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Reduced differential transform method for fourth order parabolic partial differential equations

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Abstract
In this work, reduced differential transform method is proposed for fourth order Parabolic Partial Differential Equations with variables coefficients. The approximate solution of this equation is calculated in the form of a series with easily computable terms. Exact solution can be achieved by the known forms of the series solutions. Some examples solved which shows its ability, efficiency, reliability, effectiveness and simplicity.

Keywords: partial differential equation, parabolic PDEs, two dimensional differential transform method. Mathematics subject classification

1. Introduction
It is common that most of the phenomena in the field of engineering and mathematical physics can be represented by partial differential equations (PDEs). Many problems of physical interest are described by linear partial differential equations with initial and boundary conditions. One of them is fourth order parabolic partial differential equations with variable coefficients; these equations arise in the transverse vibration problems. In recent years, several numerical methods were developed such as the variational iteration method, Adomian decomposition method [1, 4], homotopy perturbation method [3], BSpline method [2], homotopy analysis method and Laplace decomposition algorithm for these equations.
In 1986, Zhou and Pukhov have developed a so-called differential transformation method (DTM) for electrical circuit's problems [7]. The DTM is a technique that uses Taylor series for the solution of differential equations in the form of a polynomial. The implementation of the Differential Transform Method (DTM) [5, 6, 7, 8, 9] amongst others has shown reliable results to solve ordinary differential equations, partial differential equations, integro-differential equations, Systems of Volterra Integral Equations of the First Kind and delay differential equations.

2 Reduced Differential transforms method
2.1 Definition
If a function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} \tag{2.1}$$

Where the t-dimensional spectrum function of $u(x,t)$, $u(x,t)$ is the original function and $U_k(x)$ is the transformed function.
The Differential inverse transform of $U_k(x)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k \tag{2.2}$$

From (2.1) and (2.2) we write

$$u(x,t) = \sum_{k=0}^{\infty}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} t^k$$

From above it can be concluded that the concept of modified 2-dimensional DTM is derived from the power series expansion.

2.2 The table of Reduced Differential Transform operators

The fundamental mathematics operations performed by reduced differential transformation are as

Table 1

Sr.No.	Originalfunction	Transformfunction
1	$u(x,t) \pm v(x,t)$	$U_k(x) \pm V_k(x)$
2	$\alpha u(x,t)$, where α isconstant	$\alpha U_k(x)$
3	$\frac{\partial u(x,t)}{\partial t}$	$(k+1)\ U_{k+1}(x)$
4	$\frac{\partial^n u(x,t)}{\partial t^n}$	$(k+1)(k+2)\ldots(k+n)U_{k+n}(x)$
5	$\frac{\partial^n u(x,t)}{\partial x^n}$	$\frac{\partial^n}{\partial x^n} U_k(x)$
6	$x^m t^n$	$x^m \delta(k-N)$ $where \delta\ (k-n)= \begin{cases} 1 & ,ifk = n \\ 0 & ,ifk = n \end{cases}$
7	$x^m e^{\alpha t}$	$x^m \frac{\alpha^k}{K!}$
8	$e^{\alpha x} \sin(\omega t+\beta)$	$e^{\alpha x} \frac{\omega^k}{K!} \sin(\frac{\pi k}{2}+\beta)$
9	$e^{\alpha x} \cos(\omega t+\beta)$	$e^{\alpha x} \frac{\omega^k}{K!} \cos(\frac{\pi k}{2}+\beta)$

3 Numerical Applications

Example 1[2]

$$\frac{\partial^2 u(x,t)}{\partial t^2}+\left(\frac{1}{x}+\frac{x^4}{120}\right)\frac{\partial^4 u(x,t)}{\partial x^4}=0 \quad , \quad \frac{1}{2}<x<1, \quad 0<t \tag{3.1}$$

With B.Cs are $u(0.5,t)=\left(\frac{1+(0.5)^5}{120}\right)\sin t \quad , \quad u(1,t)=\frac{121}{120}\sin t$

$$\frac{\partial^2 u(0.5,t)}{\partial x^2}=0.02084\sin t \quad , \quad \frac{\partial^2 u(1,t)}{\partial x^2}=\frac{1}{6}\sin t$$

And I.Cs ae $u(x,0)=0 \quad , \quad \frac{\partial u(x,0)}{\partial x}=1+\frac{x^5}{120}$ (3.2)

Solution

The exact solution is $u(x,t)=\left(1+\frac{x^5}{120}\right)\sin t$

Applying Reduced D.T.M. on (3.1) and (3.2) we get

$$(k+1)(k+2)U_{k+2}(x)=-\left(\frac{1}{x}+\frac{x^4}{20}\right)\frac{\partial^4}{\partial x^4}U_k(x) \quad , \quad k=0,1,2,\ldots \tag{3.3}$$

$$U_0(x)=0 \quad , \quad U_1(x)=1+\frac{x^5}{120}$$

For k = 0,1,2... in (3.3) we get

$$U_2(x) = 0 \quad , \quad U_3(x) = -\frac{1}{3!}\left(1+\frac{x^5}{120}\right) \quad , \quad U_4(x) = 0 \quad , \quad U_5(x) = \frac{1}{5!}\left(1+\frac{x^5}{120}\right)$$

$$U_6(x) = 0 \quad , \quad U_7(x) = -\frac{1}{7!}\left(1+\frac{x^5}{120}\right) \quad , \quad \dots \dots$$

