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Vague generalized semi-pre closed set in topological spaces

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Abstract
In this paper the concept of vague generalized semi-pre closed sets and vague generalized semi pre-open sets are introduced and some of their properties are investigated.

Keywords: vague topology, vague generalized semi pre closed set, vague generalized semi pre- open set

1. Introduction
The concept of fuzzy sets and fuzzy operations were first introduced by Zadeh [15] in 1965. The theory of fuzzy topology was introduced by C.L Chang [4] several research were conducted on the generalizations of the notions of the fuzzy sets and fuzzy topology. In 1970 Levine [6] initiated the investigation of so-called generalized closed sets. A subset A of a topological space X is called generalized closed, briefly g-closed, if $\mathrm{cl}\,A \subseteq U$ whenever $A \subseteq U$ and U is open. Moreover A is called generalized open or g-open. If X\A is g-closed the concept has been studied extensively in recent years by a large number of Topologists. In 1986 the concept of intuitionistic fuzzy sets fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets. Gau and Buehrer [5] established the concept of vague set. The theory of vague sets are regarded as a special case of context defined fuzzy sets. The basic concepts of vague set theory and its extensions are defined in [3, 5]. In this paper I introduce the concept of vague generalized pre-closed sets and vague generalized pre-open sets and also obtain their properties and relations with counter examples.

2. Preliminaries
Definition 2.1: [5] A vague set A in the universe of discourse X is characterized by membership functions given by

1. A true membership function $t_A : X \to [0,1]$
2. A false membership function $f_A : X \to [0,1]$

Where t_A (x) is lower bound on the grade membership of x derived from the

"Evidence for x " f_A (x) is lower bound on the negation membership of x derived from the

"Evidence against x and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague A is written as

$A = \{< x, [t_A(x), 1 - f_A(x)] > / x \in X\}$ Where the interval $[t_A(x), 1 - f_A(x)]$ is called the "vague value "of x in A and is denoted by $V_A(x)$

Definition 2.2: [5] Let A and B be VSs of the form $A = \{< x, [t_A(x), 1 - f_A(x)] > / x \in X\}$ and $B = \{< x, [t_B(x), 1 - f_B(x)] > / x \in X\}$. Then

a. $A \subseteq B$ If and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all $x \in X$.
b. $A = B$ If and only if $A \subseteq B$ and $B \subseteq A$.
c. $A^c = \{< x, [f_A(x), 1 - t_A(x)] > / x \in X\}$.
d. $A \cap B = \{< x, [(t_A(x) \wedge t_B(x)), ((1 - f_A(x))) \wedge (1 - f_B(x))] > / x \in X\}$.
e. $A \cup B = \{< x, [(t_A(x) \vee t_B(x)), ((1 - f_A(x))) \vee (1 - f_B(x))] > / x \in X\}$.

For the sake of simplicity we shall use the notion $A = \{< x, [t_A(x), 1 - f_A(x)] >\}$

Instead of $A = \{< x, [t_A(x), 1 - f_A(x)] > / x \in X\}$

Definition 2.3: [10]

a. semi closed set (SCS in short) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$

b. per-closed set (PCS in short) if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$

c. α -closed set (α CS in short) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$

d. Regular closed set (RCS in short) if $A = \mathrm{cl}(\mathrm{int}(A))$

Definition 2.4: [10]

a. Generalized closed set (briefly, g-closesd) if $\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

b. Generalized semi closed set (briefly, gs-closesd) if $\mathrm{Scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X

c. α -Generalized closed set (briefly, α g-closed) if α $\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X

d. Generalized pre closed set (briefly, gp-closesd) if P $\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X

Definition 2.5: [10] A vague topology on X is a family τ of vague sets in X. Satisfying the following axioms.

a. $0,1 \in \tau$

b. $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

c. $\cup G_1 \in \tau$, for any family $\{ G_i / i \in j\} \subseteq \tau$

In this case pair (X,τ) is called vague Topological Space and any VS in τ is known as a vague open set in X. The complement A^C of a VOS in a VTS (X,τ) is called a vague closed set in X.

