International Journal of Multidisciplinary Research and Development
Online ISSN: 2349-4182, Print ISSN: 2349-5979; Impact Factor: RJIF 5.72
Received: 15-08-2020; Accepted: 30-08-2020; Published: 16-09-2020
www.allsubjectjournal.com
Volume 7; Issue 9; 2020; Page No. 71-75

Effectiveness of using geogebra on students' conceptual understanding in differential calculus for grade xi students at gongzim ugyen dorji central school

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Abstract
The integration of emerging and relevant technology in the teaching and learning of differential calculus creates a conceptually rich learning environment. From this perspective, this study aims to determine the effectiveness of a computer-assisted instruction method using GeoGebra in developing the conceptual understanding of differential calculus for grade XI students. This study employed a quasi-experiment approach with 60 students from Gongzim Ugyen Dorji Central School in Haa. The students were divided into two equal groups. Group 'A' used the GeoGebra software, while group 'B' used the conventional method to learn differential calculus. The data was collected through conceptual understanding tests. An Independent sample t-test was employed using the Statistical Package for the Social Sciences (SPSS 22.0). Findings of the study showed that students who were taught using GeoGebra outperformed those who learned through conventional methods. The results confirmed that GeoGebra software is capable of enhancing and significantly improving students' conceptual understanding of differential calculus.

Keywords: conceptual understanding, differential calculus, geogebra

1. Introduction

Calculus is one of the greatest achievements of the human mind because calculus has demonstrated the power to illuminate the most fundamental problems in mathematics, physical sciences, biological sciences, and engineering. In fact, the discovery of calculus in the seventeenth century created a major shift in many fields including mathematics. In the US and elsewhere calculus often functions as a filter, preventing large numbers of students from pursuing a STEM (science, technology, engineering, and mathematics) career after graduating from grade12 (Hagman, Rasmussen, & Kelton, 2014) [7]. The better instructional experience in calculus improves students' success in calculus learning and their continued interest in STEM careers (Hagman *et al*., 2014) [7].

The importance of STEM for a developing country like Bhutan is an essential tool for further growth and development. For any successful economy, particularly in today's quest for a knowledge-based economy, science, technology, and engineering are the basic requisites (Dema, 2018) [6]. Considering the importance of integration of technology in education, Education Blueprint 2014-2024 recommended a shift in the education system by leveraging ICT to address contemporary challenges and realize the visions and aspirations of the education system. Capitalizing on ICT will transform education by creating a platform for effective communication, enabling access to information and knowledge, and creating conceptually rich learning environments where students construct their own knowledge visualize and experiment. Moreover, the Ministry of Education launched its education ICT Master plan called iSherig, on May 24, 2019, to harness the potential benefits of ICT in teaching and learning, and to enhance the use of ICT-integrated teaching and learning resources to produce globally competent learners through the use of emerging and relevant technology. The National School Curriculum Conference Report [NSCCR] (2016) also recommended to the Royal Education Council and Ministry of Education to develop a STEM standard. Since the Differential Calculus is the backbone of STEM, it is of paramount importance to emphasize the innovative ways of teaching and learning differential calculus.

The conventional teaching and learning of differential calculus that relies on symbols and notations, has been the preferred method of calculus instruction for decades. These practices have often been responsible for students failing to understand the key concepts of differential calculus. Teachers often focused more on the computational procedures rather than promoting comprehension of underlying concepts (Lasut, 2015; Theodosis, Pamifilos, Christous, Maleev & Jones, 2007) [33, 22]. The teaching and learning of calculus had become merely a list of procedures to follow, and resulted in routine algebraic manipulations. As a result, students developed the skills of manipulating algebraically, understanding instrumentally and memorizing the formulae instead of comprehending conceptual or relational proficiencies. Thus, the traditional approach of teaching and learning calculus failed to develop a conceptual understanding of its principles (Michael, 2006) [12]. Moreover, students found the concept of derivative epistemologically difficult. For instance, students could correctly define the derivative and differentiate the algebraic functions instrumentally, but failed to interpret the meaning of derivative conceptually (Ubuz, 2001) [23]. Students find difficulty in conceptualizing and relating the rate of change to the concept of derivative (Bezuidenhout, 2001; Orhun, 2012) [3, 15] and comprehending the difference between average rate of change and instantaneous rate of change and relating to the concept of derivative Bingolbalias(cited in Sachin, Yenmez & Erbas, 2015) [21]. Additionally, students find difficulty in (i) understanding how the average rate of change approximates the instantaneous rate of change, (ii) understanding how the slope of the secant lines approximate the slope of the tangent line (Hahkioniemi, 2006) [8] and, (iii) conceptualizing the role of limit in providing the algebraic definition of derivative.

