International Journal of Multidisciplinary Research and Development
Online ISSN: 2349-4182, Print ISSN: 2349-5979, Impact Factor: RJIF 5.72
www.allsubjectjournal.com
Volume 4; Issue 4; April 2017; Page No. 63-71

Regular and totally regular property of disjunction of two fuzzy graphs

[1] Dr. K Radha, [2] M Vijaya

[1] P.G. Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli, Tamil Nadu, India
[2] Department of mathematics, Bharathidasan University Constituent College, Perambalur, Tamil Nadu, India

Abstract
In this paper, the degree and total degree of vertices in $G_1 \vee G_2$ in terms of those in G_1 and G_2 are determined for some particular cases. Using them the regular property and the totally regular property of $G_1 \vee G_2$ are studied. In general, the disjunction of two regular (totally regular) fuzzy graphs need not be a regular (totally regular) fuzzy graph. In this paper, the necessary and sufficient conditions for the disjunction of two regular (totally regular) fuzzy graphs to be regular (totally regular) under some restrictions are obtained.

Keywords: degree of a vertex, total degree, regular fuzzy graph, totally regular fuzzy graph, disjunction

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson. J. N and Peng. C. S introduced the concept of operations on fuzzy graphs.

The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Mordeson. J.N. and Peng. C.S [2]. Sunitha. M. S and Vijayakumar. A discussed about the complement of the operations of union, join, Cartesian product and composition on two fuzzy graphs. The degree of a vertex in some fuzzy graphs and the Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations union, join, Cartesian product, composition and conjunction was discussed by Nagoorgani. A and Radha. K. In this paper we study about the degree and total degree of a vertex in disjunction of two fuzzy graphs and the regular and totally regular property of disjunction of two fuzzy graphs.

First we go through some preliminaries which can be found in [1-6].

2. Basic Definitions

Definition 2.1 [1]

A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G:(σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ,(i.e.) $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of G:(σ, μ) is denoted by G*: (V, E) where $E \subseteq V \times V$.

Definition 2.2 [3]

Let $G^*: (V, E)$ be a graph. The degree $d_{G^*}(v)$ of a vertex v in G^* is the number of edges incident with v. if all the vertices of G^* have the same degree r, then G^* is called a regular graph of degree r. Here r is an integer.

Definition 2.3 [5]

Let G:(σ, μ) be a fuzzy graph. The degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$$

Definition 2.4 [6]: Let G:(σ, μ) be a fuzzy graph on G^*. The total degree of a vertex $u \in V$ is defined by

$$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u).$$

If each vertex of G has the same total degree k, then G is said to be a totally regular fuzzy graph of total degree k or a k-totally regular fuzzy graph.

Definition 2.5 [10]

Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$. The Disjunction of G_1^* and G_2^* is $G^* = G_1^* \vee G_2^* : (V, E)$ where $V = V_1 X V_2$ and $E = \{ (u_1, u_2)(u_1, v_2) / u_1 \in V_1, u_2v_2 \in E_2 \} \cup \{ (u_1, u_2)(v_1, u_2) / u_2 \in V_2, u_1v_1 \in E_1 \} \cup \{ (u_1, u_2)(v_1, v_2) / u_1v_1 \in E_1, u_2v_2 \in E_2, u_1 \neq v_1, u_2 \neq v_2 \}$

Define G: (σ, μ) by $\sigma(u_1, u_2) = \sigma_1(u_1) \vee \sigma_2(u_2)$, for all $(u_1, u_2) \in V_1 X V_2$

and

$$\mu((u_1, u_2)(v_1, v_2)) = \begin{cases} \sigma_1(u_1) \vee \mu_2(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in E_2 \\ \sigma_2(u_2) \vee \mu_1(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in E_1 \\ \mu_1(u_1, v_1) \vee \mu_2(u_2, v_2), & \text{if } u_1v_1 \in E_1, u_2v_2 \in E_2 \end{cases}$$

This is called the Disjunction of the fuzzy graphs G_1 and G_2 and is denoted by $G_1 \vee G_2$.

