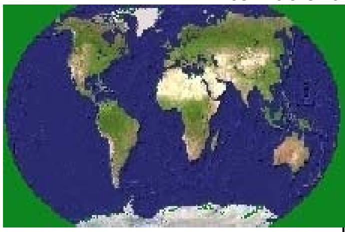
International Journal of Multidisciplinary Research and Development 2014; 1(2): 121-122



International Journal of Multidisciplinary Research and Development

IJMRD 2014; 1(2): 121-122
www.allsubjectjournal.com
Received: 12-07-2014
Accepted: 27-07-2014
e-ISSN: 2349-4182
p-ISSN: 2349-5979

Manoj Sharma
Department of Mathematics RJIT, BSF Academy, Tekanpur,

Correspondence:
Manoj Sharma
Department of Mathematics RJIT, BSF Academy, Tekanpur,

Fractional q-calculus and generalized mittag-leffler function

Manoj Sharma

Abstract
This paper is devoted to fractional q-derivative of special functions. To begin with the theorem on term by term q-fractional differentiation has been derived. fractional q-differentiation of Generalized Mittag − Leffler function has been obtained.

Keywords: Fractional integral and derivative operators, Fractional q-derivative, Generalized Mittag-Leffler function and Special functions.
Mathematics Subject Classification:- Primary33A30, Secondary 33A25, 83C99

1. Introduction:

Definition:

1.1. q-Analogue of Differential Operator

Al-Salam [3], has given the q-analogue of differential operator as

$$D_q f(x) = \frac{f(xq) - f(x)}{x(q-1)} \tag{1.1}$$

This is an inverse of the q-integral operator defined as

$$\int_x^{\infty} f(t)\, d(t:q) = x(1-q) \sum_{k=1}^{\infty} q^{-k} f(xq^{-k}) \tag{1.2}$$

Where $0 < |q| < 1$

1.2. Fractional q-Derivative of Order α:

The fractional q-derivative of order α is defined as

$$D_{x,q}^{\alpha} f(x) = \frac{1}{\Gamma_q(-\alpha)} \int_0^x (x - yq)_{-\alpha-1} f(y) d(y;q) \tag{1.2.1}$$

Where Re $(\alpha) < 0$

As a particular case of (1.2.1), we have

$$D_{x,q}^{\alpha} x^{\mu-1} = \frac{\Gamma_q(\mu)}{\Gamma_q(\mu-\alpha)} x^{\mu-\alpha-1} \tag{1.2.2}$$

1. 3 Generalized Mittag-Leffler function

Here , first the notation and the definition of the Generalized Mittag-Leffler function, introduced by Ahmad Faraj , Tariq Salim [13] has been given as

$$E_{\alpha,\beta,n}^{a_1,b_1,m}(z) = \sum_{k=0}^{\infty} \frac{(a_1)_{km}}{(b_1)_{kn}} \frac{z^k}{\Gamma(\alpha k + \beta)} \tag{1.3}$$

Here $\alpha, \beta \in C, \mathrm{Re}(\alpha) > 0,\ \mathrm{Re}(\beta) > 0$,$(a_1)_{km}$, $(b_1)_{kn}$ are the pochammer symbols and m,n are non-negative real numbers.

2. Main Results

In this section , we drive the results on term by term q-fractional differentiation of a power series. As particular case we will the fractional q-differentiation of generalized mittag-leffler function

Theorem 1: If the series $E^{a_1,b_1,m}_{\alpha,\beta,n}(z)$ converges absolutely for $|q| < \rho$ then

$$D^{\mu}_{z,q}\left\{ z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}\right\} = \sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{1}{\Gamma(\alpha k+\beta)} D^{\mu}_{z,q} z^{k+\lambda-1} \qquad (2.1)$$

Where Re $(\lambda) > 0$, Re $(\mu) < 0$, $0< |q| < 1$

Proof: Starting From the left side and using equation (1.2.1), we have

$$D^{\mu}_{z,q}\left\{ z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}\right\}$$

$$=\frac{1}{\Gamma_q(-\mu)}\int_0^z (z-yq)_{-\mu-1}\, y^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}\, d(y;q)$$

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_q(-\mu)}\int_0^1 (1-tq)_{-\mu-1}\, z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}\, d(t;q) \qquad (2.2)$$

Now the following observation are made

(i) $\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}$ converges absolutely and therefore uniformly on domain of x over the region of integration.

(ii) $\int_0^1 \left|(1-tq)_{-\mu-1}t^{\lambda-1}\right|\, d(t;q)$ is convergent,

Provided Re $(\lambda) > 0$, Re $(\mu) < 0$, $0< |q| < 1$

Therefore the order of integration and summation can be interchanged in (2.2) to obtain.

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_q(-\mu)}\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{z^k}{\Gamma(\alpha k+\beta)}\int_0^1 (1-tq)_{-\mu-1}\, t^{\lambda+k-1} d(t;q)$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_{km}}{(b_1)_{kn}}\frac{}{\Gamma(\alpha k+\beta)} D^{\mu}_{z,q} z^{k+\lambda-1}$$

Hence the statement (2.1) is proved.

References

1. Agarwal, R.P.: Fractional q-derivatives and q-integrals and certain hypergeometric transformations,
2. Ganita 27 (1976), 25-32.
3. Agarwal, R.P.: "Resonance of Ramanujan's Mathematics, 1", New Age International Pvt. Ltd. (1996), New Delhi.
4. Al-Salam, W.A.: Some fractional q-integral and q-derivatives, Proc. Edin. Math. Soc. 15 (1966), 135-140.
5. Exton, H.: q-hypergeometric functions and applications, Ellis Horwood Ltd. Halsted Press, John Wiley and Sons, (1990), New York
6. Gasper, G. and Rahman, M.: Basic Hypergeometric Series, Cambridge University Press, (1990), ambridge.
7. Manocha, H.L. and Sharma, B. L.: Fractional derivatives and summation, J. Indian Math. Soc. 38 (1974), 371-382.
8. Rainville, E.D.: Special Functions, Chelsea Publishing Company, Bronx, (1960), New York.
9. Salim, T, and Faraj, A ., A generalization of Mittag – Leffler function and integral operator associated with fractional calculus. J. Frac. Cal. appl. Vol.3, No.5, 1-13 (2012).
10. Yadav, R. K. and Purohit, S. D.: Fractional q-derivatives and certain basic hypergeometric transformations. South East Asian, J. Math.and Math. Sc. Vol. 2 No. 2 (2004), 37-46.