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Research on the solution of triple definite integral with monte Carlo method based on MATLAB software

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Abstract
Monte Carlo method is one which based on the thought of probability and statistics and usually to be used to find the numerical solution of qualitative problems in mathematics, which is of great value to solve the definite integral problem. Based on the previous research, this paper focuses on the solution using the Monte Carlo method to solve triple definite integral problems by using MATLAB statements and functions. In this process, we find a method to reduce the errors after the error analysis of the results. Therefore, some suggestions are addressed to apply the Monte Carlo method reasonably in solving triple definite integral problems.

Keywords: monte Carlo method, triple definite integral, MATLAB

1. Introduction

Monte Carlo (MC) method, also known as statistical test method or stochastic simulation method, is one which based on the thought of probability and statistics to solve the numerical solution of qualitative problems in mathematics [1]. Previous studies have verified that it can be applied to solve the problems of one and two definite integrals, and the results obtained have a good accuracy, which undoubtedly lays a good foundation for the further study of general definite integral problems [2, 4]. Meantime, the previous research also pointed out that this method could be used to solve the problems of triple integral too. However, so far, there are no relevant examples or corresponding result analysis [5, 6]. Obviously, this situation is not conducive to use this method in mathematical research to solve multiple integral problems. Therefore, this paper intends to explore the problem of applying the Monte Carlo method to solve triple definite integral problems.

2. The Solution and analysis

In previous research, a MATLAB algorithm applying the Monte Carlo method to solve double definite integrals was introduced as follows [7]:

```
function s=mtc (f, fai1, fai2, a, b, c, d, n)
if nargin<8 n=10 000; end
x=unifrnd (a, b, 1, n); y=unifrnd (c, d, 1, n);
s=0;
for k=1: n
if and(y(k)>=feval (fai1, x(k)), y(k)<=feval
(fai2, x(k)))
s=s+feval (f, x(k), y(k));
end
end
s=s/n*(b−a) *(d−c);
return
```

```
function s=mtcx (f, fai1, fai2, a, b, c, d, n)
if nargin<8 n=10 000; end
x=unifrnd (a, b, 1, n); y=unifrnd (c, d, 1, n);
s=sum (feval (f, x, y). *and(y>=feval (fai1, x), y<=
feval (fai2, x)))
s=s/n*(b−a) *(d−c);
return
```

Can this program be used to solve triple definite integrals? We constructed three common types of triple definite integral problems to explore.

Example.1 Solve the triple integral of

$$\int_0^1\int_2^3\int_4^5 xy^2z^3e^{-xyz}\,|\sin(x+y-z)|\,dxdydz$$

The characteristic of this triple definite integral is that the upper and lower limits of three variables are fixed constants, so the integral region is a rectangle.

For this triple definite integral, we modified the program above and input the following code into MATLAB:

```
function s=mtc4(g, a, b, c, d, e, f, n)
if nargin<8n=10000; end
x=unifrnd (a, b, 1, n);
y=unifrnd (c, d, 1, n);
z=unifrnd (e, f, 1, n);
s=0;
for k=1: n
s=s+feval (g, x(k), y(k), z(k));
end
s=s/n*(b-a) *(d-c) *(f-e)
return
```

```
clc; clear all;
tic
s1=mtc4(inline ('x.*y.^2. *z.^3. *exp(-x.*y.*z).
*abs(sin(x+y-z))'),0,1,2,3,4,5,10000)
toc
```

The result is
s1 = 4.0090
In order to check this result, we call the integral function in MATLAB and input the following code to calculate again:

```
tic;
fun = @ (x, y, z) x.*y.^2. *z.^3. *exp (-x.*y.*z). *abs (sin (x+y-z));
xmin = 0; xmax = 1;
ymin = 2; ymax = 3;
zmin = 4; zmax = 5;
q = integral3(fun, xmin, xmax, ymin, ymax, zmin, zmax, 'Method', 'tiled')
vpa(q)
toc
```

The result obtained is:
ans = 3.9659373360587735213300675241044

From these two values, it can be seen that the results calculated by two different methods are very similar.

Example.2 Solve the triple integral of $\int_0^1\int_{x^2}^{x}\int_{y^3}^{y^2} 1000xy^2z^3e^{-xyz}\,|\sin(x+y-z)|\,dxdydz$

The characteristic of this integral is that the upper and lower limits of the variable y are determined by x, and the upper and lower limits of z are determined by y.
For this integral, we input the following code into MATLAB for calculation:

```
function s=mtc3(g, fai1, fai2, fai3, fai4, a, b, n)
if nargin<8n=10000; end
x=unifrnd (a, b, 1, n);
c=min (feval (fai3, x)); d=max (feval (fai4, x));
y=unifrnd (c, d, 1, n);
e=min (feval (fai1, y)); f=max (feval (fai2, y));
z=unifrnd (e, f,1, n);
s=0;
for k=1: n
If z(k)>=feval (fai1, y(k)) &z(k)<=feval (fai2, y(k)) &y(k)>=feval (fai3, x(k)) & y(k)<=feval (fai4, x(k))
s=s+feval (g, x(k), y(k), z(k));
end
end
s=s/n*(b-a) *(d-c) *(f-e)
return

clc; clear all;
tic
s1=mtc3(inline('1000*x.*y.^2.*z.^3.*exp(-x.*y.*z).*abs(sin(x+y-z))'),inline('y.^3'),inline('y.^2'),inline('x.^2'),inline('x'),0,1,10000)
Toc
```

The obtained result is:
s1 = 0.230301948390407

This result is also similar to the result of calculating by calling the integral function in MATLAB, but there is a significant gap.

