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Comparison of three estimation methods for Frechet distribution

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Abstract
This paper study the performance of three different estimation methods of scale parameter, namely, least square method, weighted least square method and maximum likelihood method for two parameter Frechet distribution. The Monte- Carlo simulation is used to compare these three methods in terms of bias and mean square error. The simulation study showed that, the maximum likelihood method was the best as compared to least square and weighted least square method in terms of bias as well as mean square error.

Keywords: Frechet distribution**,** least square method, weighted least square method, maximum likelihood estimation method, mean square error

1. Introduction

The Frechet (extreme value type II) distribution is one of the probability distributions used to model extreme events. The extreme value distribution is becoming increasingly important in engineering statistics as a suitable distribution to represent phenomena with usually large maximum observations. In engineering circles, this distribution is often called the Frechet distribution. French mathematician Maurice Frechet (1927) introduced Frechet distribution. It is also known as inverse Weibull distribution.

Frechet distribution has wide range of application in modeling and analysis data, like, floods, rain fall and wind speeds. Coles and Stuart (2001) [1] applied Frechet distribution to extreme events such as annually maximum one-day rainfalls and river discharges. Frechet distribution is also used in sociological modeling Nadarajah and Kotz (2008) [2].

We consider two-parameter Frechet distribution with shape parameter α, scale parameter β. The probability density function of two parameter Frechet distribution is,

$$f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1}\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),\ x>0,\ \alpha,\ \beta > 0 \qquad (1)$$

The cumulative distribution function is given by

$$F(x) = \exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),\ x>0,\ \alpha,\ \beta > 0 \qquad (2)$$

We estimated the scale parameter β of Frechet distribution by using the three different methods, namely, least square method, weighted least square method and maximum likelihood method. The least square method and weighted least square method are commonly used methods. The performance of these three methods compared thorough Monte Carlo simulations with different sample sizes from 5 to 1000. We calculated bias and mean square error for every estimator. On the basis of Monte- Carlo simulation study we recommend the method which has minimum bias and mean square error. Kamran A. et. al. (2012) [3] compared the maximum likelihood estimator, probability weighted moment estimator and bayes estimator of scale parameter. Ivana P. et.al. (2014) [4] studied the performance of least square method, weighted least square method, maximum likelihood method and method of moments for two-parameter Weibull distribution.

2. Methods of Estimation

In this section, we describe the methods of estimation for the two parameter Frechet distribution.

2.1 Least square method

The Least squares technique was developed by Gauss (1795), Legendre (1805) and Adrain (1808) independently. It is published in the first decade of the nineteenth century. It is the most popular parameter estimation technique. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the Frechet distribution. The CDF of Frechet distribution (2) will be transformed to a linear function.

$$\ln(-\ln(F(x_i))) = \alpha \ln \beta - \alpha \ln x_i \qquad (3)$$

Comparing equation (3) with $Y_i = \beta_1 + \beta_2 X_i$, we get

$Y_i = \ln(-\ln(F(x_i)))$, $\beta_1 = \alpha \ln \beta$, $X_i = \ln x_i$ and $\beta_2 = -\alpha$

Now consider $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$ be the order statistics of $x_1, x_2 \ldots x_n$. The mean rank function is used to estimate the value of C.D.F. F(x).

$$\hat{F}(x_{(i)}) = \frac{i}{n+1} \qquad (4)$$

Thus, the regression parameter $\hat{\beta}_1$ and $\hat{\beta}_2$are estimated from the function

$$\sum_{i=1}^{n} \epsilon_i^2=\sum_{i=1}^{n}(Y_i - \beta_1 - \beta_2 \ln x_{(i)})^2 \quad (5)$$

Differentiating equation (5) partially w. r. t. β_1 and equating to zero, we get

$$\hat{\beta}_1=\frac{\sum_{i=1}^{n}\ln\left(-\ln\left(F(x_{(i)})\right)\right)+\hat{\alpha}\sum_{i=1}^{n}\ln x_{(i)}}{n}$$

Differentiating equation (5) partially w. r. t. β_2 and equating to zero, we get

$$\hat{\beta}_2 = \frac{n\sum_{i=1}^{n}\ln(-\ln(F(x_{(i)})))\ln x_{(i)} - \sum_{i=1}^{n}\ln(-\ln(F(x_{(i)})))\sum_{i=1}^{n}\ln x_{(i)}}{\left(n\sum_{i=1}^{n}(\ln x_{(i)})^2 - \left(\sum_{i=1}^{n}\ln x_{(i)}\right)^2\right)}$$

Therefore, estimates $\hat{\beta}$ and $\hat{\alpha}$ of the parameter β and α are given as

$$\hat{\alpha} = -\hat{\beta}_2 \text{ and } \hat{\beta} = \exp\left(\frac{\sum_{i=1}^{n}\ln\left(-\ln\left(F(x_{(i)})\right)\right)+\hat{\alpha}\sum_{i=1}^{n}\ln x_{(i)}}{n\hat{\alpha}}\right) \quad (6)$$