Definition 2.6: [10] Let (X,τ) be a Vague Topological Space and $A = \{\langle x, [t_A, 1-f_A]\rangle\}$ be a VS in X. Then the vague interior and a vague closure are defined by

a. $\mathrm{vint}(A) = \cup\{G/G \text{ is a VOS in X and } G \subseteq A\}$,

b. $\mathrm{vcl}(A) = \cap\{K/K \text{ is a VCS in X and } A \subseteq K\}$.

Note that for any vague set A in (X,τ), I have $\mathrm{vcl}(A^c) = (\mathrm{vint}(A))^c$ and $\mathrm{vint}(A^c) = (\mathrm{Vcl}(A))^c$.

Definition 2.7: [10] A vague set $A = \{\langle x, [t_A, 1-f_A]\rangle\}$ in a VTS (X,τ) is said to be a

a. Vague semi closed set (VSCS in short) if $\mathrm{vint}(\mathrm{vcl}(A)) \subseteq A$.

b. Vague semi open set (VSOS in short) if $A \subseteq vcl(v\mathrm{int}(A))$.

c. Vague pre-closed set (VSCS in short) if $vcl(v\mathrm{int}(A)) \subseteq A$.

d. Vague pre-open set (VSOS in short) if $A \subseteq v\mathrm{int}(vcl(A))$.

e. Vague α -closed set (V α CS in short) if $vcl(v\mathrm{int}(vcl(A))) \subseteq A$.

f. Vague α - open set (V α OS in short) if $A \subseteq v\mathrm{int}(vcl(v\mathrm{int}(A)))$.

g. Vague regular open set (VROS in short) if $A = v\mathrm{int}(vcl(A))$.

h. Vague regular closed set (VRCS in short) if $A = vcl(v\mathrm{int}(A))$.

Definition 2.8: [11] Let A be a vague set of a vague Topological Space (X,τ). Then the vague semi interior of $A(\mathrm{vs\,int}(A))$ in short) and vague semi closure of $A(\mathrm{vscl\,int}(A))$ in short) are defined by

a. $\mathrm{vsint}(A) = \cup\{G/G \text{ is a VSOS in X and } G \subseteq A\}$,

b. $\mathrm{vscl}(A) = \cap\{K/K \text{ is a VSCS in X and } A \subseteq K\}$.

Result 2.9: [11] Let A be a VS of a VTS (X,τ), then

a. $\mathrm{vscl}(A) = A \cup v\mathrm{int}(vcl(A))$

b. $\mathrm{vsint}(A) = A \cap vcl(v\mathrm{int}(A))$

Definition 2.10: [11] Let A be a vague set of a Vague Topological Space (X,τ). Then the vague alpha interior of $A(v\alpha int((A)in\ short)$ and vague alpha closure of $A(v\alpha cl((A)in\ short)$ are defined by

a. $v\alpha int(A) = \cup\{G/G \text{ is a } V\alpha OS \text{ in } X \text{ and } G \subseteq A\}$,

b. $v\alpha cl(A) = \cap\{K/K \text{ is a } V\alpha CS \text{ in } X \text{ and } A \subseteq K\}$.

Definition 2.11: [11] A Vague set A of a Vague Topological Space (X,τ) is said to be a vague generalized closed set (VGCS in short) if $vcl(A) \subseteq U$, whenever $A \subseteq U$ and U is a Vague Open Set in X.

Definition 2.12: [11] A vague set A of a Vague Topological Space (X,τ) is said to be a vague generalized semi closed set (VGSCS in short) if $vscl(A) \subseteq U$, whenever $A \subseteq U$ and U is a vague open set in X.

3. Vague Generalized Semi- Pre Closed Set

Definition 3.1: Let A be a vague set in aVague Topological Space (X,τ). Then the semi-pre interior and semi-pre closure of Aare defined as.

$VSP\,int(A) = \cup\{G/G\ is\ a VSP\ open\ in\ X\ and\ G \subseteq A\}$

$VSP\,cl(A) = \cap\{K/\ \ K\ is\ a\ VSP\ close\ in\ X\ and\ A \subseteq K\}$, Note that for any VS in (X,τ) we have $VSPcl(A^c) = (VSP\,int(A^c))$ and $VSP\,int(A^c) = (VSP\,cl(A))^c$.

