



## Game theory and its applications

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### Abstract

In the fast moving increasingly automated competitive world where Games are the aspect of life in various Disciplines namely Mathematics, Economics, International Relations and Politics, Commerce, social science, Operational Research, computer science etc. In this Paper An attempt has been made to study the game theory and the implications of game theory in modern context of society regarding real life situations which is based on two approaches i.e. Problem solving and decision making.

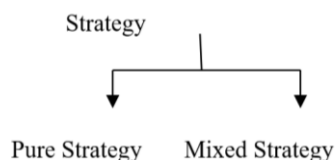
**Keywords:** competitors, rational, conflicts

### Introduction

The main foundation of research is based on two things one is innovation and other is Invention. In the modern era of competition where technology play a dominant role in the real world. Here, is the Operational research which is an art of giving bad answers to problems which otherwise have worse answers. Game theory is one of the best technique in operational research which are depend on two or more opposing parties with conflicting interest. As a Mathematical tool for the decision maker the strength of game theory is the methodology which provides for structuring and analyzing problems of strategic Choices. A competitive situation will be called a game which has following properties such as there are finite no of Participants called players with finite no of strategies available to him, Every game results is an outcome and a play of game takes place when each player employs his strategy. Therefore the main aim of this research is to look at game theory with more emphasis on the zero-sum games, dominance, mixed strategies, Nash equilibrium, and lastly on the bidding in auctions.

### Basic terminologies Used in game theory

➤ **Player:** The competitors in the game are known as players



- If the players select the same strategy each time, then it is known as pure strategy.
- If the Players use a combination of strategies and each player always guess as to which course of action is to be selected by other player then this is known as mixed

### Strategy

- **Optimum Strategy:** A course of play which puts the player in the most preferred Position is called an optimum strategy.
- **Value of the Game:** It is expected payoff of play When

all the players an optimum strategies. If value of game is zero then game is called fair and if it is non zero then it is called Unfair.

- **Payoff Matrix:** When player select their particular strategies, the payoffs can be represented in the form of matrix is called payoff Matrix.

### Research Methodology

The following methodology was followed for the present study.

### Objectives of the Study

The present study aims

- To look at game theory with more emphasis on the zero-sum games. Dominance, mixed strategies. Nash equilibrium, and lastly on the bidding in auctions.
- To acquire the knowledge of game theory by applying the different strategies.
- To illuminate the nature of conflict and cooperation.
- To work in competitive environment which aids in decision making.

### Method of Data Collection

The present study was based on Secondary Data.

- **Secondary Data:** The secondary data will be collected from published books, journals, research papers, magazines, internet etc.

### Research Type

- The study is descriptive in the sense that it is carried out with the objective of describing a particular situation.
- The study is analytical in nature as an attempt has been made to find out the cause rather than result.

### Basics of Game Theory

#### Definitions and zero sum games

Game theory is a branch of mathematics that analyzes interaction involving strategic decision making. This type of interaction is called a game. One type of game is classified as Zero-Sum.

Zero-Sum game is the algebraic sum of gains and losses of all the players is zero.

For example A simple game consisting of two players where each has two options can be represented by 2x2 matrix. Let player one's option be represented by rows and Player two's option be represented by columns.

		Player 2	
		Option 1	Option 2
Player1	Option 1	Outcome if each chooses option 1	P1 chooses 1, P2 chooses 2
	Option 2	P1 chooses 1, P2 chooses 2	Outcome if each chooses option 2

**Fig 1:** The general form of payoff matrix for a two person zero sum game.

The entries of the matrix contain possible outcomes of the game. If each player chooses option 1, the outcome of the game is the entry in position (1, 1) of the matrix representing the game.

Non Zero Sum game is where one player's gain does not strictly apply another player's loss. In this outcomes cannot be represented by only one number rather the outcomes of two persons can be expressed as an ordered pair. The first no of the ordered pair represents the payoff to player1 and the second number represents the payoff to player 2. These games are also represented with a matrix.