Understanding relations between three big ideas play a key role in developing the conceptual understanding in differential calculus. The study conducted by Sahin *et al*., (2015) highlighted that if one of the three big ideas is ignored, then the concept of derivative may not be fully understand because of the compartmentalization of the three big ideas in students' conceptual system. Students can solve the differentiation correctly, which implies procedural understanding, but they may not actually make sense of what concept of derivative conceptually means.

The GeoGebra software chosen as a teaching method has the potential to visualize the conceptual definition of derivatives and how the slope of the secant line approximates the slope of the tangent line graphically. GeoGebra is a Dynamic Mathematics Software [DMS] developed by Markus Hohenwarter in 2002 [9] for teaching and learning Mathematics from primary to the university level (Hohenwaeter, Jarvus, & lavicza, 2009) [9]. It combines many aspects of different mathematical packages, and dynamically joins Geometry, Algebra and Calculus. The numerous calculus- related interactive worksheets and methods developed by teachers and researchers are available (www.geogebra.org). Many studies have been conducted to determine the effectiveness of the use of GeoGebra in teaching and learning of differential calculus. The study by Nobre, *et al*., (2016) [13], compared the use of GeoGebra and traditional method by involving 26 high school students. The findings indicated significant differences on test scores between two groups; students achieved higher scores using the GeoGebra software. Research by Udofia & Uko (2018) [48] showed that teaching and learning of differential calculus by utilizing the GeoGebra was found to be efficient, and enhanced performance significantly.

The dynamic geometry software like GeoGebra are useful tools in teaching and learning calculus as it integrates graphical, numerical and symbolic functions, The visualization that is possible with today's dynamic software enables the student to see and explore mathematical relations and concepts that were difficult to "show" in past prior to technology. GeoGebra helps students visualize limits, to determine slopes and tangent lines of curves (Garber, Picking, Kathleen, & Espy, 2010) [5]. It provides many possibilities to help students obtain an intuitive feeling and adequately visualize the derivative concepts.

Ljubica (2009) [11] discusses the use of GeoGebra software as an innovative method of teaching and learning calculus. The study was conducted with 31 students of the Accredited Business- technical School of the Vocational Studies. The results indicated a significant impact on the students' understanding and knowledge of the differential calculus. Moreover, he further asserted that GeoGebra was a powerful tool for visualization and simulation of the fundamental concepts of differential calculus (the Slope of tangent line, connection between slope of tangent and secant line, and limits). Similarly, Aydos (2015) [2] found that GeoGebra had a positive influence on students' conceptual understanding of limit and continuity. The experimental, student group who received intervention with GeoGebra significantly performed better compared to those who received traditional/textbooks-based instructions or intervention.

Despite the supporting evidence from studies investigating the impact of GeoGebra in developing the conceptual understanding of differential calculus with different population of learner, there is limited academic discourse on the impact of GeoGebra on the conceptual understanding of differential calculus in Bhutanese contexts. Thus, this study was designed for grade 11 students of Gongzim Ugyen Dorji Central School unearth the effect of instructional activities using GeoGebra on students' performance regarding their conceptual understanding of differential calculus.

2. Materials and Method

This study uses a qausi-experimental design to examine the effectiveness of GeoGebra software on students' conceptual understanding of differential calculus. Quasi-experiment was the best approach due to the availability of the original classes of students (Wiersma, 2000) [49]. The population of sample comprised of 94 students from grade 11 mathematics at Gongzim Ugyen Dorji Central School. The cluster random sampling was adopted to select 60 students from 3 sections. The students were then divided into two groups; one was a treatment group and the other was the control. Students in the treatment group were taught calculus using the GeoGebra software. Students in the control group were taught through normal conventional learning.

