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Non-differentiable multi objective fractional minimax programming problem under $[F, \alpha, \rho, d]$-convex functions

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Abstract
In this paper, we derive some theorems and duality theorems on non-differentiable Multiobject Fractional Minimax Programming Under $[F, \alpha, \rho, d]$ convex function.

Keywords: $[F, \alpha, \rho, d]$ convex function and Duality

1. Introduction

Minimax fractional problems appear in the formulation of discrete and continous rational approximation problems with respect to the Chebyshev Norm [3], in continuous rational games [12], in multiobjective programming [15], in engineering design as well as in some portfolio selection problems discussed by Bajona-Xandri and Martinez legaz [2].

Minimax mathematical programming has been of much interest in the recent past [1, 4, 5, 11, 13, 17, 18, 19]. Schmitendorf [11] established necessary and sufficient optimality conditions for minimax problem. Tanimoto [14] applied these optimality conditions to define a dual problem and derived duality theorms, which were extended for the fractional analogue of generalized minimax problem by Yadav and Mukherjee [18].

Motivated by various concepts of generalized convexity, Liang *et al.* [8, 9] introduced a unified formulation of generalized convexity, which was called $[F, \alpha, \rho, d]$- convexity and obtained some corresponding optimality conditions and duality results for the single objective practional problems and multiobjective problems. Recently, Liang and Shi [10] obtained sufficient conditions and duality theorems for minimax fractional problem under $[F, \alpha, \rho, d]$-convexity. Lia *et al.* [7] derived necessary and sufficient conditions for nondifferentiable minimax fractional problem with generalized convexity and applied these optimality conditions to construct one parametric dual model and also discussed duality theorems, Lai and Lee [6] obtained duality theorems for two parametric-free dual models of non-differentiable minimax fractional problem involving generalized convexity assumptions. Recently, Mishra *et al.* [12] established duality results for one parametric and two parametric-free dual models of nondifferentiable minimax fractional programming problem under generalized univexity.

But no serious attempt in utilizing multiobjective fractional minimiax under $[F, \alpha, \rho, d]$-convexity. Hence in this Paper an attempt is made to fill the gap in the aim of research by developing some theorems based on $[F, \alpha, \rho, d]$ convex function.

2. Formulation

We now consider the following non-differentiable minimax fractional programming problem.

2.1. Primal Problem

$$(FP)\ \frac{\min}{x \in R^n} \sup_{y \in Y} \frac{f_i(x, y) + (x^t, Bx)^{\frac{1}{2}}}{h_i(x, y) - (x^t, Dx)^{\frac{1}{2}}},$$

$$\text{Subject to } g_j(x) \le 0,\ x \in X,$$

Where Y is a compact subset of $R^m, f_i, h_i : R^n \times R^m \to R$ are C^1 on $R^n \times R^m$ and $g_j : R^n \to R^p$ is C^1 on R^n. B and D are nxn positive semidefinite matrices.

Let $S = \{x \in X : g(x) \le 0\}$ denote the set of all feasible solutions of (FP). For each $(x, y) \in R^n \times R^m$, we define

$$\phi(x, y) = \frac{f_i\,(x, y) + (x^t, Bx)^{\frac{1}{2}}}{h_i\,(x, y) - (x^t, Dx)^{\frac{1}{2}}} \ ,$$

Such that for each $(x, y) \in s \times Y, f_i\,(x, y) + (x^t, Bx)^{\frac{1}{2}} \geq 0$ and $h_i\,(x, y) - (x^t, Dx)^{\frac{1}{2}} > 0$. For each $x \in s,$ we define

$$J(x) = \left\{ j \in J \quad g_j\,(x) = 0 \right\}. \text{ where } J = \{1, 2, ... p\}$$

$$Y(x) = \left\{ y \in Y : \frac{f_i\,(x, y) + (x^t, Bx)^{\frac{1}{2}}}{h_i\,(x, y) - (x^t, Dx)^{\frac{1}{2}}} = \sup_{z \in Y} \frac{f_i\,(x, z) + (x^t, Bx)^{\frac{1}{2}}}{h_i\,(x, z) - (x^t, Dx)^{\frac{1}{2}}} \right\},$$

$$k(x) = \left(s, t_i, \bar{y}\right) \in N \times R_+^s \times R^{ms} : 1 \leq s \leq n+1, t = (t_1, t_2 ..... t_s) \in R_+^s$$

$$\text{with } \sum_{i=1}^{s} t_i = 1, \ \bar{y} = (\bar{y}_1, \bar{y}_2 ..... \bar{y}_s) \text{ with } \bar{y}_i \in Y(x)\ (i = 1, 2, ... s)$$