		Player 2	
		Option 1	Option 2
Player1	Option 1	Outcome if each chooses option 1	P1 chooses 1, P2 chooses 2
	Option 2	P1 chooses 1, P2 chooses 2	Outcome if each chooses option 2

**Fig 2:** The general form of payoff matrix for a two person non zero sum game.

**Dominance**

Since all players are assumed to be rational in game theory, they make choices which result in the outcome they prefer most, given what their opponent do. Both players are said to have dominant strategies. In extreme case a player have two strategies A and B so that, given any combination of strategies of other player. The outcome resulting from a is better than the outcome resulting from B. Then strategy A is said to dominate strategy B. In some games examinations of which strategies are dominated results in the conclusion that rational players could only ever choose one of their strategies. The following examples illustrate this idea.

**Example: Prisoner's Dilemma**

The prisoner's is a game in a strategic form between two players. Each player has two strategies, called "cooperate" and "defect" which are labeled C and D for player I and c and d for player II respectively. (For simpler identification, upper case letters are used for strategies of player I and lower case letters for player II.)

		II	
		c	d
I	C	2	3
	D	0	1

**Fig 3:** The Prisoner's Dilemma game.

Figure 1 shows the resulting payoffs in this game. Player I choose a row, either C or D, and simultaneously player II chooses one of the columns c or d. The strategy combination (C, c) has payoff 2 for each player, and the combination (D, d) gives each player payoff 1. The combination (C, d) results in payoff 0 for player I and 3 for player II, and when (D, c) is played, player I gets 3 and player II gets 0.

Any two-player game in strategic form can be described by a table like the one in Figure 1, with rows representing the strategies of player I and columns those of player II. (A player may have more than two strategies.) Each strategy combination defines a payoff pair, like (3, 0) for (D, c), which is given in the respective table entry. Each cell of the table shows the payoff to player I at the (lower) left, and the payoff to player II at the (right) top. These staggered payoffs, due to Thomas Schelling, also make transparent when, as here, the game is symmetric between the two players. Symmetry means that the game stays the same when the players are exchanged, corresponding to a reflection along the diagonal shown as a dotted line in Figure 2. Note that in the strategic form, there is no order between player I and II since they act simultaneously (that is, without knowing the other's action), which makes the symmetry possible

		II	
		c	d
I	C	2	3
	D	0	1

**Fig 4**

Figure 2. The game of Figure 1 with annotations, implied by the payoff structure. The dotted line shows the symmetry of the game. The arrows at the left and right point to the preferred strategy of player I when player II plays the left or right column, respectively. Similarly, the arrows at the top and bottom point to the preferred strategy of player II when player I plays top or bottom.

In the Prisoner's Dilemma game, "defect" is a strategy that dominates "cooperate." Strategy D of player I dominate C since if player II chooses c, then player I's payoff is 3 when choosing D and 2 when choosing C; if player II chooses d, then player I receives 1 for D as opposed to 0 for C. These preferences of player I are indicated by the downward pointing arrows in Figure 2. Hence, D is indeed always

better and dominates C. In the same way, strategy d dominates c for player II. 9 No rational player will choose a dominated strategy since the player will always be better off when changing to the strategy that dominates it. The unique outcome in this game, as recommended to utility-maximizing players, is therefore (D, d) with payoffs (1, 1). Somewhat paradoxically, this is less than the payoff (2, 2) that would be achieved when the players chose (C, c). The story behind the name “Prisoner’s Dilemma” is that of two prisoners held suspect of a serious crime. There is no judicial evidence for this crime except if one of the prisoners testifies against the other. If one of them testifies, he will be rewarded with immunity from prosecution (payoff 3), whereas the other will serve a long prison sentence (payoff 0). If both testify, their punishment will be less severe (payoff 1 for each). However, if they both “cooperate” with each other by not testifying at all, they will only be imprisoned briefly, for example for illegal weapons possession (payoff 2 for each). The “defection” from that mutually beneficial outcome is to testify, which gives a higher payoff no matter what the other prisoner does, with a resulting lower payoff to both. This constitutes their “dilemma.” Prisoner’s Dilemma games arise in various contexts where individual “defections” at the expense of others lead to overall less desirable outcomes. Examples include arms races, litigation instead of settlement, environmental pollution, or cut-price marketing, where the resulting outcome is detrimental for the players. Its game-theoretic justification on individual grounds is sometimes taken as a case for treaties and laws, which enforce cooperation. Game theorists have tried to tackle the obvious “inefficiency” of the outcome of the Prisoner’s Dilemma game. For example, the game is fundamentally changed by playing it more than once. In such a repeated game, patterns of cooperation can be established as rational behavior when players’ fear of punishment in the future outweighs their gain from defecting today.

