



Hydromagnetic convective flow past impermeable plate in rotating system: Dusty flow with hall current

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Abstract

An analysis indicating the effect of Hall current on the unsteady boundary layer flow in an incompressible, electrically conducting, viscous liquid with solid, non-conducting, spherical and uniformly distributed dust particles bounded by infinite, impermeable, oscillating plate under the influence of transversely applied uniform magnetic field has been presented. The system rotates as a rigid body rotation. Perturbation technique is used to obtain the solutions for primary velocity and secondary velocity of liquid, dust particles and temperature field. Expressions for shear stress at the plate due to primary and secondary velocity of the liquid and dust particles are derived. In addition rate of heat transfer is also obtained. The results obtained are numerically evaluated and either plotted graphically or presented in tables followed by a discussion.

Keywords: electrically conducting, hydromagnetic convective, dusty flow, hall current

1. Introduction

The problem of hydromagnetic flow in rotating system in presence of uniform magnetic field finds application in a MHD generators /accelerators, induction flow meters, centrifugal machines, petroleum industry and so on. The problems on MHD flow taking Hall effect into account are of peculiar importance due to the fact that in presence of strong magnetic field the Hall current effect plays a significant role in determining the features of the flow pattern. Saffman [1962] initiated and investigated the fact of dust particles in the laminar flow of incompressible fluid with constant mass concentration of dust particles. Micheal and Miller [1966] have discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Reddy [1972] studied the unsteady hydrodynamic and hydromagnetic boundary layers in a homogeneous viscous fluid in rotating systems. Debnath [1972,73,74] has shown in his studies that the oscillatory boundary layer flows confined to the ultimate boundary layers are established through inertial oscillations and the propagation of diffused hydromagnetic waves. The effect of Hall current in the hydromagnetic rotating conducting fluid have been investigated by several authors including Pop [1971], Gupta [1975], Dutta and Muzamdar [1976], Venkatasiva [1979], Tiwari and Singh [1983], Singh and Singh [1992], Sengupta and Ray [1994], Singh *et al.* [1999, 2000] [1, 2, 3, 4, 5, 6, 7]. Recently, Singh [2003] [26, 27, 31, 32] has studied hydromagnetic convective flows with mass transfer taking Hall current, viscous dissipation and Joule heating. Further Singh [2003] [26, 27, 31, 32] has also studied the same problem taking into account thermal diffusion. The present study deals with an analysis indicating the effect of Hall current on the unsteady boundary layer flow generated impulsively in an incompressible electrically conducting viscous liquid with solid non-conducting, spherical, equal shape and size and uniformly distributed dust particles bounded by infinite oscillating impermeable plate under the influence of uniform magnetic field.

2. Formulation of the Problem

Equations for the dusty flow are:

$$\frac{\partial u}{\partial t} - (u \cdot \nabla)u + 2\Omega u = -\nabla p + \frac{1}{\rho} \vec{J} \times \vec{B} + \mathcal{G} \nabla^2 u + \frac{KN_0}{\rho} (v - u) \quad (1)$$

$$\text{div } v = 0 \quad (2)$$

$$m \left[\frac{\partial v}{\partial t} + (v \cdot \nabla)v + 2\Omega v \right] = K(u - v) \quad (3)$$

$$\frac{\partial N}{\partial t} + \text{div}(N_0 v) = 0 \quad (4)$$

In Cartesian coordinate system (x, y, z) , we assume x and y -axes in the plane of the porous plate and z -axis normal to the plate with velocity components (u, v, w) in these directions respectively. In this system unsteady flow of an electrically conducting incompressible viscous liquid with uniformly distributed non-conducting, spherical dust particles past an infinite plate is considered. A uniform magnetic field B_0 is acting normal to the plate and the Hall effect is taken into consideration. The magnetic Reynold number is considered to be small so that the induced magnetic field is neglected. The dusty liquid as well as the plate is in a state of solid body rotation with constant angular velocity Ω about the z -axis normal to the plate. In addition to above the analysis is based on the following assumptions:

1. The dust particles are solid, non-conducting, spherical in shape, identical in size and symmetrically uniformly distributed in the liquid.
2. Chemical reactions, mass transfer and radiation between the particles are not considered.
3. The temperature is uniform within the particles and the interaction between the particles is ignored.
4. The number density of the dust particles is assumed constant throughout the motion.
5. The dust concentration is assumed small so that it is not disturbing the continuity and electromagnetic effects.