For an experimental group, one GeoGebra file was used during instruction to introduce and develop the concept of derivative in relation to the limit, average rate of change, and slope of tangent. On other hand, the relation between the three big ideas or how the slope of secant line approximates the slope of tangent line was delineated with a static graph drawn on the board. The visualization possible with GeoGebra helped to develop the relational or conceptual understanding in differential calculus.

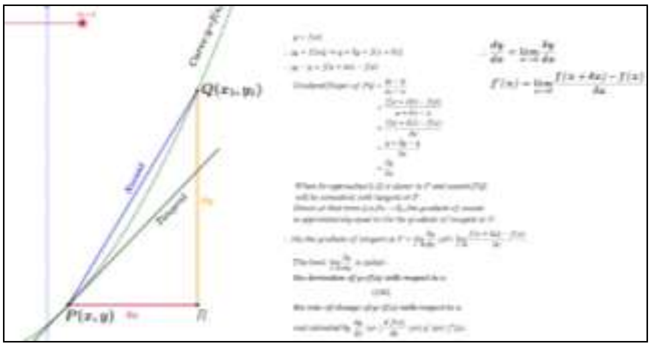

Fig 1: Screenshot of Relation Between the slope of secant line, limit and tangent line

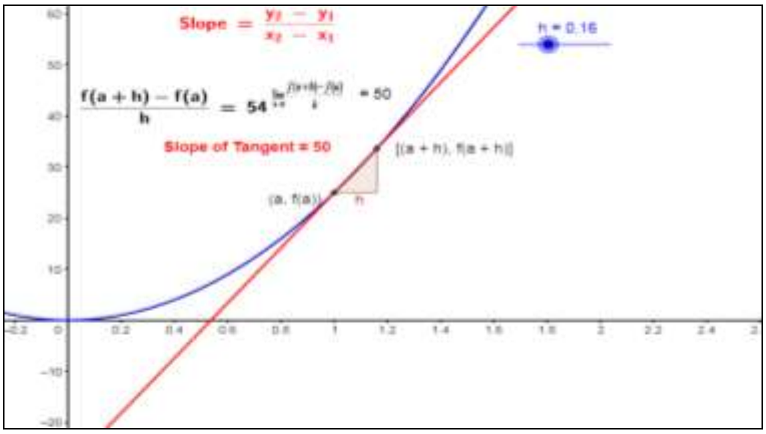

Fig 2: Screenshot How Slope of Secant line approximates the Slope of tangent line

Data were collected by using a pre and post conceptual understanding test for each group. The test items which consisted of six questions were modified from BHSEC Mathematics Book-1 for Class XI. Instrument was piloted to ensure its reliability and validity. The average Item objective Congruence (IOC) was 0.89, validating the appropriateness for the study. The reliability of the instrument was proven high with item reliability of 0.88. Comparative statistical analysis was done using the t-test. The independent sample t-test was used to compare the learning achievement between the control and experiment group. The inferential t-test with p<0.005 level of significance, the mean and standard deviation were used to infer the results.

3. Results

Before conducting the inferential *t-test*, the normality test were conducted using the

Kolmogorov-Smironov test as shown in the table 1. The findings of Kolmogorov-Smironov analysis for the degree of normality assumptions was satisfied for both pre-post-test of EG and CG (P> 0.05). Additionally Levene's test for equality of variances of scores for two groups (EG& CG) were conducted as shown in the table 3.1

Table 1: Test of normality with One-Sample Kolmogorov-Smirnov Test

	Pretest-EG	Posttest-EG	Pretest-CG	Posttest-CG
N	32	32	30	30
Kolmogorov-Smirnov Z	.978	.944	.986	1.406
Sig. (2-tailed)	.294	.335	.285	.038

Table 2: Levene's test for equality of variances

		F	df	Sig.
Pre-test	Equal variances assumed	6.571	62	0.78
	Equal variance not assumed		58.99	
	Equal variance assumed	17.714	62	
Post-test	Equal variance not assumed		42. 73	0.00

The Levene's test indicated that the assumption of homogeneity of variance of pre-test was met as p value is greater than 0.05$(F(1,62) = 6.571, p = 0.78)$. However, the variation of post-test scores for both the groups was not same as p-value is less than 0.05 ($(F(1,42.73) = 17.714, p = 0.00)$. Thus, the assumption of equal variances had been violated.

