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The analysis on the specific properties of asymptotic normality of t distribution

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Abstract
The t distribution is asymptotically normal, which has been proved by the method of finding its limit value. In this paper, with the introduction of the difference function and the use of Matlab software, asymptotic normality of t distribution is verified more intuitive, and more specific properties of its asymptotic process are analyzed further.

Keywords: t distribution, asymptotic normality, matlab

1. Introduction

In probability statistics, t distribution function is a kind of important distribution function. This distribution has the property of asymptotic normality, that is, when the degree of freedom n tends to infinity, the t distribution function approaches the standard normal distribution. This conclusion has been proved by the method of finding the limit (Yang and Li, 1994 [1]; Yan *et al*., 2013 [2]; Li and Meng, 2004 [3]; Liang, 1995 [4]; Wang, 2007 [5]; Peng *et al*., 2012 [6]; Zhang, 1996) [7], but the detailed description of this process was not given. Therefore, what specific properties this approximation process has is not clear. This paper intends to introduce the difference function between the t distribution function and the standard normal distribution, and use Matlab to draw its function image to analyze the specific properties of its approximation process in detail.

2. Visual Verification of Asymptotic Normality of t Distribution Function

It has been proved that t distribution functions are asymptotically normal, but there is no intuitive description of this. So at first, we use Matlab to analyze its asymptotic normality by inputting the following code in Matlab and drawing the probability density function of standard normal distribution and t distribution of degrees of freedom from 1 to 100.

```
Clear all; clc;
x=-4:0.1:4;
n=linspace (1,100,100);
Axis ([-4 4 0 0.41]);
Ylabel ('$t (n) $','interpreter’,’ latex', 'FontSize', 18);
Xlabel ('x')
For i=1:100
A (i,:) = tpdf(x, n (i)); end
Plot(x, A);
Hold on; z=normpdf (x, 0, 1)
Plot(x, z,'color','r','linewidth', 2 .3);
Title ('t distribution of degrees of freedom from 1 to 100 and standard normal distribution');
Legend ('n ranges from 1 to100');
```

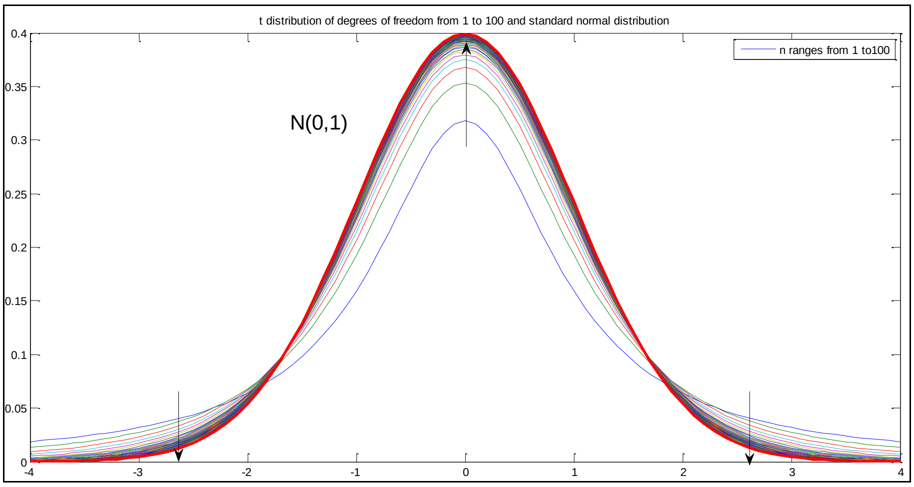

Fig 1: Asymptotic normality of t distribution

The results are shown in Figure 1. From the Figure 1, it could be seen that when the degree of freedom n of t distribution function increases, the probability distribution function rises near 0 and decreases on the other two sides. It approaches to the standard normal distribution function overall. Therefore, the t distribution function does have the property of asymptotic normality.

3. The Analysis on the Specific Properties by Using Difference Function

3.1. Basic Properties of Difference Function

If the value of random variable is x, the probability density function of standard normal distribution is $f_N(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, and the probability density function of t distribution with degrees of freedom n is $f_t(x,n)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$. In this way, the difference function expression between the standard normal distribution and the t distribution with degrees of freedom n is as follows:

$$\varphi(x,n)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}-\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$$

Let us input the following code to draw the function image when the parameter n changes from 1 to 100.

```
Clc; clear all;
x = -4:0.01:4;
n = linspace (1,100);
Axis ([-4 4 -0.2 0.41]);
Ylabel ('$t (n)-N (0, 1) $','interpreter','latex', 'FontSize', 18);
Xlabel ('x')
Z = normpdf(x, 0, 1)
For I = 1:100
A (i,:) = tpdf(x, n (i))-z; end
Plot (x, A);
Title ('Difference function with parameter from 1 to 100');
Legend ('parameter from 1 to 100');
```