The equations governing the dusty flow under the usual Boussinesq approximation in non-dimensional form are:

$$\frac{\partial^2 U_1}{\partial z^2} - \frac{E}{2} \frac{\partial U_1}{\partial t} - \left[iE + \frac{M^2}{1-im} \right] U_1 - \frac{l}{\tau} (U_1 - V_1) - G_r T = 0 \tag{5}$$

$$\frac{E}{2} \frac{\partial V_1}{\partial t} + iEV_1 - \frac{l}{\tau} (V_1 - U_1) = 0 \tag{6}$$

$$\frac{\partial^2 T}{\partial z^2} - P_r \frac{\partial T}{\partial t} - \alpha_0 T = 0 \tag{7}$$

Now introducing the following non-dimensional variables and parameters used in above equations and boundary conditions are:

$$z^* = \frac{zU^*}{g}, \quad t^* = \Omega t, \quad U_1^* = \frac{U_1}{U^*}, \quad V_1^* = \frac{V_1}{U^*}, \quad \tau^* = \frac{\tau U^*}{g},$$

$$T_1^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad n_1 = \frac{n_1'}{\Omega}, \quad n_2 = \frac{n_2'}{\Omega}, \quad m = \omega_e \tau_e \text{ (Hall parameter),}$$

$$P_r = \frac{\mu C_p}{K_T} \text{ (Prandtl number),} \quad E = \frac{2\Omega g}{U^* 2} \text{ (Rotation parameter),} \quad M = \frac{\sigma g B_0^2}{\rho U^*} \text{ (Magnetic parameter),}$$

$$G_r = \frac{g g \beta (T_w - T_\infty)}{U^{*3}} \text{ (Grashof number),}$$

$$\tau = \frac{M}{K} \text{ (Relaxation time parameter)}$$

$$\text{and} \quad l = \frac{m N_0}{\rho} \text{ (Mass concentration of dust particles)}$$

The non-dimensional boundary conditions relevant to the problem are:

$$U_1(z, t) = -\frac{U}{U^*} + \left[a e^{-n_1 t} + b e^{-n_2 t} \right], \quad V_1(z, t) = -\frac{V}{U^*} + \left[a^* e^{-n_1 t} + b^* e^{-n_2 t} \right],$$

$$T(z, t) = -\frac{T}{T^*} + \left[c e^{-n_1 t} + d e^{-n_2 t} \right] \quad \text{at} \quad z = 0$$

$$U_1(z,t) \rightarrow 0, \quad V_1(z,t) \rightarrow 0, \quad T(z,t) \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty \quad (8)$$

3. Solution of the Problem

To obtain the solution we assume:

$$\begin{aligned}
 U_1(z,t) &= f_1(z) + f_2(z)e^{-n_1t} + f_3(z)e^{-n_2t} \\
 V_1(z,t) &= g_1(z) + g_2(z)e^{-n_1t} + g_3(z)e^{-n_2t} \\
 T(z,t) &= h_1(z) + h_2(z)e^{-n_1t} + h_3(z)e^{-n_2t}
 \end{aligned} \quad (9)$$

Substituting (9) in (5)-(7), we get following coupled equations:

$$f_1''(z) - \frac{E}{2} f_1'(z) - D_1 f_1(z) - \frac{l}{\tau} [f_1(z) - g_1(z)] = G_r h_1(z) \quad (10)$$

$$f_2''(z) - \frac{E}{2} f_2'(z) - D_1 f_2(z) - \frac{l}{\tau} [f_2(z) - g_2(z)] = G_r h_2(z) \quad (11)$$

$$f_3''(z) - \frac{E}{2} f_3'(z) - D_1 f_3(z) - \frac{l}{\tau} [f_3(z) - g_3(z)] = G_r h_3(z) \quad (12)$$

$$g_1(z) = (A_1 - iB_1) f_1(z) \quad (13)$$

$$g_2(z) = \frac{1}{\left(A_2 + \frac{iE\tau}{l}\right)} f_2(z) \quad (14)$$

$$g_3(z) = \frac{1}{\left(A_3 + \frac{iE\tau}{l}\right)} f_3(z) \quad (15)$$

$$h_1''(z) - \alpha_0 h_1(z) = 0 \quad (16)$$

$$h_2''(z) + n_1 P_r h_2(z) - \alpha_0 h_2(z) = 0 \quad (17)$$

$$h_3''(z) + n_2 P_r h_3(z) - \alpha_0 h_3(z) = 0 \quad (18)$$

The boundary conditions (8) are transformed to:

$$\begin{aligned}
 f_1 &= -\frac{U}{U^*}, \quad f_2 = a, \quad f_3 = b, \quad g_1 = -\frac{V}{U^*}, \quad g_2 = a^*, \\
 g_3 &= b^*, \quad h_1 = -\frac{T}{T^*}, \quad h_2 = c, \quad h_3 = d \quad \text{at} \quad z = 0 \\
 f_1 = f_2 = f_3 &\rightarrow 0, \quad g_1 = g_2 = g_3 \rightarrow 0, \quad h_1 = h_2 = h_3 \rightarrow 0 \\
 &\text{as} \quad z \rightarrow \infty
 \end{aligned} \quad (19)$$

The solutions of (10)-(18) satisfying corresponding boundary conditions (19), we get:

$$f_1(z) = \left(-\frac{U}{U^*} - D_3 \right) e^{-\beta_1 z} + D_3 e^{-\sqrt{\alpha_0} z} \tag{20}$$

$$f_2(z) = (a - D_5) e^{-\beta_2 z} + D_5 e^{-A_4 z} \tag{21}$$

$$f_3(z) = (b - D_7) e^{-\beta_3 z} + D_7 e^{-A_5 z} \tag{22}$$

$$g_1(z) = (A_1 - iB_1) \left[\left(-D_3 - \frac{V}{U^* (A_1 - iB_1)} \right) e^{-\beta_1 z} + D_3 e^{-\sqrt{\alpha_0} z} \right] \tag{23}$$

$$g_2(z) = \frac{l}{(A_2 l - iE\tau)} \left[\left(\frac{a(A_2 l - iE\tau)}{l} - D_5 \right) e^{-\beta_2 z} + D_5 e^{-A_4 z} \right] \tag{24}$$

$$g_3(z) = \frac{l}{(A_3 l - iE\tau)} \left[\left(\frac{b(A_3 l - iE\tau)}{l} - D_7 \right) e^{-\beta_3 z} + D_7 e^{-A_5 z} \right] \tag{25}$$

$$h_1(z) = -\frac{T}{T^*} e^{-\sqrt{\alpha_0} z} \tag{26}$$

$$h_2(z) = c e^{-A_4 z} \tag{27}$$

$$h_3(z) = d e^{-A_5 z} \tag{28}$$

Substituting the values of $f_1(z)$, $f_2(z)$, $f_3(z)$, $g_1(z)$, $g_2(z)$, $g_3(z)$, $h_1(z)$, $h_2(z)$ and $h_3(z)$ in the relation shown in (9) we get:

$$T(z, t) = -\frac{T}{T^*} e^{-\sqrt{\alpha_0} z} + c e^{-A_4 z} e^{-n_1 t} + d e^{-A_5 z} e^{-n_2 t} \tag{29}$$

$$U(z, t) = u_1 + iu_2 \tag{30}$$

$$V(z, t) = v_1 + iv_2 \tag{31}$$

where

$$u_1 = p_3 e^{-\sqrt{\alpha_0} z} + \left\{ e^{-R_2 z} (F_2 \cos S_2 z - q_5 \sin S_2 z) + p_5 e^{-A_4 z} \right\} e^{-n_1 t} + \left\{ e^{-R_3 z} (F_3 \cos S_3 z - q_7 \sin S_3 z) + p_7 e^{-A_5 z} \right\} e^{-n_2 t} + e^{-R_1 z} (F_1 \cos S_1 z - q_3 \sin S_1 z)$$

$$u_2 = q_3 e^{-\sqrt{\alpha_0} z} - \left\{ e^{-R_2 z} (q_5 \cos S_2 z + F_2 \sin S_2 z) - q_5 e^{-A_4 z} \right\} e^{-n_1 t} - \left\{ e^{-R_3 z} (q_7 \cos S_3 z - F_3 \sin S_3 z) - q_7 e^{-A_5 z} \right\} e^{-n_2 t} - e^{-R_1 z} (q_3 \cos S_1 z + F_1 \sin S_1 z)$$