Notation

The relation $\sigma_1 \leq \mu_2$ means that $\sigma_1(u) \leq \mu_2(e), \forall\, u \in V_1$ and $\forall\, e \in E_2$ where σ_1 is a fuzzy subset of V_1 and μ_2 is a fuzzy subset of E_2.

Lemma 2.6 [4]

If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$.

3. Degree of a vertex in $G_1 \vee G_2$

For any $(u_1, u_2) \in V_1 \times V_2$, its degree in $G_1 \vee G_2$ is

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1v_1 \in E_1,\, u_2v_2 \in E_2} \mu((u_1 v_1)(u_2, v_2))$$

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1) + \sum_{u_1v_1 \in E_1,\, u_2v_2 \in E_2} \mu_1(u_1 v_1) \vee \mu_2(u_2, v_2) \quad ---\rightarrow (3.1)$$

In the following theorems, we find the degree of vertices in $G_1 \vee G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem 3.1

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $d_{G_1 \vee G_2}(u_1, u_2) = \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_2}(u_2) d_{G_1^*}(u_1)$.

Proof : Since $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. From (3.1),

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \sigma_1(u_1) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) + \sum_{u_1v_1 \in E_1,\, u_2v_2 \in E_2} \mu_2(u_2, v_2)$$
$$= \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_2}(u_2) d_{G_1^*}(u_1)$$

Corollary 3.2

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $d_{G_1 \vee G_2}(u_1, u_2) = \sigma_1(u_1) d_{G_2^*}(u_2) + t d_{G_2}(u_2) d_{G_1^*}(u_1)$

Proof: The proof follows from theorem 3.1.

Theorem 3.3

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $d_{G_1 \vee G_2}(u_1, u_2) = \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_1}(u_1) d_{G_2^*}(u_2)$

Proof: Since $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$. From (3.1),

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \sigma_1(u_1) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) + \sum_{u_1 \neq v_1, u_2 \neq v_2,\, u_1v_1 \in E_1,\, u_2v_2 \in E_2} \mu_1(u_1, v_1)$$

$$= \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_1}(u_1) d_{G_2^*}(u_2)$$

Corollary 3.4

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $d_{G_1 \vee G_2}(u_1, u_2) = \sigma_2(u_2) d_{G_1^*}(u_1) + t d_{G_1}(u_1) d_{G_2^*}(u_2)$

Proof: The proof follows from theorem 3.3.

Theorem 3.5

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \leq \mu_2$, then $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_2}(u_2)[1 + d_{G_1^*}(u_1)] + \sigma_2(u_2) d_{G_1^*}(u_1)$

Proof: We have $\sigma_1 \leq \mu_2$. Hence $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. From (3.1),

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2 = V_2, u_1v_1 \in E_1} \sigma_2(u_2) + \sum_{u_1 \neq v_1, u_2 \neq v_2, u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_2(u_2, v_2)$$

$$= d_{G_2}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_2}(u_2) d_{G_1^*}(u_1)$$
$$= d_{G_2}(u_2)[1 + d_{G_1^*}(u_1)] + \sigma_2(u_2) d_{G_1^*}(u_1)$$

Corollary 3.6

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \leq \mu_2$, then $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_2}(u_2) + t d_{G_2}(u_2) d_{G_1^*}(u_1)$

Proof: The proof follows from theorem 3.5.

Corollary 3.7

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \leq \mu_2$ and σ_2 is a constant function then $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_2}(u_2)[1 + d_{G_1^*}(u_1)] + c_2 d_{G_1^*}(u_1)$

Proof: The proof follows from theorem 3.5.