2.2 Weighted Least Square method

To estimate the parameter by using weighted least square method for Frechet distribution. We use the following regression function with regression parameter $\hat{\beta}_1$ and $\hat{\beta}_2$

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} w_i (Y_i - \beta_1 - \beta_2 \ln x_{(i)})^2 \quad (7)$$

Where w_i is the weighted factor proposed by Bergman and it is given by

$$w_i = \left[\left(1-\hat{F}(x_{(i)})\right)\ln\left(1-\hat{F}(x_{(i)})\right)\right]^2, \; i=1,2,\ldots,n$$

To estimate the regression parameter $\hat{\beta}_1$, differentiating equation (7) partially w. r. t. β_1 and equate to zero, we get

$$\hat{\beta}_1=\frac{\sum_{i=1}^{n} w_i\ln(-\ln\left(F(x_{(i)})\right)+\hat{\alpha}\sum_{i=1}^{n} w_i\ln x_{(i)}}{\sum_{i=1}^{n} w_i} \quad (8)$$

To estimate the regression parameter$\hat{\beta}_2$, differentiating equation (7) partially w. r. t. β_2 and equate to zero, we get

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln(-\ln(F(x_{(i)}))\ln x_{(i)} - \sum_{i=1}^{n} w_i \ln(-\ln(F(x_{(i)}))\sum_{i=1}^{n} w_i \ln x_{(i)}}{\left(\sum_{i=1}^{n} w_i \sum_{i=1}^{n}(\ln x_{(i)})^2 - \left(\sum_{i=1}^{n}\ln x_{(i)}\right)^2\right)} \quad (9)$$

Therefore, estimates $\hat{\beta}$ and $\hat{\alpha}$ of the parameter β and α are given as

$$\hat{\alpha}=-\hat{\beta}_2 \text{ and } \hat{\beta}= \exp\left(\frac{\sum_{i=1}^{n} w_i\ln(-\ln\left(F(x_{(i)})\right)+\hat{\alpha}\sum_{i=1}^{n} w_i\ln x_{(i)}}{\hat{\alpha}\sum_{i=1}^{n} w_i}\right) \quad (10)$$

Maximum likelihood Estimation method

The method of maximum likelihood provides estimators that have both a reasonable intuitive basis and many desirable statistical properties. The method is very broadly applicable and is simple to apply. Once a maximum-likelihood estimator is derived, the general theory of maximum-likelihood estimation provides standard errors, statistical tests, and other results useful for statistical inference. The likelihood function for equation (1) is,

$$L=\left(\frac{\alpha}{\beta}\right)^n \prod_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{-(\alpha+1)} \exp\left(-\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{-\alpha}\right) \quad (11)$$

Taking logarithm on both sides of equation (11), we get

$$\ln L=n\ln\alpha - n\ln\beta - (\alpha+1)\sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{-\alpha} \quad (12)$$

Differentiate equation (12) with respect to β and equating to zero, we get

$$\hat{\beta} = \left(\frac{n}{\sum_{i=1}^{n}(x_i)^{-\alpha}}\right)^{1/\alpha} \quad (13)$$

3. Monte Carlo simulation

The performance of three methods, namely, least square method (LS), weighted least square method (WLS) and maximum likelihood estimation method (MLE) for Frechet distribution is compared using the Monte-Carlo simulation. We generated N=100000 samples of sizes n = 5, 10, 20, 30, 50, 100, 200, 300, 500 and 1000 from Frechet distribution for α=(1, 2) and β=(0.5, 1.5). The efficiency of the methods is compare based on the mean square error (MSE) and Bias. The Bias of an estimator is the difference between an estimator's expectation and the true value of the parameter being estimated. i.e. Bias($\hat{\beta}$) = E($\hat{\beta}$) – β.

The MSE of an estimator is one way of quantifying the difference between an estimator and the true value of the quantity being estimated. The Mean square error is defined as the sum of the variance and the squared bias of an estimator. i. e. MSE ($\hat{\beta}$) = Var ($\hat{\beta}$) + $(\text{Bias}(\hat{\beta}))^2$.

4. Result and Discussion

The results of the Monte Carlo simulation study are presented to assess the performance of the estimators of scale parameter β. The obtained results are reported in terms of bias and MSE of the considered methods in Tables 1-2.

In tables-1, for β=0.5 and table-2, for β=1.5, the comparison shows that the mean square error and bias of maximum likelihood estimator is smaller as compared to least square and weighted least square estimator. The bias of least square method is more as compared to other two estimation methods. As sample size increases the bias of all three methods decreases. For sample size n= 1000 the all three methods have smaller bias as compared to other sample sizes for both values of α=1 and α=2.

