must be determined. For this, the function must be derived and equal to zero, then the criterion of the second derivative to know if the time found is a maximum should be applied. After saying if there is a maximum, we check whether the sample will suffer any damage.
4. **Solve the problem:** Let us determine the critical point of

$$T(t) = 60 + 12sin\left(\frac{\pi t}{10} - \frac{5}{6}\right)$$

Then

$$T(t)' = \left(\frac{6}{5}\right)cos\left(\frac{\pi t}{10} - \frac{5}{6}\right) = 0$$

The cosine function has its zeros at $n\pi / 2$; with

$$n = 1,3,5,7, \ldots\ \pi t / 10 - 5 / 6 = n\pi / 2$$

For $n = 1$ we have

$$\left(\pi t \frac{\square}{\square} 10\right) - \left(\frac{5}{6}\right) = \pi \frac{\square}{\square} 2t = 5 + 50 / 6\pi = 7.65$$

We find $T(t)''$ and evaluate

$$T(t)'' = -\left(\frac{6\pi^2}{5}\right) sin\left(\frac{\pi t}{10} - \frac{5}{6}\right) T(7.65)'' = -11.84 < 0$$

So $t = 7.65$ is a maximum. Now

$$T(7.65) = 60 + 12sin\left(\frac{\pi * 7.65}{10} - \frac{5}{6}\right) = 72$$

So the maximum is. $7.65, 71.99$. Since $72 < 75$ then the sample will not be damaged. The graph of the function is:

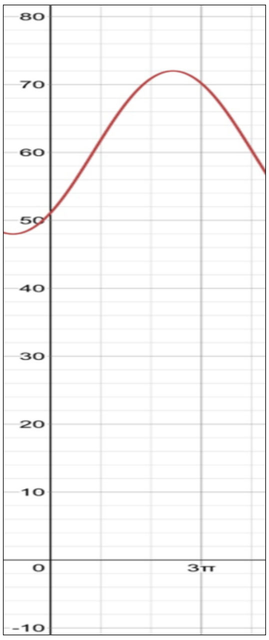

Fig 1: Behavior of $T(t) = 60 + 12sen\left(\frac{\pi t}{10} - \frac{5}{6}\right)$;.

5. **Extend:** What would you recommend to avoid damaging the sample?

That the maximum temperature is not exceeded or that the sample is conditioned with an additive, such as alcohols or glycols, with the intention that the sample resists high temperatures.
For this problem, the difficulty that the student had was the following: in step 3 of the R&K methodology, they did not describe the strategy to follow but made a list, in addition they failed to indicate which criterion of the derivative they would use to determine if the critical point it was a maximum or a minimum. In step 4, which consists of solving, the student calculated the critical point and said that it was a maximum for having observed it in the graph, but did not demonstrate it using one of the derivative criteria, since in case of using the second the derivative criterion they had had to calculate the second derivative and, in step 5, which is to be extended, they did not elaborate a question in such a way that it would give relevant information about the problem, what they did was reproduce the question in part A) of the problem.

4.1 Observation guide results

After analyzing the results of the performance evidence observation guides of each student, most of the deficiencies were found in step 4 (solve) and 5 (extend) of the Rudnick and Krulik algorithm. In item 4, 14% of the students correctly calculated the derivative of the function, as well as the application of the drift criterion. On the other hand, on the first opportunity given to them, 69% improved their calculations performance, obtaining a good academic result. In step 5, 27% prepared the question properly allowing them to have more information about the problem. Now, from those who presented the first opportunity, 81% of them managed to improve the question.

Table 1: Performance evidence (PE) grades

# Student	Grade, PE	Grade, PE (1st oportunity)	Grade, PE (2nd oportunity)
1	9		
2	9		
3	9		
4	9.8		
5	9.8		
6	9.8		
7	3	8	
8	3	8	
9	5	9.5	
10	3	9.5	
11	4	9.5	
12	5	9.5	
13	4	9.5	
14	4	9.5	
15	3	9.5	
16	3	9.5	
17	5	10	
18	5	10	
19	4	10	
20	1	5	8
21	2	4	8
22	1	4	8
23	2	5	8

The grade obtained from the performance evidence shows that from the 23 students, only 6 of them approved the evidence, this is equivalent to say that 26% of the population was competent in the evidence; their grades were very good (see table 1), the rest (74%) failed. Then the remaining 17 students were given advice in solving the problems, to strengthen their knowledge and thus have the right to take the evidence again. Here, it was obtained that 13 students passed (56.5%), which indicates that more practicing in solving problems in contexts improves their mathematical logic skills, the other students who did not pass (17.5%) were given a second chance with their respective consultancies and got to pass the evidence.

5. Discussion

The grades got by the students when taking the performance evidence, indicated that the strategy of using problems in contexts focused on petroleum engineering was good, since in that session 26% passed the exam adequately, which indicated that these students had acquired the necessary skills to solve such problems.
When the other students had their first opportunity, they showed that they had improved their ability to solve their performance evidence, since 56.5% of them had approved the evidence obtaining an improvement in the understanding of their problems; The rest were given a second chance and it was observed that they were also competent although it

was noted that they still have certain deficiencies in developing the solution of the problems.

Implementing the problems in contexts allowed students to realize how they can apply the derivative in the oil industry and working in a team helped them to express their ideas freely, making the students to improve some generic skills such as team work and interpersonal communication

6. Conclusions

Most of the students consider that the Differential and Integral Calculus subject is complicated, since they state that it is difficult for them to interpret the derivative and its usefulness in engineering, which causes them to have a negative attitude in the classroom.