Theorem 3.8

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \leq \mu_1$ then$d_{G_1 \vee G_2}(u_1, u_2) = d_{G_1}(u_1)[1 + d_{G_2^*}(u_2)] + \sigma_1(u_1) d_{G_2^*}(u_2)$

Proof: We have $\sigma_2 \leq \mu_1$. Hence $\sigma_1 \geq \mu_2$ and $\mu_2 \leq \mu_1$. From (3.1),

$$d_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1, u_2v_2 \in E_2} \sigma_1(u_1) + \sum_{u_2 = V_2, u_1v_1 \in E_1} \mu_1(u_1, v_1) + \sum_{u_1 \neq v_1, u_2 \neq v_2, u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1, v_1)$$

$$= \sigma_1(u_1) d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{G_2^*}(u_2) d_{G_1}(u_1)$$
$$= d_{G_1}(u_1)[1 + d_{G_2^*}(u_2)] + \sigma_1(u_1) d_{G_2^*}(u_2)$$

Corollary 3.9

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \leq \mu_1$ then $d_{G_1 \vee G_2}(u_1, u_2) = t d_{G_1}(u_1) d_{G_2^*}(u_2) + d_{G_1}(u_1)$

Proof: The proof follows from theorem 3.8.

Corollary 3.10

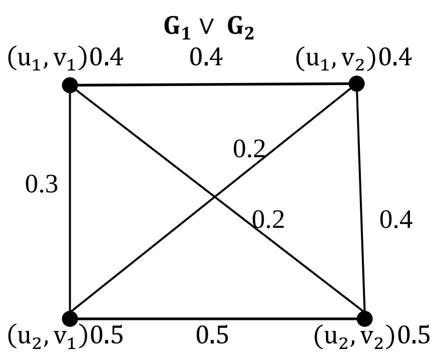
Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \leq \mu_1$ and σ_1 is a constant function then $d_{G_1 \vee G_2}(u_1, u_2) = c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1)[1 + d_{G_2^*}(u_2)]$

Proof: The proof follows from theorem 3.8.

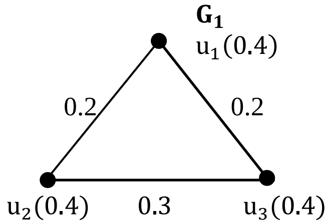
4. Regular Property of Disjunction

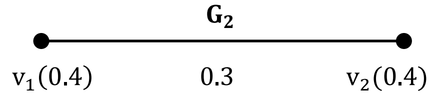
In general, there does not exist any relationship between the regular property of G_1 and G_2 and the regular property of $G_1 \vee G_2$.

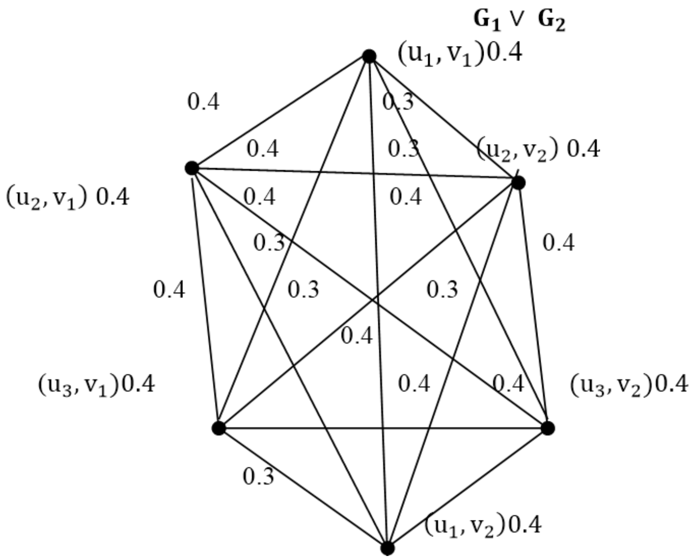
(a) If G_1 and G_2 are regular fuzzy graphs, then $G_1 \vee G_2$ need not be a regular fuzzy graph.