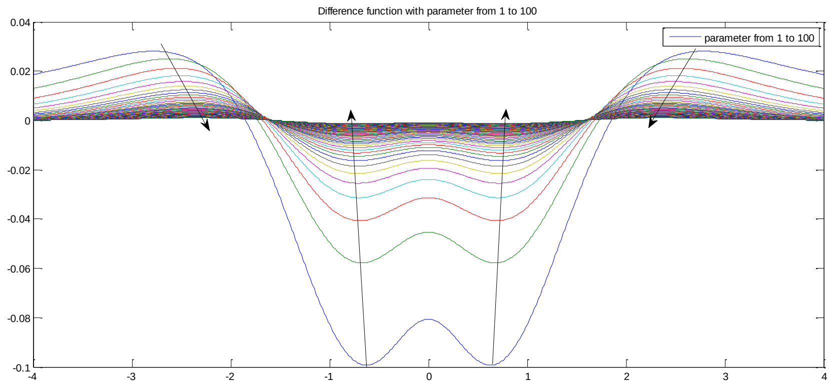

Figure 2 The images of difference function with parameter change

The results are shown in Figure 2. It can be seen from Figure 2 that when the parameter n is fixed, the difference function is symmetrical about y axis, which indicates that both the standard normal distribution function and the t distribution functions are symmetrical functions with y axis.

Additionally, when the parameter n is fixed, it is easy to see that a difference function curve has five extreme points. And because of its symmetry, one of the extreme points must be $x=0$, but it is not the maximum point. Therefore, although the maximum points of t distribution and standard normal distribution are obtained at $x=0$, the maximum value of their difference is not obtained at $x=0$, but at other extreme points.

When the parameter n increases from 1 to 100, the difference function image approaches X-axis gradually, the whole function becomes flat and the range of the function becomes smaller and smaller, which directly reflects that when n increases, the t-distribution tends to the standard normal distribution.

3.2. The Minimum and Maximum Value of Difference Function

To analyze the properties of the difference function further, we input the following code to find the minimum and maximum value of the difference function when the parameter n changes from 1 to 100.

```
clc;clear all;
MAX= []; MIN= [];
x=-4:0.01:4;
n=linspace (1,100);
Axis ([-4 4 -0.2 0.41]);
Ylabel ('$t (n)-N (0, 1) $','interpreter','latex', 'FontSize', 18);
Xlabel ('x')
z = normpdf (x, 0, 1)
For i=1:100
A (i,:) = tpdf(x, n(i))-z;
MAX (i) = max (A (i,:));
MIN (i) = min (A (i,:)); end
Vpa (MAX, 5)
Vpa (MIN, 5)
```

The results are

Table 1: When n increases from 1 to 100, the maximum value of the difference function is

0.028099	0.024998	0.021185	0.018149	0.015804	0.013969	0.012502	0.011309
0.010320	0.0094873	0.0087783	0.0081667		0.0076344	0.0071667	0.0067529
0.0063839	0.006053	0.0057548	0.0054844		0.0052381	0.0050129	0.0048064
0.0046162	0.0044404	0.0042774	0.0041260		0.0039849	0.0038531	0.0037297
0.0036141	0.0035054	0.003403	0.0033064		0.0032151	0.0031288	0.0030469
0.0029692	0.0028954	0.0028251	0.0027582		0.0026944	0.0026334	0.0025752
0.0025195	0.0024661	0.0024150	0.0023650		0.0023188	0.0022735	0.0022300
0.0021881	0.0021477	0.0021088	0.0020713		0.0020351	0.0020001	0.0019663

0.0019337	0.0019021	0.0018715	0.0018419	0.0018132	0.0017854	0.0017584
0.0017323	0.0017069	0.0016822	0.0016582	0.0016349	0.0016123	0.0015903
0.0015688	0.0015480	0.0015276	0.0015078	0.0014886	0.0014698	0.0014514
0.0014335	0.0014161	0.0013991	0.0013825	0.0013662	0.0013504	0.0013349
0.0013198	0.001305	0.0012905	0.0012763	0.0012625	0.001249	0.0012357
0.0012227	0.001210	0.0011976	0.0011854	0.0011734	0.0011617	0.0011502
0.001139						