Definition 3.2:A vague set A in a VTS (X,τ) is said to be a vague generalized semi-pre closed set $VSPcl\ (A) \subseteq U$ whenever $A \subseteq U$ and U is vague open set in ((X,τ) the complement A^c of a VGSPCS in a VTS (X,τ) is called a vague generalized semi pro-open set in X.

Theorem 3.3: Every VCS is VGSPCS but not conversely.

Proof: Let A be a VCS in (X,τ). Let $A \subseteq U$ and U is VOS in (X,τ), since $VSPcl\ (A) \subseteq Vcl(A)$ and A is a VCS in (X,τ) $VSPcl(A) \subseteq Vcl(A) = A \subseteq U$. Therefore A is a VGSPCS.

Theorem 3.4: Every VGCS is VGSPCS but not conversely.

Proof: Let A be a VGCS in X. let $A \subseteq U$ and U is VOS in (X,τ). Since $VSPcl(A) \subseteq Vcl(A)$ and by hypothesis $VSPcl(A) \subseteq U$ therefore A is a VGSPCS in X.

Theorem 3.5: Every VRCS is VGSPCS but not conversely.

Proof: Let A be a VRCS in X**.** Vague Regular closed set $A = Vcl(V\,int(A))$

$Vcl(A) = Vcl(Vcl(V\,int(A))), Vcl(A) = Vcl(V\,int(A)), Vcl(A) = A$, Therefore A is VCS

By hypothesis 3.3,A is VGSPCS in (X,τ)

Example 3.6: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT where $G = \{\langle x,[0.2,0.7],[0.4,0.6]\rangle\}$

Then the vague set $A = \{\langle x,[0.1,0.5],[0.3,0.5]\rangle\}$ is a VGSPCS in X. But not VCS, VGCS, VRS.

Theorem 3.7: Every VSCS is VGSPCS but not conversely.

Proof: Let A be a VSCS in (X,τ).Let $A \subseteq U$ and U is VOS in (X,τ) Since $VSPcl(A) \subseteq A$ $VSPcl(A) \subseteq Vcl(A)$ and A is VSCS in X. $VSPcl(A) \subseteq Vcl(A) = A \subseteq U$

Therefore A is VGSPCS.

Example 3.8: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT where $G = \{\langle x,[0.6,0.7],[0.4,0.6]\rangle\}$

Then the VS $A = \{\langle x,[0.5,0.8],[0.5,0.7]\rangle\}$ is a VGSPCS in X but not VCS.

Remark 3.9: The union of any two VGSPCS is not a VGSPCS in general as seen from the following example.

Example 3.10: Let $X=\{a,b\}$ and let $\tau=\{0,G,1\}$ is a VT where $G=\{\langle x,[0.6,0.7],[0.3,0.6]\rangle\}$

Then the VS $A=\{\langle x,[0.7,0.4],[0.4,0.8]\rangle\}$ and $B=\{x,[0.4,0.8],[0.4,0.2]\}$ is a VGSPCS in X. But $\mathrm{Vint}(\mathrm{Vcl}(A))\supseteq A$ $(A\cup B)$ is not VGSPCS.

Remark 3.11: Theintersection of any two VGSPCS is not a VGSPCS in general as seen from the following example.

Example 3.12: Let $X=\{a,b\}$ and let $\tau=\{0,G,1\}$ is a VT where $G=\{\langle x,[0.5,0.6],[0.2,0.4]\rangle\}$

Then the VS $A=\{\langle x,[0.6,0.8],[0.7,0.7]\rangle\}$ and $B=\{x,[0.3,0.6],[0.4,0.5]\}$ is a VGSPCS in X. $(A\cap B)$ is not VGSPCS.

Theorem 3.13: For a VS A the following conditions are equivalent

a. A is a VOS and a VGSPCS

b. A is a VROS

Proof: $1\Rightarrow 2$ Let A be a VOS and a VGSPCS. Then $\mathrm{SPcl}(A)\subseteq A$

This implies that $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))\subseteq A$ .since A is a VOS $\mathrm{int}(A)=A$ .Therefore $\mathrm{int}(\mathrm{cl}(A))\subseteq A$. Since A is a VOS it is a VSPOS. Hence $A\subseteq \mathrm{int}(\mathrm{cl}(A))$. Therefore $A=\mathrm{int}(\mathrm{cl}(A))$.Hence A is a VROS.