(b) If $G_1 \vee G_2$ is a regular fuzzy graph, then G_1 or G_2 need not be a regular.







In this section, we obtain Characterizations for $G_1 \vee G_2$ to be regular in some particular cases using the formulae obtained in the previous section.

Theorem 4.1

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that G_2^* is a regular graph, σ_1 and σ_2 are constant functions, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_2 is a regular fuzzy graph and G_1^* is a regular graph.

Proof: Let $\sigma_i(u_i) = c_i$ for all $u \in V_i$, where c_i is a constant, $i = 1,2$.

Let G_2^* be $r_2 -$ regular graph.

Assume that $G_1 \vee G_2$ is a regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$, $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_1 \vee G_2}(v_1, v_2)$

From theorem 3.1,

$$c_2\, d_{G_1^*}(u_1) + c_1\ r_2 + d_{G_2}(u_2) d_{G_1^*}(u_1) = c_2\, d_{G_1^*}(v_1) + c_1\ r_2 + d_{G_2}(v_2) d_{G_1^*}(v_1)$$
$$\Rightarrow c_2\, d_{G_1^*}(u_1) + d_{G_2}(u_2) d_{G_1^*}(u_1) = c_2\, d_{G_1^*}(v_1) + d_{G_2}(v_2) d_{G_1^*}(v_1) \quad ---\rightarrow (4.1)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$.

From (4.1), $c_2\, d_{G_1^*}(u) + d_{G_2}(u_2) d_{G_1^*}(u) = c_2\, d_{G_1^*}(u) + d_{G_2}(v_2) d_{G_1^*}(u)$
$$\Rightarrow d_{G_2}(u_2) d_{G_1^*}(u) = d_{G_2}(v_2) d_{G_1^*}(u)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for every u_2, v_2 in V_2. Thus G_2 is a regular fuzzy graph.

Similarly fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$.

From (4.1), $c_2\, d_{G_1^*}(u_1) + d_{G_2}(v) d_{G_1^*}(u_1) = c_2\, d_{G_1^*}(v_1) + d_{G_2}(v) d_{G_1^*}(v_1)$
$$\Rightarrow [c_2 + d_{G_2}(v)] d_{G_1^*}(u_1) = [c_2 + d_{G_2}(v)] d_{G_1^*}(v_1)$$
$$\Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$$

This is true for every u_1, v_1 in V_1. Thus G_1^* is a regular graph.

Conversely, Let G_2 be a k_2- regular fuzzy graph and G_1^* be a r_1- regular graph.
Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from theorem 3.1,

$$d_{G_1 \vee G_2}(u_1, u_2) = c_2\, d_{G_1^*}(u_1) + c_1\ r_2 + d_{G_2}(u_2) d_{G_1^*}(u_1)$$
$$= (c_2 + k_2) r_1 + c_1\ r_2$$

Hence $G_1 \vee G_2$ is a regular fuzzy graph.

Theorem 4.2 : Let $G_1{:}(\sigma_1, \mu_1)$ and $G_2{:}(\sigma_2, \mu_2)$ be two fuzzy graphs such that G_2^* is a regular graph, σ_1 is a constant function, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and G_1^* is a regular graph.
Proof: The proof of the theorem follows from theorem 4.1.

Theorem 4.3 : Let $G_1{:}(\sigma_1, \mu_1)$ and $G_2{:}(\sigma_2, \mu_2)$ be two fuzzy graphs such that G_1^* is a regular graph, σ_1 and σ_2 are constant functions, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_1 is a regular fuzzy graph and G_2^* is a regular graph.
Proof: Let $\sigma_i(u_i) = c_i$ for all $u \in V_i$, where c_i is a constant, $i = 1,2$.
Assume that $G_1 \vee G_2$ is a regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$, $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_1 \vee G_2}(v_1, v_2)$

From theorem 3.3,

$$c_1 d_{G_2^*}(u_2) + c_2\ r_1 + d_{G_1}(u_1) d_{G_2^*}(u_2) = c_1 d_{G_2^*}(v_2) + c_2\ r_1 + d_{G_1}(v_1) d_{G_2^*}(v_2)$$
$$\Rightarrow [c_1 + d_{G_1}(u_1)] d_{G_2^*}(u_2) = [c_1 + d_{G_1}(v_1)] d_{G_2^*}(v_2) \qquad ---\rightarrow (4.2)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$.