The implementation of the problems in contexts was carried out in a group of the first semester of Petroleum Engineering of the UPGM and the classes were given by a professor who has 16 years of teaching experience, she observed that the students improved their attitude in the classes during the development of the learning unit: Application of the derivative, since at the beginning of the course they were apathetic because they only saw how the derivative formulas were used for purely mathematical problems.

The teacher also analyzed through the observation guide how the students were improving their mathematical logic skills to solve problems in contexts, since when they presented their evidence only 26% of the population passed the performance evidence. After giving them advice and allowing them to present again, their academic performance improved considerably in such a way that 83% of the population was approved and, finally, the rest of the students managed to pass in the second opportunity, thus achieving that no student will fail and above all that their academic performance will improve.

7. Reference.

1. ANUIES. Reportes de calidad de las Instituciones de Educación Superior en México, 2002. Obtenido de http://www.ANUIES.mx
2. Artigue M. La enseñanza de los principios del cálculo: problemas epistemológicos, cognitivos y didácticos. México: Grupo Editorial Iberoamérica. 1995, 97-140.
3. Baker BHC. On transformation of functions. Proc. 23th Conf. of the Int. Group for the Psychology of Mathemarics Education, Utah, 2001, 18-21.
4. Barcelata S, Rodríguez C. Guía técnica para la selección de estrategias y técnicas de enseñanza y aprendizaje. Coordinación de Universidades Politécnicas. México, 2009.
5. Barrera FS. Cualidades y procesos matemáticos importantes en la resolución de problemas: un caso hipotético de suministro de medicamentos. En Ministerio de Eduacación Nacional (Ed.). Seminario Nacional de Formación de Docentes: Uso de las Nuevas Tecnologías en el Aula de Matemáticas. Santa Fe de Bogotá, Colombia, 2002, 166-185.
6. Benitíz Mojica DL. Situaciones Problemáticas en Contexto en el Aprendizaje del Cálculo. El Cálculo y su Enseñanza, 1990, 33-43.
7. Bloom BS, Broder LJ. Problem-solving processes of college students. Supplementary Educational Monographs, B, 109. American Psychological Association, 1950.
8. Camarena P. Especialidad en docencia de la ingeniería matemática en electrónica. México: ESIME-IPN, 1990.
9. Camarena P. Etapas de la matemática en el contexto de la ingeniería (Reporte técnico de investigación). México: ESIME-IPN, 2000.
10. Cruz RM. La enseñanza de la Matemática a través de la Resolución de Problemas. La Habana: Educación Cubana, 2006.
11. Cuevas A. Sistemas Tutoriales Inteligentes. Investigaciones en Matemática Educativa. México: Iberoamérica, 1996.
12. Cuevas CV, Francois P. El cálculo y su enseñanza. Cinvestav-IPN, 2013.
13. Dolores C. Una introducción a la derivada a través de la variación. México: Grupo Editorial Iberoamérica, 1999.
14. Farfán RM. El curso de precálculo: un enfoque gráfico. Publicaciones Latinoamericanas en Matemática Educativa. 1991; 5(1):206-211.
15. Farfán RM. Ingeniería didáctica en precálculo. Acerca de la puesta en escena de los resultados de investigación en el sistema de enseñanza. Publicaciones Latinoamericanas en Matemática Educativa. 1994; 8(1):457-462.
16. Hermon P, McCartan C, Cunningham G. Enhancing the Educational Development of Individuals in Group Projects. Queen's University Belfast, Proceedings of the 5th International CDIO Conference, Singapore Polytechnic, Singapore, 2009.
17. Krulik S, Rudnick JA. Problem Solving: A Handbook for Elementary School Teachers. 2 Ed. Massachusetts: Allyn and Bacon, Inc, 1988, 248.
18. Moreno M. El papel de la didáctica en la enseñanza del cálculo: evolución, estado actual y retos futuros. Córdoba, España: Universidad de Córdova: En A. Maz, B. Gómez & M. Torralba (Eds.), 2005.
19. Reif F. Teaching problem solving: A scientific approach. Obtenido de The Physics Teacher. 1981; 19:310-316: http://dx.doi.org/10.1119/1.2340790
20. Relime LZ. El cálculo en carreras de ingeniería: un estudio cognitivo. SciELO Scientific Electronic Library Online, 2007, 145-175.
21. Rubí Vázquez, Gloria Elena MM. Problemática persistente en el aprendizaje de Cálculo caso de la Facultad de Ciencias, UABC. El calculo y su enseñanza, 2010.
22. Salinas PAJ. Elementos del cálculo. Reconstrucción conceptual para el aprendizaje y la enseñanza. México: Trillas, 2002.
23. Sandoval M, García M, Velázquez G. Desarrollo de habilidades en factorización usando problemas en contextos. Congreso Internacional de Investigación de la AcademiaJournals.com, 2014, 1958-1961.
24. Schoenfeld A. Reflections on Problem Solving Theory and Practice. Obtenido de Time. 2015; 10(1-2):9. http://www.math.umt.edu/tmme/vol10no1and2/1-Schoenfeld _pp9_34.pdf
25. Steen LA. Calculus for a new century: A pump, not a filter. Washington, D.C.: Mathematical Association of America, Notes, 1987, 8.
26. Tall D. Functions and Calculus, International Handbook of Mathematics Education, chapter 8. Kluwer Academic Publishers, Netherlands, 1996.