### Mixed Strategies

Turocy and von Stengel (2001) outlined that a game in strategic form does not always have a Nash equilibrium in which each player definitely chooses one of the strategies. But players base their random selection of strategies on certain probabilities. Mixed strategies are defined as a probability distribution over the set of actions. However Rubinstein (1991) alternatively viewed mixed strategy as a belief held by all other players regarding a player's actions. Presented below is an illustration of mixed strategies equilibrium by an example of drunk driving; the police choose to set up checkpoints with probability 1/3. Assume if a player drinks Cola, he will get 0. If a player drinks Wine, he will get -2 with probability 1/3 and 1 with probability 2/3. Kockesen (n.d) also assumed that the value is the expected payoff;

$$\frac{1}{3} \times (-2) + \frac{2}{3} \times 1 = 0$$

The player is indifferent whether to drink Wine or Cola with any probability. If a player drinks Wine with probability of 1/2 and gets to the police check points, he gets an expected payoff of -1 and if he does not;

$$\frac{1}{2} \times (-2) + \frac{1}{2} \times 0 = -1$$

It is also outlined that the police are also indifferent about setting up checkpoints and any mixed strategy. This results on mixed strategy equilibrium.

### Nash Equilibrium

The Nash equilibrium is a game theoretic solution concept that is normally applied in economics. As previously outlined, Nash equilibrium was introduced by John Nash in 1950 and has emerged as one of the fundamental concepts of game theory (Kerk, n.d.). Nash equilibrium is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy (Osborne, 2002). However, Myerson (1999) viewed the concept of equilibrium as one of the most important and elegant ideas in game theory. Myerson (1999) also pointed out that a game can have many Nash equilibriums, and some of these equilibriums may be unreliable compared to what should be the outcome of a game. Some studies reflect that Nash equilibrium is concern about the actions that will be chosen by players in a strategic game (Osborne, 2002). Players have to know precisely what their opponents will choose (de Bruin, 2009). To do so, players should not base on the assumption that all players are rational. They rather focus on the basis of statistical information about previous game playing situations, if such information is available and 8 reliable. Osborne (2002) also outlined an example where there is interaction between buyers and sellers. A buyer usually transacts only once with any given seller, or interacts repeatedly but anonymously. Each player chooses her action given her belief about the other players' actions.

### Bidding in Auctions

The design and analysis of auctions is one of the triumphs of game theory. Auction theory was pioneered by the economist William Vickrey in 1961. Its practical use became apparent in the 1990s, when auctions of radio frequency spectrum for mobile telecommunication raised billions of dollars. Economic theorists advised governments on the design of these auctions, and companies on how to bid (see McMillan, “Selling spectrum rights,” *Journal of Economic Perspectives* Vol. 8, 1994, pages 145–162). The auctions for spectrum rights are complex. However, many principles for sound bidding can be illustrated by applying game-theoretic ideas to simple examples. This section highlights some of these examples; see Milgrom, “Auctions and bidding: a primer” (*Journal of Economic Perspectives* Vol. 3, 1989, pages 3–22) for a broader view of the theory of bidding in auctions. Second-price auctions with private values The most familiar type of auction is the familiar open ascending-bid auction, which is also called an English auction. In this auction format, an object is put up for sale. With 34 the potential buyers present, an auctioneer raises the price for the object as long as two or more bidders are willing to pay that price. The auction stops when there is only one bidder left, who gets the object at the price at which the last remaining opponent drops out. A complete analysis of the English auction as a game is complicated, as the extensive form of the auction is very large. The observation that the winning bidder in the English auction pays the amount at which the last remaining opponent drops out suggests a simpler auction format, the second-price auction, for analysis. In a second-price auction, each