Since f and h are continuously differentiable and Y is compact in R^m, it follows that for each $x^* \in s,\ y(x^*) \neq \phi$, and for any $\bar{y}_i \in Y(x^*)$, we have a positive constant

$$k_0 = \phi(x^*, \bar{y}_i) = \frac{f_i\,(x^*, \bar{y}_i) + (x^{*t}, Bx^*)^{\frac{1}{2}}}{h_i\,(x^*, \bar{y}_i) - (x^{*t}, Dx^*)^{\frac{1}{2}}}$$

We shall need the following generalized Schwartz inequality.

Let B be a positive semidefinite matrix of order n. Then for all $x, w \in R^n$,

$$x^t\, Bw \leq (x^t\, Bx)^{\frac{1}{2}}\, (w^t\, Bw)^{\frac{1}{2}} \qquad (21.1.1)$$

We observe that equality holds if $Bx = \lambda Bw$ for some $\lambda \geq 0$. Evidently, if $(w^t\ Bw)^{\frac{1}{2}} \leq 1,$ we have $x^t\ Bw \leq \left(x^t\ Bw\right)^{\frac{1}{2}}$.

If the functions f_i, g_i and h_i in problem (FP) are continuously differentiable with respect to $x \in R^n$.

2.2. Dual Formulation

$$\text{(D)} \qquad \max_{(s, t^*, \bar{y}) \in K(z)} \ \sup_{(z, \mu, v, w) \in H(s, t_i^*, \bar{y})} \frac{\sum_{i=1}^{s} t_i^* \left\{ f_i\,(z, \bar{y}_i) + z^t Bw \right\} + \sum_{j=1}^{p} \mu_j g_j\,(z)}{\sum_{i=1}^{s} t_i^* \left\{ h_i\,(z, \bar{y}_i) - z^t DV \right\}}$$

Where $H(s, t_i^*, \bar{y})$ denote the set of all $(z, \mu, v, w) \in R^n \times R_+^p \times R^n \times R^n$

Satisfying

$$\nabla \left(\frac{\sum_{i=1}^{s} t_i^* \left\{ f_i\,(z, \bar{y}_i) + z^t Bw \right\} + \sum_{j=1}^{p} \mu_j g_j\,(z)}{\sum_{i=1}^{s} t_i^* \left\{ h_i\,(z, \bar{y}_i) - z^t DV \right\}} \right) = 0$$

$$\begin{cases} w^t\ Bw \le 1, & (z^t\ Bz)^{\frac{1}{2}} = z^t\ Bw, \\ v^t\ Dv \le 1, & (z^t\ Dz)^{\frac{1}{2}} = z^t\ Dv \end{cases}$$

If the set $H(s, t^*, \bar{y})$ is empty, we define supremum over it to be $-\infty$. For convenience, we use the notation:

$$\psi(\cdot) = \left[\sum_{i=1}^{s} t_i^* \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}\right]\left[\sum_{i=1}^{s} t_i^* \left\{f_i(\cdot, \bar{Y}_i) + (\cdot)^t, Bw\right\} + \sum_{j=1}^{p} \mu_j g_j(\cdot)\right]$$

$$-\left[\sum_{i=1}^{s} t_i^* \left\{f_i(z, \bar{y}_i) + z^t, Bw)\right\} + \sum_{j=1}^{p} \mu_j g_j(z)\right]\left[\sum_{i=1}^{s} t_i^* \left\{h_i(\cdot, \bar{Y}_i) - (\cdot)^t Dv\right\}\right]$$

Suppose that

$\sum_{i=1}^{s} t_i^* \left\{f_i(z, \bar{y}_i) + z^t Bw\right\} + \sum_{j=1}^{p} \mu_j g_j(z) \ge 0$ and $\sum_{i=1}^{s} t_i^* \left\{h_i(z, \bar{y}_i) - z^t Dv\right\} > 0,$ For all $(s, t^*, \bar{y}) \in k(z)$, $(z, \mu, w, v) \in H(s, t^*, \bar{y})$.

3. Definition

Let R^n be the n-dimensional Euclidean space and X and open set in R^n.