3.1 Comparison of Pre-test and post-test scores of Conceptual Understanding Test between the Groups

An independent sample t-test was conducted to determine the difference in conceptual test scores between the experimental and control group as shown in the table 3.3.

Table 3: Independent Sample t-test

Test	Group	Mean	Mean Difference	Standard Deviation	Sig. (2 tailed)	Effect Size
Pre-test	Control	7.58	0.391	6.10	0.778	0.006
	Experimental	7.59		5.96		
Post-test	Control	23.83	33.125	10.52	0.000	0.91
	Experimental	56.95		4.66		

An independent- samples t-test was conducted to compare the pre-test scores for EG and CG. There was so significant difference in scores for pre-test for Experimental (M= $7.59, SD = 5.96$) and Control group, M= $7.58, SD = 6.10; t(62) = -0.283,$ $p = 0.778$(two tailed). The magnitude of the differences in the means (MD= $0.391, 95\% CI$:-3.147 to 2.366) was very small (eta squared= 0.006). This means that the data were homogeneous and treatment could be applied to these groups to identify differences caused by the treatment.

An independent sample t-test conducted for comparison of post-test scores between experimental and control group showed significant mean difference between EG (M=56.95, SD =4.66) and CG (M $- 23.83, SD - 10.52); t(42.730) - 16.276, p - 0.000$ ((two tailed). The magnitude of the differences in the means (mean difference= $33.125, 95\%\ CI: 29.057\ to\ 37.193$)was very high (eta squared =0.91).This indicates that there was a statistically significant difference in post-test scores between experimental and control group. The test scores of an experimental group were significantly higher than the test scores of the control group.

4. Discussion and Conclusion

The results of the independent sample t-test indicated that there were significant differences in conceptual understanding on the topic differential calculus between the experimental and control group. Students who were taught with GeoGebra aided instructions outperformed those who were taught with conventional teaching. This is because GeoGebra aided instruction supported students' learning meaningfully and conceptually. Moreover, it had potential to visualize and concretize abstract nature of differential calculus. This finding is consonance with the study done by Ocal (2017) [14] that GeoGebra aided instructions on students' achievement regarding their conceptual and procedural knowledge on the application of derivative revealed positive effects on the students' performance regarding the conceptual knowledge.

Conceptual understanding in differential calculus can define as understanding the main concepts and big ideas underlying the concept of the derivative, namely the rate of change, the slope of tangent line or secant line, and the limit, and how these concepts are related to each other (Sahin *et al*., 2015). In contrasts, conventional teaching and learning of differential calculus focused more on manipulation of algebraic formulae, and operate the concept of differential calculus mechanically. Ryan (1992) [20] described as "the rush to rule" where relational understanding is ignored and paid lots of attention to an instrumental understanding. Students were hardly taught differential calculus using graphs and visualization tools. Thus, GeoGebra helped to enhance the conceptual understanding of differential calculus through dynamic visualization and graphic visualization. Finding of this study are in favor of the statements by Parrot and Eu (2009) [16] who claimed that Integration of technology in teaching and learning calculus enhances the conceptual understanding through visualization and graphic representation.

The other possible reasons for significant improvement in post-test scores were attributed to potential of GeoGebra to make the connection between symbolic, visual, numeric representations, concept image and concept definition of differential calculus. The study conducted by Huang (2015) in Taiwan for the first-year calculus students at university, revealed that the development of visualization ability of students, increases students' performance in solving the problems in the definite integral. The findings also indicated that students taught through multi-dimensional approaches, like the visualization of concepts graphically and algebraically, were able to connect the concept image and concept definition, leading to significant improvement in their learning. This is in line with my study where concepts were developed with computer-assisted graphing tools.

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