$$\begin{aligned}
 v_1 &= e^{-R_1 z} (F_4 \cos S_1 z + F_5 \sin S_1 z) + F_6 e^{-\sqrt{\alpha_0} z} \\
 &\quad + \left\{ e^{-R_2 z} (F_7 \cos S_2 z + F_8 \sin S_2 z) + F_9 e^{-A_4 z} \right\} e^{-n_1 t} \\
 &\quad + \left\{ e^{-R_3 z} (F_{10} \cos S_3 z + F_{11} \sin S_3 z) + F_{12} e^{-A_5 z} \right\} e^{-n_2 t} \\
 v_2 &= e^{-R_1 z} (F_5 \cos S_1 z - F_4 \sin S_1 z) - F_5 e^{-\sqrt{\alpha_0} z} \\
 &\quad + \left\{ e^{-R_2 z} (F_8 \cos S_2 z - F_7 \sin S_2 z) - F_8 e^{-A_4 z} \right\} e^{-n_1 t} \\
 &\quad + \left\{ e^{-R_3 z} (F_{11} \cos S_3 z - F_{10} \sin S_3 z) - F_{11} e^{-A_5 z} \right\} e^{-n_2 t}
 \end{aligned}$$

4. Skin-Friction

The skin-friction (τ_{f_p}) at the plate $z = 0$ due to primary velocity and skin-friction (τ_{f_s}) due to secondary velocity are obtained as follows:

$$\begin{aligned}
 \tau_{f_p} &= \left(\frac{\partial u_1}{\partial z} \right)_{z=0} = -q_3 S_1 - R_1 F_1 - p_3 \sqrt{\alpha_0} - (q_5 S_1 + R_2 F_2 + A_4 p_5) e^{-n_1 t} \\
 &\quad - (q_7 S_3 + R_3 F_3 + A_5 p_7) e^{-n_2 t} \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{f_s} &= \left(\frac{\partial u_2}{\partial z} \right)_{z=0} = R_1 q_3 - F_1 S_1 - q_3 \sqrt{\alpha_0} - (F_2 S_2 - R_2 q_5 + A_4 q_5) e^{-n_1 t} \\
 &\quad - (F_3 S_3 - R_3 q_7 - A_5 q_7) e^{-n_2 t} \tag{33}
 \end{aligned}$$

The skin-friction (τ_{d_p}) at the plate $z = 0$ due to primary dust particle velocity and skin-friction (τ_{d_s}) due to secondary dust particle velocity are obtained as follows:

$$\begin{aligned}
 \tau_{d_p} &= \left(\frac{\partial v_1}{\partial z} \right)_{z=0} = F_2 S_1 - R_1 F_4 - F_6 \sqrt{\alpha_0} + (F_8 S_2 - R_2 F_7 - A_4 F_9) e^{-n_1 t} \\
 &\quad + (F_{11} S_3 - R_3 F_{10} - A_5 F_{12}) e^{-n_2 t} \tag{34}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{d_s} &= \left(\frac{\partial v_2}{\partial z} \right)_{z=0} = F_5 \sqrt{\alpha_0} - F_5 S_1 - R_1 F_5 + (F_8 A_4 - S_2 F_7 - R_2 F_8) e^{-n_1 t} \\
 &\quad + (F_{11} A_5 - S_3 F_{10} - S_3 F_{11}) e^{-n_2 t} \tag{35}
 \end{aligned}$$

5. Rate of heat Transfer

The rate of heat transfer in terms of Nusselt number (N_u) is:

$$N_u = \left(\frac{\partial T}{\partial z} \right)_{z=0} = -d_1 \sqrt{\alpha_0} - c A_3 e^{-n_1 t} - d A_5 e^{-n_2 t} \tag{36}$$

Table 1: Effects of various parameters on skin-friction due to primary velocity of liquid and dust particles

P_r	M	m	G_r	E	τ_{p_f}	τ_{p_d}
0.71	0.5	0.3	8.0	1.0	1.52617	1.64628
7.00	0.5	0.3	8.0	1.0	0.48753	0.49782
0.71	1.0	0.3	8.0	1.0	1.34827	1.35846
0.71	0.5	0.6	8.0	1.0	1.67184	1.69374
0.71	0.5	0.3	12.0	1.0	3.11763	3.22557
0.71	0.5	0.3	8.0	2.0	1.54874	1.56881

(at $\alpha_0 = 1.0, l = 0.4, \tau = 0.5, n_1 = 0.2, n_2 = 0.3, a = 0.2, b = 0.4,$

$$a^* = 0.3, b^* = 0.5, t = 1.0, \frac{U}{U^*} = -1 \text{ and } \frac{V}{V^*} = -1)$$

Table 2: Effects of various parameters on skin-friction due to secondary velocity of liquid and dust particles