The result of calculating by calling integral function in MATLAB is as follows:
ans =0.18980385215351819327977977991395

Example.3 Solve the triple integral of $\int_0^1\int_{x^2}^{x}\int_{x^2+y^3}^{x+y^2} 1000xy^2z^3e^{-xyz}\,|\sin(x+y-z)|\,ds$

The characteristic of this integral is that the upper and lower limits of y are determined by x, and the upper and lower limits of z are determined by both x and y. Thus this integral is more complicated than the two above.
For this integral, we input the following code into MATLAB for calculation:

```
function s=mtc2(g, fai1, fai2, fai3, fai4, a, b, n)
if nargin<8 n=10000; end
x=unifrnd (a, b, 1, n);
c=min (feval (fai3, x)); d=max (feval (fai4, x));
y=unifrnd (c, d, 1, n);
e=min (feval (fai1, x, y)); f=max (feval (fai2, x, y));
z=unifrnd (e, f, 1, n);
s=0;
for k=1: n
If z(k)>=feval (fai1, x(k), y(k)) & z(k)<=feval (fai2, x (k), y(k)) &y(k)>=feval (fai3, x(k)) &y(k)<=feval (fai4, x(k))
s=s+feval(g,x(k),y(k),z(k));
end
end

clc; clear all;
tic
s1=mtc2(inline('1000*x.*y.^2.*z.^3.*exp(-x.*y.*z).*abs(sin(x+y-z))'),inline('x.^2+y.^3'),inline('x+y.^2'),inline('x.^2'),inline('x'),0,1,10000)
Toc
```

The obtained result is
s1 =1.256538582203898

This result is very similar to the result of calculating by using calling integral function in MATLAB. The result of calling integral function in MATLAB is:
ans =1.3061311231571459678946212079609

From the calculations of the three examples above, it is known that it is feasible to use Monte Carlo method to solve triple definite integrals, but there are some errors, and sometimes the errors are large.

3. Analysis of the errors

From the three examples above, it can be seen that when we use the Monte Carlo method to solve the triple definite integral problems, the errors always exist. So how do the errors generate? And how can the errors be reduced? To find the answer, we analyzed the above code.
In the process of solving the three problems above, we just let the program loop 10000 times. Is the number of loops too small? Then we modified the program to make it loop 30000 times, 50000 times and 100000 times respectively, and analyzed the values obtained. The results are as follows:

Table 1: The results of triple definite integral problems

Results of the first definite integral problem					
Loop times	MC results	Time/s	True value	Error	Error ratio
10000	4.009	2.809714	3.965937336	-0.043062664	0.01085813
30000	3.985942397	8.451509	3.965937336	-0.020005061	0.00504422
50000	3.958062831	12.751919	3.965937336	0.007874505	0.001985535
100000	3.970102669	25.048057	3.965937336	-0.004165333	0.001050277
Results of the second definite integral problem					
Loop times	MC results	Time/s	True value	Error	Error ratio
10000	0.230301948	2.962749	0.189803852	-0.040498096	0.213368147
30000	0.213450841	7.936696	0.189803852	-0.023646989	0.124586454
50000	0.202568895	13.743624	0.189803852	-0.012765043	0.067253865
100000	0.1792	26.357448	0.189803852	0.010603852	0.055867423
Results of the third definite integral problem					
Loop times	MC results	Time/s	True value	Error	Error ratio
10000	1.256538582	3.513604	1.306131123	0.049592541	0.037969037
30000	1.379284149	10.287431	1.306131123	-0.073153026	0.056007414
50000	1.248016575	16.928619	1.306131123	0.058114549	0.044493656
100000	1.264222709	33.459989	1.306131123	0.041908414	0.032085917

From the table 1, it can be seen that with the increasing of the number of cast points, the error between the triple integral value obtained by using the Monte Carlo method and the result calculated by calling the integral function in MATLAB becomes smaller and smaller, that is to say, its accuracy is improved.

However, as can be seen from the above table, when the number of cast points increases, the time of calculation by MATLAB software also increases gradually, and the increase rate is large.

4. Conclusions

From the calculation of the three triple definite integral problems above, it can be concluded that:

1. It is feasible to apply the Monte Carlo method to solve the problems of triple definite integral problems;
2. When the Monte Carlo method is applied to solve the triple definite integral problems, the errors can be gradually reduced by increasing the number of cast points and making the program loop for more times.
3. When the number of cast points increases, the calculating time of MATLAB software will multiply, so that it becomes very long.

Therefore, we think that the Monte Carlo method does is one method to solve multiple integrals, but it is absolutely not a very convenient and easy one. Therefore, we suggest that this method is better to be used to solve those complex multiple definite integral problems which are difficult to be solved by traditional methods. Additionally, to improve the calculating program and find a more convenient and fast method to solve multiple definite integrals is obviously a further research direction in the future.

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