Table 2: The minimum value of difference function is

0.099264		0.057795	0.040667	0.031345	0.025493	0.021479
0.018556	0.016332	0.014584	0.013174	0.012012	0.011039	0.010212
0.0094994	0.0088801	0.0083365	0.0078556	0.0074272	0.0070431	0.0066967
0.0063828	0.006097	0.0058357	0.0055959	0.0053751	0.005171	0.0049818
0.004806	0.0046422	0.0044892	0.0043459	0.0042115	0.0040852	0.0039662
0.003854	0.0037479	0.0036475	0.0035524	0.003462	0.0033762	0.0032945
0.0032167	0.0031425	0.0030716	0.0030038	0.002939	0.0028769	0.0028174
0.0027603	0.0027055	0.0026527	0.0026021	0.0025533	0.0025063	0.002461
0.0024173	0.0023751	0.0023344	0.0022951	0.002257	0.0022202	0.0021846
0.0021501	0.0021167	0.0020843	0.0020529	0.0020224	0.0019928	0.0019641
0.0019361	0.001909	0.0018826	0.0018569	0.0018319	0.0018076	0.0017839
0.0017609	0.0017384	0.0017165	0.0016951	0.0016743	0.0016539	0.0016341
0.0016147	0.0015958	0.0015773	0.0015592	0.0015416	0.0015243	0.0015075
0.001491	0.0014748	0.001459	0.0014435	0.0014284	0.0014136	0.001399
0.0013848	0.0013709	0.0013572				

From the data obtained, it can be seen that when the parameter of the difference function increases from 1 to 100, the maximum value of the difference function changes from 0.028099 to 0.001139, and the minimum value changes from -0.099264 to -0.0013572. Obviously, their absolute values are reduced greatly.

3.3. Visual Representation of the Maximum and Minimum Value of Difference Function

In order to make the change process of the maximum and minimum value of the difference function with n more intuitive, we input the following code and draw the image of the function when n increases from 1 to 100.

```
clc;clear all;
syms E F;
X1= []; X2= [];SUP=[];INF=[];
x= -4:0.001:4;
n=Linspace (1,100);
z=normpdf(x, 0, 1)
 For i=1:100
A (i,:) =tpdf(x, n (i))-z;
SUP (i) =max (A (i,:));
INF (i) =min (A (i,:));
E =find (A (i,:) =SUP (i))
X1(i) = (E (: 2)-4000)*0.01
F= find (A (i,:) =INF (i))
X2(i) = (F (:, 2)-4000)*0.01end
plot(n,SUP,'o-','color','r','linewidth',2);
Hold on;
Plot (n, INF,'s-','color','b','line width', 2);
Title ('the maximum and minimum value of the difference function with parameters from 1 to100');
Legend ('The maximum value',' the minimum value');
Xlabel ('n')
Grid on;
```

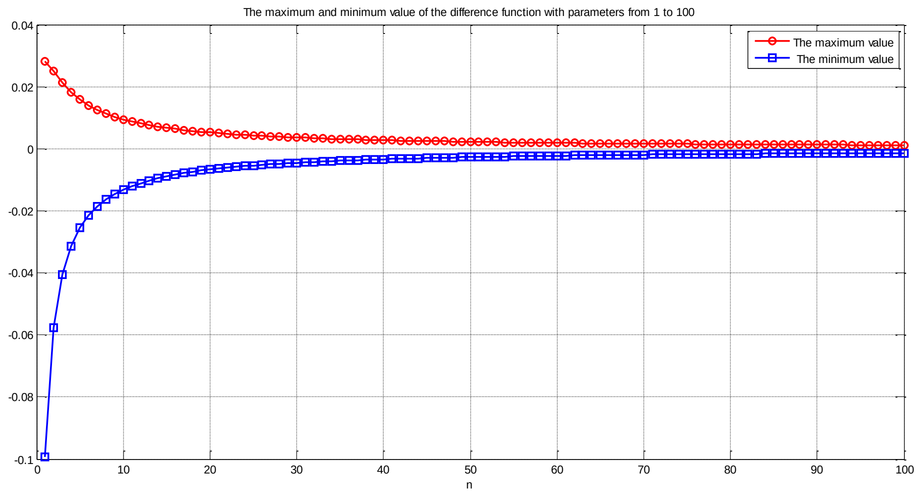

Fig 3: The maximum and minimum value of difference function with

The results are shown in Figure 3. It can be seen from Figure 3 that the maximum and minimum values of the difference function tend to 0 when parameter n increases. However, when n increases, the change speed slows down.

So it reflects that when the degree of freedom n increases, the approaching speed of t distribution to the standard normal distribution becomes slower.

3. Conclusions

1. When the degree of freedom n tends to infinity, the distribution of t tends to the standard normal distribution, and this approach is in a form that the probability t distribution function of rises near 0, and decreases on the other two sides;
2. When the degree of freedom n tends to infinity, the distribution of t tends to the standard normal distribution, but its approaching speed slows down gradually.
3. When the degree of freedom n tends to infinity, the difference between t distribution and standard normal distribution is different;
4. When the degree of freedom n tends to infinity, the approaching speed of t distribution to standard normal distribution at different points is not the same.

4. References

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