

A brief note on the units of linear dimension of the Vedic Śulvasūtras

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Abstract

The Śulvasūtras give the construction rules of the sacrificial altars, these alters are made in bricks in very complicated shape and size, in fact all the construction ware done with rope and pegs. The Śulva (rope) was used to solve propositions about the construction of various rectilinear figures; combination, transformation, and application of areas; menstruation of areas and volumes; squaring of the circle and vice versa and many others geometrical figures. That's why the Śulvasūtras give the units of measurements. The units like *aṅgulam*, *padam*, *prakramam*, *prādeśam* and many others used in the Śulvasūtras. The terms reveal that these were coined from body measures being commonly used in daily life, and became quite popular in social life. Although they are derived from the basic unit "Puruṣa", [size of the *Yajamāna*] standardization measures are also taken. An *aṅgulam* is a standard unit of measurement; it is equal to fourteen grains of the *aṅu* and it is equal to thirty-four *tilam* (sesame) grains put together with their broad sides.

Keywords: śulvasūtra, baudhāyana, mānava, Āpastamba, kātyayana, aṅgulam

1. Introduction

The Śulvasūtras are a small part of the *Kalpa Sūtras*; give the geometric basis for sacrificial structures like altars and hearths. In Vedic India the religious ceremonies and sacrifices were performed on altars. The altars are raised platforms made of bricks. There were two types of sacrifices namely perpetual and timely. The design of the altar for each one is different and of intricate in design. The design and construction of these altars gave rise to a new branch of knowledge known as Śulvasūtras. The origin of the word can be traced in the word *Śulva* (rope) and *Sūtra* (aphorisms). This paper has presented a brief outline of the Śulvasūtras.

2. Period of the Śulvasūtras

Scholars are of different opinion regarding the time of composition of the Śulvasūtras. Kāśīkar^[1] is of opinion that Baudhāyana Śulvasūtra and Vādhula Śulvasūtra was composed in 800 – 500 B.C., Mānava Śulvasūtra and Āpastamba Śulvasūtra in 650 – 300 B.C., Satyāśāḍha Śulvasūtra, Varāha Śulvasūtra and Kātyayana Śulvasūtra in 300 B.C. – 400 A.D. According to Satya Prokash^[2] Baudhāyana Śulvasūtra is in 800 B.C., Mānava Śulvasūtra in 750 B.C., and Āpastamba Śulvasūtra in 600 B.C. and Kātyayana Śulvasūtra in 200 B.C. According to Dr. K. D. Dvivedi Baudhāyana Śulvasūtra in 900 – 850 B.C., Āpastamba Śulvasūtra in 720 B.C., Kātyayana Śulvasūtra in 400 B.C. According to P. V. Kane^[3] Perhaps the Baudhāyana Śulvasūtra is the oldest, formulated around 800 B.C. and the other Śulvasūtras was composed in the next 400 years or.

3. A brief discussion on the Śulvasūtras

The root word 'Śulv' or 'Sulb' means to measure and *Śulva* means measuring rope or cord or string. But the sūtras

themselves use the word *rajju* not *Śulva* in this sense. The Kātyāyana Śulvasūtra opens with "*rajjusamāsam vakṣyāmah*" (manipulation with cords will now be expounded). This cord could work as a straight edge (stretched tight), a compass (drawn around a point), and more; such as getting proportional lengths, by doubling the string over itself the desired number of times. A pole was also often used, for functions such as the making of circles and sometimes in marking corners of rectangles, so that the cord or rope could be stretched around the poles to make various shapes or measurements.

The Śulvasūtras are a collection of early Hindu religious documents. These Sūtras contained the instructions for constructing special altars (*vedi*, *citi*, *maṇḍapa*) to the sacrifice. The Śulvasūtras describe how different sacrificial altars are required to be erected. These altars, made out of brick, could be very complicated in shape and size, and often required the use of mathematical formulas. These Śulvasūtras looked upon as the earliest literature on Indian geometry. About the Śulvasūtras, Mr. A. C. Burnell says that "we must look to the Śulva portion of the Kalpa Sūtras for the earliest beginnings of geometry among the Brāhmaṇas."

The Śulvasūtras belong to the Yajurveda. There is neither any Śulvasūtras belonging to Ṛgveda or to Sāmveda and Atharvaveda. Many texts have not come down to us. Scholars are of different opinion regarding the numbers of Śulvasūtras. Gaurinath Sastri^[4] says – "Among the Śulvasūtras of Black YV mention may be made of Baudhāyana, Āpastamba, and Hiranyakeśin belonging to Taittirīya, Kāthaka to the Kāthaka and Mānava and Varāha to the Maitrāyaṇi School. The White YV possesses Kātyāyana-Sulvasūtra". Sukumari Bhattacharji^[5] says – "Altogether we have seven Śulvasūtras by Baudhāyana, Kātyāyana, Āpastamba, Satyāśāḍha, Kātyāyana, Varāha and Maitrāyaṇī." Ravi P. Agarawal and Syamal K. Sen says – "only seven Śulvasūtras are extant, named for the sages

¹. Cāra Śulbasūtra, Dr. Raghunath Purusottam Kulakarni, M.S.R.V.V.P.