$2\Rightarrow 1$ Let A be a VROS. Therefore $A=\mathrm{int}(\mathrm{cl}(A))$. Since every VROS in a VOS and $A\subseteq A$ Thisimplies that $\mathrm{int}(\mathrm{cl}(A))\subseteq A$, Thatis $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))\subseteq A$, Therefore A is a VSPCS. Hence A is a VGSPCS.

Theorem 3.14: For a VOS A in (X,τ) the following conditions are equivalent

a. A is VCS

b. A is a VGSPCS and VQ-set

Proof: $1\Rightarrow 2$ Since A is a VCS it is a VGSPCS now

$\mathrm{int}(\mathrm{cl}(A))=\mathrm{int}(\mathrm{cl}(A))=\mathrm{int}(A)=A=\mathrm{cl}(A)=\mathrm{cl}(\mathrm{int}(A))$, By hypothesis. Hence A is a VQ-set

$2\Rightarrow 1$ Since A is a VOS and a VGSPCS by hypothesis

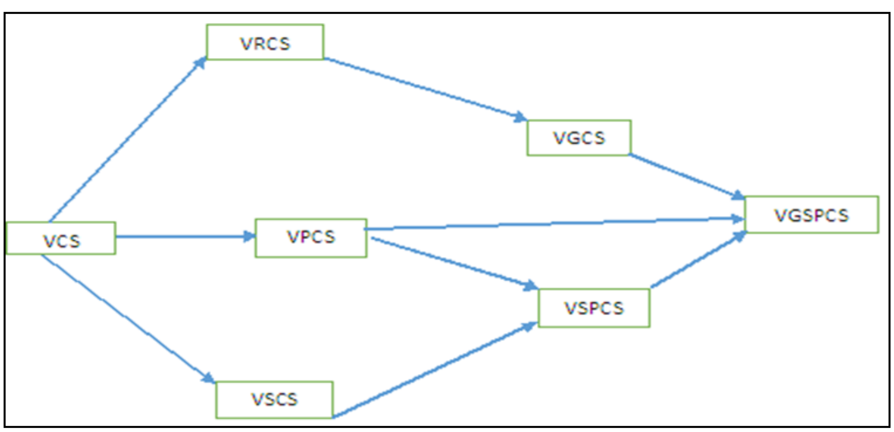
A is a VROS, $\therefore A=\mathrm{int}(\mathrm{cl}(A))=\mathrm{cl}(\mathrm{int}(A))=\mathrm{cl}(A)$ By hypothesis hence A is a VCS

Theorem 3.15: A VS A of a VTS in (X,τ) is a VGSPOS

If and only if $F\subseteq \mathrm{SPint}(A)$ whenever V is a VCS and $V\subseteq A$.

Proof: Necessity: Suppose A is a VGSPOS let V be a VCS such that $F\subseteq A$ then V^c is a VOS and $A^c\subseteq F^c$ by hypothesis A^c is a VGSPCS we have $\mathrm{SPcl}(A^c)\subseteq F^c$ therefore, $F\subseteq \mathrm{SPint}(A)$

Sufficiency: Let U be a VOS such that $A^c\subseteq U$ then $U^c\subseteq \mathrm{SPint}(A)$ Therefore $\mathrm{SPcl}(A^c)\subseteq U$ and A^c a VGSPCS. Hence is a VGSPOS.



4. Vague Generalized Semi Pre Open Set

Definition 4.1: A vague set A is a said to be a vague generalized semi pre-open set (VGSPOS) in (X,τ) then the complement A^c is a VGSPCS in (X,τ). The family of all VGSPOS's of a VTS (X,τ) is denoted by VGSPO(X).

Example 4.2: Let $X=\{a,b\}$ and let $\tau=\{0,G,1\}$ is a VT on X. Where $G=\{\langle[0.5,0.6],[0.4,0.6]\rangle\}$ Then the VS $A=\{\langle x,[0.5,0.7],[0.4,0.6]\rangle\}$ is a VGSPOS in X.

Theorem 4.3: For any $VTS(X,\tau)$ we have the following

1. Every VOS is a VGSPOS
2. Every VSOS is a VGSPOS
3. Every V α OS is a VGSPOS
4. Every VPOS is a VGSPOS. But the conversely not true in general.