From (4.2), $[c_1 + d_{G_1}(u)] d_{G_2^*}(u_2) = [c_1 + d_{G_1}(u)] d_{G_2^*}(v_2)$
$$\Rightarrow d_{G_2^*}(u_2) = d_{G_2^*}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2^* is a regular graph.

Similarly fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$.

From (4.2), $[c_1 + d_{G_1}(u_1)] d_{G_2^*}(v) = [c_1 + d_{G_1}(v_1)] d_{G_2^*}(v) \Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.

Conversely, Let G_1 be a k_1-regular fuzzy graph and G_2^* be a r_2-regular graph.

Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from theorem 3.3,

$$d_{G_1 \vee G_2}(u_1, u_2) = c_1 r_2 + k_1 r_2 + c_2 r_1$$
$$= (c_1 + k_1) r_2 + c_2 r_1$$

Hence $G_1 \vee G_2$ is a regular fuzzy graph.

Theorem 4.4
Let $G_1{:}(\sigma_1, \mu_1)$ and $G_2{:}(\sigma_2, \mu_2)$ be two fuzzy graphs such that G_1^* is a regular graph, σ_2 is a constant function, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and G_1^* is a regular graph.
Proof: The proof of the theorem follows from theorem 4.3.

Theorem 4.5
Let $G_1{:}(\sigma_1, \mu_1)$ and $G_2{:}(\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$ and σ_2 is a constant function. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_2 is a regular fuzzy graph and G_1^* is a regular graph.
Proof: Let $\sigma_2(u_2) = c_2$ for all $v \in V_2$, where c_2 is a constant.
Assume that $G_1 \vee G_2$ is a regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$, $d_{G_1 \vee G_2}(u_1, u_2) = d_{G_1 \vee G_2}(v_1, v_2)$

From Corollary 3.7,

$$d_{G_2}(u_2)\left[1+d_{G_1^*}(u_1)\right]+c_2 d_{G_1^*}(u_1)=d_{G_2}(v_2)\left[1+d_{G_1^*}(v_1)\right]+c_2 d_{G_1^*}(v_1) \qquad --\rightarrow (4.3)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 X V_2$.

From (4.3), $d_{G_2}(u_2)\left[1+d_{G_1^*}(u)\right]+c_2 d_{G_1^*}(u)=d_{G_2}(v_2)[1+d_{G_1^*}(u)]+c_2 d_{G_1^*}(u)$

$$\Longrightarrow d_{G_2}(u_2)=d_{G_2}(v_2)$$

This is true for every u_2, v_2 in V_2. Thus G_2 is a regular fuzzy graph.
Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 X V_2$.

From (4.3), $d_{G_2}(v)\left[1+d_{G_1^*}(u_1)\right]+c_2 d_{G_1^*}(u_1)=d_{G_2}(v)[1+d_{G_1^*}(v_1)]+c_2 d_{G_1^*}(v_1)$

$$\Longrightarrow d_{G_2}(v)+d_{G_2}(v)d_{G_1^*}(u_1)+c_2 d_{G_1^*}(u_1)=d_{G_2}(v)+d_{G_2}(v)d_{G_1^*}(v_1)+c_2 d_{G_1^*}(v_1)$$

$$\Longrightarrow \left[c_2+d_{G_2}(v)\right]d_{G_1^*}(u_1)=\left[c_2+d_{G_2}(v)\right]d_{G_1^*}(v_1)$$

$$\Longrightarrow d_{G_1^*}(u_1)=d_{G_1^*}(v_1)$$

This is true for every u_1, v_1 in V_1. Thus G_1^* is a regular graph.
Conversely, let G_2 be a k_2 −regular fuzzy graph and G_1^* be a r_1 −regular graph.
Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from Corollary 3.7,

$$d_{G_1 \vee G_2}(u_1, u_2)=k_2[1+r_1]+c_2 r_1$$

Hence $G_1 \vee G_2$ is a regular fuzzy graph.