Definition (3.1): A functional $F: X \times X \times R^n \to R$ is said to be sublinear if $\forall x, \bar{x} \in X$

(i) $F(x, \bar{x}; a_1 + a_2) \le F(x, \bar{x}; a_1) + F(x, \bar{x}; a_2)\ \forall\ a_1, a_2 \in R^n$

(ii) $F(x, \bar{x}; \beta a) = \beta F(x, \bar{x}; a)\ \forall \beta \in R_+$ And $\forall a \in R^n$.

By (ii) it is clear that $F(x, \bar{x}, 0) = 0$.

Definition (3.2)

[8, 9] given an open set $x \subset R^n$, a number $\rho \in R$, and two functions $\alpha: X \times X \times \to R_+ \setminus \{0\}$ and $d(\cdot,\cdot) X \times X \to R,$ a differentiable function ζ over x is said to be (F, α, ρ, d) - convex at $\bar{x}$, if for any $x \in X$, $F: X \times X \times \to R^n \to R$ is sublinear, and ζ (x) satisfies the following condition.

$$\zeta(x) - \zeta(\bar{x}) \ge F\left(x, \bar{x}; \alpha(x, \bar{x}) \nabla \zeta(\bar{x})\right) + \rho d^2(x, \bar{x})$$

Definition (3.3)

Given an open set $x \subset R^n$, a number $\rho \in R$, and two functions $\alpha: X \times X \times \to R_+ \setminus \{0\}$ and $d(\cdot,\cdot) X \times X \to R,$ a differentiable function ζ over x is said to be (F, α, ρ, d) -Pseudo convex at $\bar{x}$, if for any $x \in X$, there exists a sublinear functional $F: X \times X \times R^n \to R$ such that $\mu(x) < \mu(\bar{x}) \Rightarrow F\left(x, \bar{x}; \alpha(x, \bar{x}) \nabla G(\bar{x})\right) - \rho^2 d^2(x, \bar{x})$

Further μ is said to be strictly $F(\alpha, \rho, d)$-Pseudo convext at $\bar{x}$ if for any $x \in X$, there exists a sublinear functional $FiX \times X \times R^n \to R$

Such that

$F\left(x, \bar{x}; \alpha(x, \bar{x}) \nabla \zeta(\bar{x})\right) \ge -\rho d^2(x, \bar{x}) \Rightarrow \zeta(x) > \zeta(\bar{x})$.

4. Necessary optimality condition

If x^* is a solution of problem (FP). Assuming $z_{\bar{y}}(x^*)$ to be empty, there exist $(s, t^*, \bar{y}) \in k(x^*), w, v \in R^n$ and $\mu^* \in R_+^p$ satisfying

$$\nabla\left(\frac{\sum_{i=1}^{s} t_i^*\left\{f(x^*,\bar{y}_i)+x^{*t}Bw\right\}+\sum_{j=1}^{p}\mu_j^* g_j(x^*)}{\sum_{i=1}^{s} t_i^*\left\{h(x^*,\bar{y}_i)-x^{*t}DV\right\}}\right)=0$$

$$\sum_{j=1}^{p}\mu_j^* g_j(x^*)=0$$

$$t_i^*\in R_+^s\ (i=1,2,.....s),\quad \sum_{i=1}^{s} t_i^*=1$$

$$w^t\ Bw\le 1,\quad (x^{*t}\ Bx^*)^{\frac{1}{2}}=x^{*t}\ Bw,$$

$$v^t\ Dv\le 1,\quad (x^{*t}\ Dx^*)^{\frac{1}{2}}=x^{*t}\ Dv$$

5. Duality Theorems

5.1. Theorem (Weak Duality): Suppose that x and $(z,\mu,v,w,s,t,\bar{y})$ are respectively the feasible solution for (FP) and (D). Also assume that $\psi(\cdot)$ is $[F,\alpha,\rho,d]$-Pseudo Convex at z and the inequality $\frac{\rho}{\alpha(x,z)}\ge 0$.
Holds, then

$$\sup_{y\in Y}\frac{f_i\ (x,y)+(x^t,Bx)^{\frac{1}{2}}}{h_i\ (x,y_i)-(x^*,Dx)^{\frac{1}{2}}}\ \ge\ \frac{\sum_{i=1}^{s} t_i^*\left\{f_i\ (z,\bar{y}_i)+z^tBw\right\}+\sum_{j=1}^{p}\mu_j g_j(z)}{\sum_{i=1}^{s} t_i^*\left\{h_i\ (z,\bar{y}_i)-z^tDV\right\}}$$