Proof: Straight forward

The converse of the above statements need not be true which can be seen from the following example.

Example 4.4: $X=\{a,b\}$ and $G=\{\langle x,[0.6,0.7],[0.5,0.7]\rangle\}$ then, $\tau=\{0,G,1\}$ is a vague topology in X. The vague set $A=\{\langle x,[0.5,0.6],[0.4,0.7]\rangle\}$ is a VGSPOS ix (X,τ) but not VOS.

Example 4.5: $X=\{a,b\}$ and $G=\{\langle x,[0.5,0.6],[0.6,0.7]\rangle\}$ then, $\tau=\{0,G,1\}$ is a vague topology in X. The vague set $A=\{\langle x,[0.6,0.8],[0.4,0.7]\rangle\}$ is a VGSPOS ix (X,τ) but not VSOS.

Example 4.6: $X=\{a,b\}$ and $G=\{\langle x,[0.4,0.5],[0.4,0.7]\rangle\}$ then, $\tau=\{0,G,1\}$ is a vague topology in X. The vague set $A=\{\langle x,[0.3,0.5],[0.4,0.6]\rangle\}$ is a VGSPOS ix (X,τ) but not VαOS.

Example 4.7: $X=\{a,b\}$ and $G=\{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$ then, $\tau=\{0,G,1\}$ is a vague topology in X. The vague set $A=\{\langle x,[0.2,0.4],[0.2,0.5]\rangle\}$ is a VGSPOS ix (X,τ) but not VPOS.

5. Application of Vague Generalized Semi-Pre Closed Set

Definition 5.1: If every VGSPCS in (X,τ) is a VSPCS in (X,τ) then the space can be called as a vague generalized semi pre-closed $T_{1/2}$.

Theorem 5.2: A VTS (X,τ) is a vague semi pre topological space if and only if VSPO(X) = VGSPO(X).

Proof: Necessity: Let A be a VGSPOS in (X,τ). Then A^c is a VGSPCS in (X,τ) by hypothesis is A^C is a VSPCS in (X,τ) and therefore A is a VSPOS in (X,τ) hence VSPOS(X)=VSPO(X)

Sufficiency: Let A be a VSPCS in (X,τ). Then A^c is a VGSPOS in (X,τ) by hypothesis is A^C is a VSPOS in (X,τ) and therefore A is a VSPCS in (X,τ) hence (X,τ) is a VSPT space.

Theorem 5.3: Let (X,τ) be a VTS and X is a VSPT then the following conditions are equivalent.

a. $A\in VGPO(X)$
b. $A\subseteq Vcl(Vint(Vcl(A)))$
c. $Vcl(A)\in VRS(X)$

Proof: $1\Rightarrow 2$ Let A be a VGSPOS then since X is a VSPT space A is a VSPOS therefore $A\subseteq cl(int(A))$

$2\Rightarrow 3$ If $A\subseteq Vcl(Vint(Vcl(A)))$ then $Vcl(A)=Vcl(Vint(Vcl(A)))$ this implies, $V\,cl(A)\in VRC(X)$

$3\Rightarrow 1$ Since $cl(A)$ is a VRCS $V\,cl(A)=Vcl(Vint(Vcl(A)))$ and since, $A\subseteq Vcl(A), A\subseteq Vcl(Vint(Vcl(A)))$ Therefore A is a VSPOS. Hence $A\in VGSPO(X)$

Theorem 5.4

a. $A\in VGSPC(X)$

b. $\mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A))) \subseteq A$

c. $\mathrm{Vint}(A) \in \mathrm{VRO}(X)$

Proof: $1 \Rightarrow 2$ Let A be a VGPCS then since X is a VSPT space A is a VSPCS there fore $\mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A))) \subseteq A$

$2 \Rightarrow 3$ If $\mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A))) \subseteq A$ then $\mathrm{Vint}(A) = \mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A)))$ this implies, $\mathrm{Vint}(A) = \mathrm{VRO}(X)$

$3 \Rightarrow 1$ Since $\mathrm{Vint}(A)$ is a VROS $\mathrm{Vint}(A) = \mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A)))$ and since $\mathrm{Vint}(A) \subseteq A$, $\mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(A))) \subseteq A$ Therefore A is a VSPCS , Hence $A \in \mathrm{VGSPC}(X)$

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