P_r	M	m	G_r	E	τ_{p_f}	τ_{p_d}
0.71	0.5	0.3	8.0	1.0	-1.52845	-1.64473
7.00	0.5	0.3	8.0	1.0	-0.48728	-0.49482
0.71	1.0	0.3	8.0	1.0	-1.34624	-1.35743
0.71	0.5	0.6	8.0	1.0	-1.67376	-1.69548
0.71	0.5	0.3	12.0	1.0	-3.11478	-3.22627
0.71	0.5	0.3	8.0	2.0	-1.54827	-1.56786

(at $\alpha_0 = 1.0, l = 0.4, \tau = 0.5, n_1 = 0.2, n_2 = 0.3, a = 0.2, b = 0.4,$

$$a^* = 0.3, b^* = 0.5, t = 1.0, \frac{U}{U^*} = -1 \text{ and } \frac{V}{V^*} = -1)$$

Table 3: Effects Of P_r and α_0 on rate of heat transfer in terms of Nusselt number (N_u)

P_r	α_0	N_u
0.71	0.5	-1.74875
7.00	0.5	-7.67283
11.4	0.5	-11.87145
0.71	1.0	-1.6471
0.71	0.0	-1.78108
0.71	-1.0	-2.18586

$$\text{(at } c = 0.2, d = 0.3, \frac{T}{T^*} = -1 \text{ and } t = 1.0)$$

6. Discussion and Conclusions

To get physical insight into the problem the equations of momentum and energy are solved and distributions are plotted numerically. The effects of Prandtl number (P_r), Magnetic parameter (M), Hall parameter (m), Grashof number (G_r) and Rotation parameter (E) on primary velocity of liquid and dust particles and secondary velocity of liquid and dust particles are discussed numerically through graphs. The effects of Prandtl number (P_r) and Heat source parameter (α_0) on temperature field is discussed through.

To be realistic, the numerical values of Prandtl number are chosen to be $P_r = 0.71, P_r = 7.00$ and $P_r = 11.4$, which respectively correspond to air, water at 20°C and water at 4°C. The numerical values of Grashof number are chosen for $G_r > 0$, which corresponds to cooling case of the impermeable plate. The numerical values of the remaining parameters, although chosen arbitrarily, are in agreement with those of researchers of the field.

The effects of Prandtl number (P_r), Magnetic parameter (M), Hall parameter (m), Grashof number (G_r) and Rotation parameter (E) on skin-friction due to primary velocity of liquid and dust particles; skin-friction due to secondary velocity of liquid and dust particles are numerically presented in the Table-1 and Table-2. The effects of Prandtl number (P_r) and heat source parameter (α_0) on rate of Heat transfer in terms of Nusselt number (N_u) is shown in Table-3. In the case of velocities and skin-frictions due to primary and secondary velocities, the effects are observed at $\alpha_0 = 1.0, l = 0.4,$

$\tau = 0.5$, $n_1 = 0.2$, $n_2 = 0.3$, $a = 0.2$, $b = 0.4$, $a^* = 0.3$, $b^* = 0.5$, $t = 1.0$, $\frac{U}{U^*} = -1$ and $\frac{V}{V^*} = -1$ while in the case of temperature field the effects are observed at $c = 0.2$, $d = 0.3$, $\frac{T}{T^*} = -1$ and $t = 1.0$ for the purpose of

discussion and conclusions. The conclusions of the study are drawn on the basis of these fixed numerical values and variation in values of the remaining parameters. The conclusions of the study are as follows:

1. The primary velocity of the liquid and dust particles increases near the plate and after attaining a maximum value it decreases as z increases.
2. An increase in G_r or m increases the primary velocity of the liquid and dust particles.
3. An increase in P_r , M or E decreases the primary velocity.
4. The secondary velocity of the liquid and dust particles decreases near the plate and after attaining a minimum value it increases as z increases.
5. An increase in P_r , M or E increases the secondary velocity of the liquid and dust particles.
6. An increase in G_r or m decreases the secondary velocity of the liquid and dust particles.
7. An increase in P_r or S decreases the temperature field.
8. An increase in G_r , m or E increases the skin-frictions due to primary velocity of liquid and dust particles.
9. An increase in P_r or M decreases the skin-frictions due to primary velocity of liquid and dust particles.
10. An increase in G_r , m or E decreases the skin-frictions due to secondary velocity of liquid and dust particles.
11. An increase in P_r or M increases the skin-frictions due to secondary velocity of liquid and dust particles.
12. An increase in P_r decreases the rate of heat transfer in terms of Nusselt number (N_u) while an increase in α_0 increases the rate of heat transfer in terms of Nusselt number (N_u).

7. References

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