Using Inverse Differential Transform we have

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

$$u(x,t) = U_0(x)t + U_1(x)t + U_2(x)t^2 + U_3(x)t^3 + U_4(x)t^4 + \dots$$

$$u(x,t) = \left(1+\frac{x^5}{120}\right)t - \frac{1}{3!}\left(1+\frac{x^5}{120}\right)t^3 + \frac{(1}{5!}\left(1+\frac{x^5}{120}\right)t^5 - \frac{(1}{7!}\left(1+\frac{x^5}{120}\right)t^7 + \dots$$

$$u(x,t) = \left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots\right)\left(1+\frac{x^5}{120}\right)$$

in closed form is

$$u(x,t) = \left(1+\frac{x^5}{120}\right)\sin t$$

Example 2 [2]

$$\frac{\partial^2 u(x,t)}{\partial t^2} + \left(\frac{x}{\sin x} - 1\right)\frac{\partial^4 u(x,t)}{\partial x^4} = 0 \quad , \quad 0 < x < 1, \quad 0 < t \tag{3.4}$$

With B.Cs are

$$u(0,t) = 0 \quad , \quad u(1,t) = e^{-t}(1-\sin 1)$$

$$\frac{\partial^2 u(0,t)}{\partial x^2} = 0 \quad , \quad \frac{\partial^2 u(1,t)}{\partial x^2} = e^{-t}\sin 1$$

And I.Cs are

$$u(x,0) = x - \sin x \quad , \quad \frac{\partial u(x,0)}{\partial x} = -x + \sin x \tag{3.5}$$

Solution

The exact solution is $u(x,t) = (x - \sin x)e^{-t}$

Applying Reduced D.T.M. on (3.4) and (3.5) we get

$$(k+1)(k+2)U_{k+2}(x) = -\left(\frac{x}{\sin x} - 1\right)\frac{\partial^4}{\partial x^4}U_k(x) \quad , \quad k = 0,1,2,\dots$$

$$U_0(x) = x - \sin x \quad , \quad U_1(x) = -x + \sin x \tag{3.6}$$

For $k = 0, 1, 2...$ In (3.6) we get

$$U_2(x) = \frac{1}{2!}(x - \sin x) \quad , \quad U_3(x) = -\frac{1}{3!}(x - \sin x) \quad , \quad U_4(x) = \frac{1}{4!}(x - \sin x) \quad , \quad \dots \dots$$

Using Inverse Differential Transform we get

$$u(x,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \dots\right)(x - \sin x)$$

$$u(x,t) = e^{-t}(x - \sin x)$$

Example 3 [2, 3]

$$\frac{\partial^2 u(x,t)}{\partial t^2} + (1+x)\frac{\partial^4 u(x,t)}{\partial x^4} = \left(x^3 + x^4 - \frac{6}{7!}x^7\right)\cos t \quad , \quad 0 < x < 1, \quad 0 < t \qquad (3.7)$$

with B.Cs are

And I.Cs are

$$u(0,t) = 0 \quad , \quad u(1,t) = \frac{6}{7!}\cos t$$

$$\frac{\partial^2 u(0,t)}{\partial x^2} = 0 \quad , \quad \frac{\partial^2 u(1,t)}{\partial x^2} = \frac{1}{20}\cos t$$

$$u(x,0) = \frac{6}{7!}x^7 \quad , \quad \frac{\partial u(x,0)}{\partial x}$$

$$= 0 \qquad (3.8)$$

Solution

The exact solution is $u(x,t) = \frac{6}{7!}x^7 \cos t$

Applying Reduced D.T.M. on (3.7) and (3.8) we get

$$(k+1)(k+2)U_{k+2}(x) = -(1+x)\frac{\partial^4}{\partial x^4}U_k(x) + \left(x^3 + x^4 - \frac{6}{7!}x^7\right)\frac{1}{k!}\cos\left(\frac{\pi k}{2}\right) \quad , k = 0, 1, 2, .$$

$$U_0(x) = \frac{6}{7!}x^7 \quad , \quad U_1(x) = 0 \qquad (3.9)$$

For $k = 0,1,2,\dots$ in (3.9) we get

$$U_2(x) = -\frac{6}{7!2!}x^7 \quad , \quad U_3(x) = 0 \quad , \quad U_4(x) = \frac{6}{7!4!}x^7 \quad , \quad U_5(x) = 0 \quad , \quad U_6(x) = -\frac{6}{7!6!}x^7 \quad , \quad \dots\dots$$

Using Inverse Differential Transform we get

$$u(x,t) = \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \dots\right)\frac{6}{7!}x^7$$

$$u(x,t) = \frac{6}{7!}x^7 \cos t$$

4 Conclusion

In this work, reduced differential transform has been applied to obtain the solution of fourth order Parabolic Partial Differential Equations with variables coefficients. This method reduces the computational difficulties of the other existing methods. This method is simple in calculations, time saving, straight forward, computational efficiency, high accuracy and gives better results than other existing methods. The objective of this paper is to present a powerful, simple and efficient technique for finding the exact solutions for large classes of problems.

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