Proof: By means of a contradiction, suppose that

$$\sup_{y\in Y}\ \frac{f_i\ (x,y)+(x^t\ Bx)^{\frac{1}{2}}}{h_i\ (x,y)-(x^t\ Dx)^{\frac{1}{2}}}<\frac{\sum_{i=1}^{s} t_i\ \left\{f_i\ (z,\bar{y}_i)+z^tBw\right\}+\sum_{j=1}^{p}\mu_j g_j(z)}{\sum_{i=1}^{s} t_i\ \left\{h_i\ (z,\bar{y}_i)-z^tDv\right\}}$$

For $y\in Y$. If we replace y by $\bar{y}_i$ in the above in equality and sum up after multiplying by t_i, then we have

$$\left[\sum_{i=1}^{s} t_i\left\{f_i\ (x,\bar{y}_i)+(x^t\ Bx)^{\frac{1}{2}}\right\}\right]\left[\sum_{i=1}^{s} t_i\ \left\{h_i\ (z,\bar{y}_i)-z^tDv\right\}\right]$$

$$<\left[\sum_{i=1}^{s} t_i\ \left\{f_i\ (z,\bar{y}_i)+z^tBw\right\}+\sum_{j=1}^{p}\mu_j g_i\ (z)\right]\left[\sum_{i=1}^{s} t_i\ \left\{h_i\ (x,\bar{y}_i)-(x^t\ Dx)^{\frac{1}{2}}\right\}\right]$$

Using the generalized Schwartz inequality and (2.2.2) we get

$$\phi(x)\le\left[\sum_{i=1}^{s} t_i\ \left\{h_i\ (z,\bar{y}_i)-z^tDv\right\}\right]\left[\sum_{i=1}^{s} t_i\ \left\{f_i\ (x,\bar{y}_i)+(x^t\ Bx)^{\frac{1}{2}}+\sum_{j=1}^{p}\mu_j g_j(x)\right\}\right]$$

$$-\left[\sum_{i=1}^{s} t_i\ \left\{f_i\ (z,\bar{y}_i)+z^tBw\right\}+\sum_{j=1}^{p}\mu_j g_j(x)\right]\left[\sum_{i=1}^{s} t_i\ \left\{h_i\ (x,\bar{y}_i)-(x^t\ Dx)^{\frac{1}{2}}\right\}\right]$$

$$<\sum_{j=1}^{p}\mu_j g_j(x)\sum_{i=1}^{s} t_i\ \left\{h_i\ (z,\bar{y}_i)-z^tDv\right\}$$

Since $\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\} > 0 \ \sum_{j=1}^{p} \mu_j g_j(x) \le 0,$

It follows that $\psi(x) < 0 = \psi(z)$

As ψ (.) is (F, α, ρ, d) - Pseudo Convex at z. Therefore $F\left(x, z; \alpha(x, z)\nabla\psi(z)\right) < -\rho d^2(x, z),$ that is

$$F(x, z; \alpha(x, z)\left[\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}\right]\nabla\left[\sum_{i=1}^{s} t_i \left\{f_i(z, \bar{y}_i) + z^t Bw + \sum_{j=1}^{p} \mu_j g_j(z)\right\}\right]$$

$$-\left[\sum_{i=1}^{s} t_i \left\{f_i(z, \bar{y}_i) \times z^t Bw\right\} + \sum_{j=1}^{p} \mu_j g_j(z)\right]\nabla\left[\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}\right] < -\rho d^2(x, z)$$

On multiplying the above inequality by

$\dfrac{1}{\alpha(x, z)\left[\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}\right]^2}$ and using the sublinearity of F, we have

$$F(x, z; \nabla\left(\frac{\sum_{i=1}^{s} t_i \left\{f_i(z, \bar{y}_i) + z^t Bw\right\} + \sum_{j=1}^{p} \mu_j g_j(z)}{\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}}\right) < \frac{-\rho d^2(x, z)}{\alpha(x, z)\left[\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}\right]^2}$$

Using the fact that $\dfrac{\rho}{\alpha(x, z)} \ge 0,$

We have

$$F\left(x, z; \nabla\left(\frac{\sum_{i=1}^{s} t_i \left\{f_i(z, \bar{y}_i) + z^t Bw\right\} + \sum_{j=1}^{p} \mu_j g_j(z)}{\sum_{i=1}^{s} t_i \left\{h_i(z, \bar{y}_i) - z^t Dv\right\}}\right)\right) < 0 \qquad (5.1.1)$$

In the right of (2.2.1) in equality (5.1.1) contradicts F(x, z; 0) =0.