Theorem 4.6
Let $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \leq \mu_2$ and σ_2 is a constant function, then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and G_1^* is a regular graph.
Proof: The proof of the theorem follows from Corollary 3.6.

Theorem 4.7
Let $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \leq \mu_1$ and σ_2 is a constant function. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_1 is a regular fuzzy graph and G_2^* is a regular graph.
Proof: Let $\sigma_1(u_1)=c_1$ for all $u \in V_1$, where c_1 is a constant.
Assume that $G_1 \vee G_2$ is a regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$, $d_{G_1 \vee G_2}(u_1, u_2)=d_{G_1 \vee G_2}(v_1, v_2)$

From corollary 3.10,

$$d_{G_1}(u_1)\left[1+d_{G_2^*}(u_2)\right]+c_1 d_{G_2^*}(u_2)=d_{G_1}(v_1)\left[1+d_{G_2^*}(v_2)\right]+c_1 d_{G_2^*}(v_2) \qquad --\rightarrow (4.4)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 X V_2$.

From (4.4), $d_{G_1}(u)\left[1+d_{G_2^*}(u_2)\right]+c_1 d_{G_2^*}(u_2)=d_{G_1}(u)\left[1+d_{G_2^*}(v_2)\right]+c_1 d_{G_2^*}(v_2)$

$\Longrightarrow d_{G_1}(u)+d_{G_1}(u)d_{G_2^*}(u_2)+c_1 d_{G_2^*}(u_2)=d_{G_1}(u)+d_{G_1}(u)d_{G_2^*}(v_2)+c_1 d_{G_2^*}(v_2)$

$$\Longrightarrow \left[c_1+d_{G_1}(u)\right]d_{G_2^*}(u_2)=\left[c+d_{G_1}(u)\right]d_{G_2^*}(v_2)$$

$$\Longrightarrow d_{G_2^*}(u_2)=d_{G_2^*}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2^* is a regular graph.
Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 X V_2$.

From (4.4),

$$d_{G_1}(u_1)\left[1+d_{G_2^*}(v)\right]+c_1 d_{G_2^*}(v)=d_{G_1}(v_1)\left[1+d_{G_2^*}(v)\right]+c_1 d_{G_2^*}(v)$$

$$\Rightarrow d_{G_1}(u_1)=d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.
Conversely, let G_1 be a k_1 −regular fuzzy graph and G_2^* is a r_2 −regular graph.

Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from corollary 3.10, $d_{G_1 \vee G_2}(u_1, u_2) = k_1[1 + r_2] + c_1 r_2$. Hence $G_1 \vee G_2$ is a regular fuzzy graph.

Theorem 4.8
Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \le \mu_1$ and σ_2 is a constant function. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph and G_2^* is a regular graph.
Proof: The proof follows from corollary 3.9.