potential buyer privately submits, perhaps in a sealed envelope or over a secure computer connection, his bid for the object to the auctioneer. After receiving all the bids, the auctioneer then awards the object to the bidder with the highest bid, and charges him the amount of the second-highest bid. Vickrey's analysis dealt with auctions with these rules. How should one bid in a second-price auction? Suppose that the object being auctioned is one where the bidders each have a private value for the object. That is, each bidder's value derives from his personal tastes for the object, and not from considerations such as potential resale value. Suppose this valuation is expressed in monetary terms, as the maximum amount the bidder would be willing to pay to buy the object. Then the optimal bidding strategy is to submit a bid equal to one's actual value for the object. Bidding one's private value in a second-price auction is a weakly dominant strategy. That is, irrespective of what the other bidders are doing, no other strategy can yield a better outcome. (Recall that a dominant strategy is one that is always better than the dominated strategy; weak dominance allows for other strategies that are sometimes equally good.) To see this, suppose first that a bidder bids less than the object was worth to him. Then if he wins the auction, he still pays the second-highest bid, so nothing changes. However, he now risks that the object is sold to someone else at a lower price than his true valuation, which makes the bidder worse off. Similarly, if one bids more than one's value, the only case where this can make a difference is when there is, below the new bid, another bid exceeding the own value. The bidder, if he wins, must then pay that price, which he prefers less than not winning the object. In all other cases, the outcome is the same. Bidding one's true valuation is a simple strategy, and, being weakly dominant, does not require much thought about the actions of others. 35 While second-price sealed-bid auctions like the one described above are not very common, they provide insight into Nash equilibrium of the English auction. There is a strategy in the English auction which is analogous to the weakly dominant strategy in the second price auction. In this strategy, a bidder remains active in the auction until the price exceeds the bidder's value, and then drops out. If all bidders adopt this strategy, no bidder can make himself better off by switching to a different one. Therefore, it is a Nash equilibrium when all bidders adopt this strategy. Most online auction websites employ an auction which has features of both the English and second-price rules. In these auctions, the current price is generally observable to all participants. However, a bidder, instead of frequently checking the auction site for the current price, can instead instruct an agent, usually an automated agent provided by the auction site, to stay in until the price surpasses a given amount. If the current bid is by another bidder and below that amount, then the agent only bids up the price enough so that it has the new high bid. Operationally, this is similar to submitting a sealed bid in a second-price auction. Since the use of such agents helps to minimize the time investment needed for bidders, sites providing these agents encourage more bidders to participate, which improves the price sellers can get for their goods.

## Conclusion

This paper has viewed that game theory is not simply a matter of mathematics but concerns the real world, in the sense that involves decision-making by several players that

also affect the interest of other players. But it does not mean that the purpose of game theory is to predict behavior in the same sense as in sciences but it is capable of such things. Players are arranged in their preferences, their information, the strategic actions available to them, and how these influence their payoffs (returns). In situations where there are more than two players, a decision by player 1 does also affect the interest of other players (player 2). Game theory covers many aspects such as economics, political science, and psychology, as well as logic and biology. Game theory also views Nash equilibrium as a basic concept of the subject, but in situations of strategic games players base their random selection of strategies using certain probabilities. The subject looks at different scenarios that involve decision making and Nash equilibrium is said to be the most effective concept that deals with that with the famous application of Prisoner's Dilemma is also said to be the most common example to illustrate game theory. The game theory is viewed as analysis of the concepts used in social reasoning when dealing with situations of decision making and conflicts. Much highlight has been given on auction games, where a set of players, a set of strategies available to each player, and a payoff vector corresponding to each combination of strategies. Game theory builds an abstract between theory and real life situation. In real life situation, game theory can be observed in many marketing industries and sale industries, in some through the use of auctions and also in people's behavior, the way they make the everyday choices sort of includes the game theory especial when an individual has to make a choice. Big industries also apply game theory in most of the management strategies, when a certain decision or development has to be taken. Some game strategies do come into practice. This theory is of much use in many fields of competitive scenarios and can even be suitable in any economic related field where decision making is of importance.

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