Theorem (5.2) (Strong Duality)

Suppose that $\bar{x}$ is optimal for (FP) and $\nabla g_j(\bar{x}), j \in J(\bar{x})$ is linearly independent. Then there exist $(\bar{s}, \bar{t}, \bar{y}^*) \in k(\bar{x})$ and $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}) \in H(\bar{s}, \bar{t}, \bar{y}^*)$ such that $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ is feasible for (D).

Further, if the weak duality (theorem 5.1) holds for all feasible $(z, \mu, v, w, s, t, \bar{y})$ of (D), then $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ is optimal for (D) and the two objectives have the same extreme values.

Proof: Since $\bar{x}$ is optimal for (FP) and $\nabla g_j(\bar{x}), j \in J(\bar{x})$ is linearly independent. Then by necessary condition there exist $(\bar{s}, \bar{t}, \bar{y}^*) \in k(\bar{x})$ and $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}) \in H(\bar{s}, \bar{t}, \bar{y}^*)$ such that $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ is feasible for (D) and the two objective values are equal. The optimality of $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ for (FP) thus follows from weak duality (Theorem 5.1).

Theorem (5.3) (Strict Converse Duality)

Let $\bar{x}$ and $(\bar{z}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ be optimal solutions for (FP) and (D) respectively. Also suppose that ψ (.) is strictly $[F, \alpha, \rho, d]$-Pseudo Convex at $\sum$, for all $(\bar{s}, \bar{t}, \bar{y}^*) \in k(\bar{x})$, $(\bar{z}, \bar{\mu}, \bar{w}, \bar{v}) \in H(\bar{s}, \bar{t}, \bar{y}^*)$, and the inequality $\frac{\rho}{\alpha(x, z)} \ge 0$ holds, and $\nabla g_j(\bar{x}), j \in J(\bar{x})$ is linearly independent. Then $\bar{z} = \bar{x};$ that is, $\bar{z}$ is optimal for (FP).

Proof: We shall assume that $\bar{z} \neq \bar{x};$ and exhibit a contradiction. Since $(\bar{z}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ is feasible for (D), it follows that

$$\nabla\left(\frac{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w}\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})}{\sum_{i=1}^{s} \bar{t}_i \left\{h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v}\right\}}\right) = 0$$

The above inequality along with the sublinearity of F and $\frac{\rho}{\alpha(\bar{x}, \bar{z})} \geq 0$

$$F(\bar{x}, \bar{z}; \nabla\left[\frac{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w}\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})}{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v}\right\}}\right] = 0 \geq \frac{-\rho d^2(\bar{x}, \bar{z})}{\alpha(\bar{x}, \bar{z})}$$

Which together with the sublinearity of F and $\alpha(\bar{x}, \bar{z}) > 0$ yields

$$F\left(\bar{x}, \bar{z}; \alpha(\bar{x}, \bar{z})\right) \nabla \left[\frac{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w}\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})}{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v}\right\}}\right] \geq -\rho d^2(\bar{x}, \bar{z})$$

Using strict $[F, \alpha, \rho, d]$ - Pseudo convexity of ψ (.), we obtain $\psi(\bar{x}) > \psi(\bar{z})$

Since $\psi(\bar{z})$ =0, then we have $\psi(\bar{x}) > 0,$ that is

$$\left[\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v}\right\}\right] \left[\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{x}, \bar{y}_i^*) + (\bar{x}^t B\bar{w})\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{x})\right]$$

$$> \left[\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w}\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})\right] \left[\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + (\bar{z}^t B\bar{w})\right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})\right]$$

$$\left[\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{h_i(\bar{x}, \bar{y}_i^*) - (\bar{x}^t D\bar{v})\right\}\right] \qquad (5.3.1)$$