The mean square error of the weighted Lest Square Method is larger than the mean square error of the maximum likelihood estimation method for each sample size. The mean square error of the least square method is the largest.

As sample size increases the mean square error decreases for each methods and hence precision of parameter increases. It is also note that, the bias and MSE of β decreases as α change from 1 to 2.

Table 1: Simulated bias and MSE when β=0.5

n	Methods	α=1		α=2	
		Bias	MSE	Bias	MSE
5	LS	0.1641	0.1961	0.0712	0.0326
	WLS	0.1599	0.1911	0.0698	0.0263
	MLE	0.1243	0.1451	0.0416	0.0205
10	LS	0.0845	0.0496	0.0545	0.0202
	WLS	0.0790	0.0483	0.0519	0.0191
	MLE	0.0546	0.0416	0.0193	0.0080
20	LS	0.0595	0.0303	0.0423	0.0143
	WLS	0.0546	0.0260	0.0395	0.0137
	MLE	0.0265	0.0162	0.0097	0.0036
30	LS	0.0369	0.0170	0.0156	0.0104
	WLS	0.0320	0.0151	0.0158	0.0100
	MLE	0.0170	0.0098	0.0063	0.0022
50	LS	0.0222	0.0111	0.0125	0.0094
	WLS	0.0199	0.0103	0.0136	0.0088
	MLE	0.0101	0.0055	0.0038	0.0013
100	LS	0.0159	0.0101	0.0099	0.0100
	WLS	0.0123	0.0091	0.0102	0.0084
	MLE	0.0050	0.0026	0.0019	0.0006
200	LS	0.0099	0.0068	0.0087	0.0082
	WLS	0.0087	0.0058	0.0081	0.0072
	MLE	0.0025	0.0013	0.0010	0.0003
300	LS	0.0056	0.0055	0.0065	0.0032
	WLS	0.0065	0.0025	0.0061	0.0030
	MLE	0.0016	0.0008	0.0006	0.0002
500	LS	0.0036	0.0025	0.0035	0.0025
	WLS	0.0028	0.0015	0.0031	0.0024
	MLE	0.0010	0.0005	0.0004	0.0001
1000	LS	0.0024	0.0012	0.0013	0.0012
	WLS	0.0019	0.0009	0.0010	0.0010
	MLE	0.0005	0.0003	0.0002	0.0001

Table 2: Simulated bias and MSE when β=1.5

n	Methods	α=1		α=2	
		Bias	MSE	Bias	MSE
5	LS	0.3988	1.3351	0.1942	0.2370
	WLS	0.3901	1.3203	0.1958	0.2196
	MLE	0.3728	1.3062	0.1248	0.1847
10	LS	0.2635	0.4666	0.0883	0.0961
	WLS	0.2596	0.4634	0.0852	0.0889
	MLE	0.1638	0.3741	0.0580	0.0718
20	LS	0.0994	0.1613	0.0388	0.0527
	WLS	0.0888	0.1575	0.0353	0.0625
	MLE	0.0794	0.1457	0.0290	0.0320
30	LS	0.0618	0.0993	0.0277	0.0399
	WLS	0.0520	0.0857	0.0203	0.0407
	MLE	0.0511	0.0879	0.0188	0.0202
50	LS	0.0403	0.0573	0.0231	0.0215
	WLS	0.0399	0.0524	0.0199	0.0229
	MLE	0.0304	0.0496	0.0113	0.0118
100	LS	0.0285	0.0555	0.0103	0.0182
	WLS	0.0245	0.0536	0.0058	0.0179
	MLE	0.0150	0.0236	0.0056	0.0058
200	LS	0.0205	0.0405	0.0097	0.0103
	WLS	0.0195	0.0360	0.0065	0.0099
	MLE	0.0076	0.0115	0.0029	0.0028

300	LS	0.0125	0.0122	0.0065	0.0075
	WLS	0.0120	0.0122	0.0045	0.0071
	MLE	0.0049	0.0076	0.0018	0.0019
500	LS	0.0065	0.0069	0.0043	0.0043
	WLS	0.0071	0.0066	0.0033	0.0039
	MLE	0.0030	0.0045	0.0011	0.0011
1000	LS	0.0012	0.0058	0.0023	0.0016
	WLS	0.0013	0.0045	0.0019	0.0013
	MLE	0.0015	0.0023	0.0006	0.0006

5. Conclusions

From the result of simulation study, we observe that if sample size increases the mean square error and bias of each method decreases. Mean square error decreases as values of α increase. The maximum likelihood estimator has smaller MSE as well as bias as compared to least square and weighted least square estimator. Thus we can say that maximum likelihood method outperforms least square and weighted least square method for Frechet distribution.

6. References

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