5. Total Degree of a vertex in $G_1 \vee G_2$

$$td_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1v_1 \in E_1,\, u_2v_2 \in E_2} \mu((u_1 v_1)(u_2, v_2))$$
$$td_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1)$$
$$+ \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1 v_1) \vee \mu_2(u_2, v_2) + \sigma_1(u_1) \vee \sigma_2(u_2) \quad \text{---}\!\!\rightarrow (5.1)$$

In the following theorems, we find the total degree of vertices in $G_1 \vee G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem 5.1
Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \ge \mu_2$, $\sigma_2 \ge \mu_1$ and $\mu_1 \le \mu_2$. Then $td_{G_1 \vee G_2}(u_1, u_2) = \sigma_1(u_1) d_{G_2^*}(u_2) + td_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_2(u_2)$.
Proof: Since $\sigma_1 \ge \mu_2$, $\sigma_2 \ge \mu_1$ and $\mu_1 \le \mu_2$. Then $\sigma_1 \le \sigma_2$ .From (5.1),

$$td_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \sigma_1(u_1) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) + \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_2(u_2, v_2) + \sigma_1(u_1) \vee \sigma_2(u_2)$$
$$= \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \vee \sigma_2(u_2)$$
$$= \sigma_1(u_1) d_{G_2^*}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_1^*}(u_1) d_{G_2}(u_2) + \sigma_2(u_2)$$
$$= \sigma_1(u_1) d_{G_2^*}(u_2) + d_{G_1^*}(u_1) td_{G_2}(u_2) + \sigma_2(u_2)$$

Theorem 5.2
Let $G_1 : (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \ge \mu_2$, $\sigma_2 \ge \mu_1$ and $\mu_2 \le \mu_1$. Then $td_{G_1 \vee G_2}(u_1, u_2) = \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_2^*}(u_2) td_{G_1}(u_1) + \sigma_1(u_1)$.
Proof: Proof is similar to the proof of theorem 5.1.

Theorem 5.3
Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \le \mu_2$. Then $td_{G_1 \vee G_2}(u_1, u_2) = td_{G_2}(u_2)[1 + d_{G_1^*}(u_1)]$.
Proof: We have $\sigma_1 \le \mu_2$. Hence $\sigma_2 \ge \mu_1$ and $\mu_1 \le \mu_2$. Also $\sigma_1 \le \sigma_2$. From (5.1),

$$td_{G_1 \vee G_2}(u_1, u_2) = \sum_{u_1 = V_1,\, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2 = V_2,\, u_1v_1 \in E_1} \sigma_2(u_2) + \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_2(u_2, v_2) + \sigma_1(u_1) \vee \sigma_2(u_2)$$
$$\Rightarrow td_{G_1 \vee G_2}(u_1, u_2) = d_{G_2}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \vee \sigma_2(u_2)$$
$$= d_{G_2}(u_2) + [\sigma_2(u_2) + d_{G_2}(u_2)] d_{G_1^*}(u_1) + \sigma_2(u_2)$$
$$= td_{G_2}(u_2)[1 + d_{G_1^*}(u_1)]$$

Theorem 5.4
Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \le \mu_1$ then $td_{G_1 \vee G_2}(u_1, u_2) = td_{G_1}(u_1)[1 + d_{G_2^*}(u_2)]$
Proof: Proof is similar to the proof of theorem 5.3.

6. Totally Regular Property of Disjunction
In general, there does not exist any relationship between the totally regular property of G_1 and G_2 and the totally regular property of $G_1 \vee G_2$.
In this section, we obtain Characterizations for $G_1 \vee G_2$ to be totally regular in some particular cases.

Theorem 6.1

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that G_2^* is a regular graph, σ_1 and σ_2 are constant functions, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. Then $G_1 \vee G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and G_1^* is a regular graph.

Proof: Let $\sigma_i(u_i) = c_i$ for all $u \in V_i$, where c_i is a constant, $i = 1,2$.

Let G_2^* be $r_2 -$ regular graph.

Assume that $G_1 \vee G_2$ is a totally regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$, $td_{G_1 \vee G_2}(u_1, u_2) = td_{G_1 \vee G_2}(v_1, v_2)$

From theorem 5.1,

$$c_1\ r_2 + td_{G_2}(u_2)d_{G_1^*}(u_1) + c_2 = c_1\ r_2 + td_{G_2}(v_2)d_{G_1^*}(v_1) + c_2$$
$$\Rightarrow td_{G_2}(u_2)d_{G_1^*}(u_1) = td_{G_2}(v_2)d_{G_1^*}(v_1) \ ---\rightarrow (6.1)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \ X\ V_2$.