The relations (2.1), (2.2.2), (5.3.1) and

$\sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{x}) \leq 0$ Imply

$$\sup_{y \in Y} \frac{f_i(\bar{x}, y) + (\bar{x}^t \bar{B}x)^{\frac{1}{2}}}{h_i(\bar{x}, y) - (\bar{x}^t D\bar{x})^{\frac{1}{2}}} > \frac{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w}\right\} + \sum_{j=1}^{p} \mu_j g_j(\bar{z})}{\sum_{i=1}^{s} \bar{t}_i \left\{h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v}\right\}} \qquad (5.3.2)$$

Since $\bar{x}$ is optimal for (FP) and $g_j(\bar{x}),\ j \in J(\bar{x})$ is linearly independent, by strong duality (Theorem 5.2), there exist $(\bar{s}, \bar{t}, \bar{y}^*) \in k(\bar{x})$ and $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}) \in H(\bar{s}, \bar{t}, \bar{y}^*)$ such that $(\bar{x}, \bar{\mu}, \bar{v}, \bar{w}, \bar{s}, \bar{t}, \bar{y}^*)$ turns to be an optimal solution of (D) and

$$\sup_{y \in Y} \frac{f_i(\bar{x}, y) + (\bar{x}^t \bar{B} x)^{\frac{1}{2}}}{h_i(\bar{x}, y) - (\bar{x}^t D\bar{x})^{\frac{1}{2}}} = \frac{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{ f_i(\bar{z}, \bar{y}_i^*) + \bar{z}^t B\bar{w} \right\} + \sum_{j=1}^{p} \bar{\mu}_j g_j(\bar{z})}{\sum_{i=1}^{\bar{s}} \bar{t}_i \left\{ h_i(\bar{z}, \bar{y}_i^*) - \bar{z}^t D\bar{v} \right\}}$$

Which contradicts the fact of (5.3.2). Hence $\bar{x} = \bar{z}$.

References

1. Ahmad I. Optimality conditions and duality in-fractional minimax programming involving generalized ρ-invexity, Inter. j manag. sys. 2003; 19:165-180.
2. Bajona–Xandri C, Martinez–Legaz JE. Lower sub differentiability in minimax fractional programming, optimization, 1998.
3. BarrodFale I. Best rational approximation and strict quasi convexity, SIAM J NUMER. Anal. Loc, 1973, 8-12.
4. Bector CR, Bhatia BL. Sufficient optimality condition and duality for minimax problems. Utilitas Mathematical. 1985; 27:229-247.
5. Chandra S, Kumar V. Duality in fractional minimax programming. J Aust. Math. Soc. Ser. A. 1995; 58:376-386.
6. Lai HC, Lee JC. On duality theorems for a nondifferentiable minimax fractional programming. Journal of computational and applied mathematics. 2002; 146:115-126.
7. Lai HC, Lee JC, Tanaka K. Necessary and sufficient condition for minimax fractional programming. Journal mathematical Analysis and Appications. 1999; 230(2):311-328.
8. Liang ZA, Shi ZW. Optimality conditions and duality for a minimax fractional programming with generalized convexity, J math. Anal. Appl. 2003; 227:474-488.
9. Liang ZA, Huang HX, Pardalas PM. Efficiency conditions and duality for a class of multi-objective programming problems J. Global optim. 2003; 27:1-25.
10. Liang ZA, Shi ZW. Optimality conditions and duality for a minimax fractional programming with generalized convexity, J. math. Anal. Appl. 2003; 227:474-488.
11. Schmitendorf WE. Necessary conditions and sufficient conditions for static minimax problems, J. Math. Anal. Appl. 1977; 57:683-693.
12. Soleimani-damanesh M. Infinite (Semi-infinite) problems to characterize the optimality of non-linear optimization problems. Eur. J Oper. Res. 2008; 188:49-56.
13. Stancu-Minasian IM. Fractional programming: Theory methods and applications, Kluwer, Dordrecht, 1997.
14. Tanimoto S. Duality for a class of non-differentiable mathematical programming problems. J. Math. Anal. Appl. 1981; 79:283-294.
15. Varalakshmi G, Reddy PRS. Multi-objective fractional minimax problem involving locally lipschitz functions v–invexty. International conference on statistical science, OR & IT, Tirupati; OR, 2007, 47.
16. Yadav SR, Mukhrjee RN. Duality for fractional minimax programming problems. J. Aust. Math. Soc. Ser. B. 1990; 31:484-492.

Zhang J, Mond B. Duality for a nondifferentiable programming problem. Bull. Austral. Math. Soc. 1997; 55:29-4.