Then from (6.1), $\Rightarrow td_{G_2}(u_2)d_{G_1^*}(u) = td_{G_2}(v_2)d_{G_1^*}(u)$

$$\Rightarrow td_{G_2}(u_2) = td_{G_2}(v_2)$$

This is true for every u_2, v_2 in V_2. Thus G_2 is a totally regular fuzzy graph.

Similarly fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \ X\ V_2$.

From (6.1) , $td_{G_2}(v)d_{G_1^*}(u_1) = td_{G_2}(v)d_{G_1^*}(v_1) \Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$.

This is true for every u_1, v_1 in V_1. Thus G_1^* is a regular graph.

Conversely, Let G_2 be a $k_2 -$ totally regular fuzzy graph and G_1^* be a $r_1 -$ regular graph

Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from theorem 5.1,

$$td_{G_1 \vee G_2}(u_1, u_2) = c_1\ r_2 + k_2 r_1 + c_2$$

Hence $G_1 \vee G_2$ is a totally regular fuzzy graph.

Theorem 6.2

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that G_1^* is a regular graph, σ_1 and σ_2 are constant functions, $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$. Then $G_1 \vee G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph and G_2^* is a regular graph.

Proof: Proof is similar to the proof of theorem 6.1.

Theorem 6.3

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$. Then $G_1 \vee G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and G_1^* is a regular graph.

Proof: Assume that $G_1 \vee G_2$ is a totally regular fuzzy graph.

Then for any two vertices $(u_1, u_2), (v_1, v_2)$ in $G_1 \vee G_2$,

$td_{G_1 \vee G_2}(u_1, u_2) = td_{G_1 \vee G_2}(v_1, v_2)$

From theorem 5.3, this gives

$$td_{G_2}(u_2)\left[1 + d_{G_1^*}(u_1)\right] = td_{G_2}(v_2)\left[1 + d_{G_1^*}(v_1)\right] \ ---\rightarrow (6.2)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \ X\ V_2$.

From (6.2), $td_{G_2}(u_2)\left[1 + d_{G_1^*}(u)\right] = td_{G_2}(v_2)[1 + d_{G_1^*}(u)]$

$$\Rightarrow td_{G_2}(u_2) = td_{G_2}(v_2)$$

This is true for every u_2, v_2 in V_2. Thus G_2 is a totally regular fuzzy graph.

Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \ X\ V_2$.

From (6.2), $td_{G_2}(v)\left[1 + d_{G_1^*}(u_1)\right] = td_{G_2}(v)[1 + d_{G_1^*}(v_1)] \Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$

This is true for every u_1, v_1 in V_1. Thus G_1^* is a regular graph.

Conversely, let G_2 be a $k_2 -$ totally regular fuzzy graph and G_1^* be a $r_1 -$ regular graph.

Then for any vertex (u_1, u_2) of $G_1 \vee G_2$, from theorem 5.3, $td_{G_1 \vee G_2}(u_1, u_2) = k_2[1 + r_1]$.

Hence $G_1 \vee G_2$ is a regular fuzzy graph.

Theorem 6.4
Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs sch that $\sigma_2 \leq \mu_1$. Then $G_1 \vee G_2$ is a regular fuzzy graph if and only if G_1 is a totally regular fuzzy graphs and G_2^* is a regular graph.
Proof: Proof is similar to the proof of theorem 6.3.

Conclusion
In this paper, we have found the degree and total degree of vertices in disjunction of two fuzzy graphs and we have shown that the disjunction of two regular (totally regular) fuzzy graph need not be regular (totally regular) fuzzy graph. We have obtained necessary and sufficient condition for the disjunction of two fuzzy graphs to be regular and totally regular in some particular cases.

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