². Cāra Śulbasūtra, Dr. Raghunath Purusottam Kulakarni, M.S.R.V.V.P.

³. History of the Dharmasāstras, Introduction- P.V. Kane

⁴. A History of Vedic Literature, Gaurinath Sastri, 2006

⁵ Literature in the Vedic Age, Vol-II, Calcutta

who wrote them: Apastamba, Baudhayana (born 3200 B.C.), Katyayana, Manava, Maitrayana, Varāha and Vidhula” Dr. Raghunath Purusottam Kulakarni enumerates eight Śulvasūtras— Baudhāyana, Āpastamba, Satyāśāḍha, Mānava, Vādhula, Maitriyaṇī and Varāha Śulvasūtra in Black YV and White YV is Kātyāyana Śulvasūtra. In these Śulvasūtras the rules of construction of sacrificial altar is not very different between them; they all use the same technical terms for the same kinds of geometrical measurements of altars. But the numbers of available Śulvasūtras are four, which are mathematically very significant – Baudhāyana Śulvasūtra, Mānava Śulvasūtra, Āpastamba Śulvasūtra and Kātyāyana Śulvasūtra.

3.1 Baudhāyana Śulvasūtra

The Baudhāyana Śulvasūtra is the oldest Śulvasūtra and it belongs to the school of the Taittirīya Saṁhitā. Dvarakanath Yajva is the main commentator of this Śulvasūtra, has divided it into three Adhyāyas – “*vyākṛtaḥ adhyāyaḥ*”, “*dharmavācaka*” and “*upapatti samanvitaḥ*”. Dr. Caland and Prof. Thībo have also classified the BSS into ten adhyāyas – adhyāya I-113 sūtras, adhyāya II-83 sūtras, adhyāya III-61 sūtras, adhyāya IV-117 sūtras, adhyāya V-36 sūtras, adhyāya VI-22 sūtras, adhyāya VII-16 sūtras, adhyāya VIII-17 sūtras, adhyāya IX-33 sūtras and adhyāya X- 21 sūtras.

The Baudhāyana Śulvasūtra describes the units of measurements, drawing of a square with the help of only cord (*Rajju*) and pole (*Śanku*), to draw an oblong, description of *dvikarṇī*, *trīya karṇī*, the Baudhāyana theorem^[6] which known as Pythagorean theorem, method to combine two squares, method to deduct one smaller square from a larger one, to turn a square into an oblong and an oblong to a square, to turn a square into a triangle of equal area, to turn a square into a rhombus or double triangle, to turn a square into a circle of equal area, to turn a circle into a square of equal area, a relation between the diagonal of a square and the side of the square, the concept of *saviṣeṣa*, distance between *Āhavanīya agni* and *Gārhapatya agni*, various type of *vedī*, different varieties and quality of bricks. And also describe the *Śyenaciti* that is the altar of the shape of falcon, its body, tail, head, wings, lengthening of wings; sixteen type of bricks for piling the *Ratha-Cakra citi*, two types of *Droṇaciti*, construction rules of the *Śmaśāna citi*, *Kūrma citi* or the altar of the shape of a *tortoise*, either with angular limbs or with circular limbs, laying of bricks, shaping of the head and feet, different types of bricks etc.

3.2 Āpastamba Śulvasūtra

The Āpastamba Śulvasūtra is divided into six paṭalas in 21 khaṇḍas. In the 1st paṭala one to three khaṇḍas, 2nd paṭala four to five khaṇḍas, 3rd paṭala six to tenth khaṇḍas, 4th paṭala eleven to fourteenth khaṇḍas, 5th paṭala fifteen to seventeen khaṇḍas and sixth paṭala is eighteen to twenty one khaṇḍas. The ĀSS deals how can draw a square, the diagonal of an oblong produces by itself both the areas with the two sides of the oblong produce separately i.e. Pythagorean theorem^[7], description of *dvikarṇī*, *trikarṇī*, method to combine two or more square which area is same or different, to turn a square

in to an oblong and an oblong in to a square of equal area, place of the Agni, the concept of *saviṣeṣa*^[8], various type of *vedī* such as *Saumiki vedī*, *Mhāvedī*, *Sautrāmaṇivedī* etc. The Āpastamba Śulvasūtra describe the tape for measuring the hearths and goes on with different subjects. Among them we have the geometrical solution of the architectural problem of fitting a circle within a square and vice versa. Later sections deal with the different altars for the major sacrifices like *Soma*, *Darśapūrṇamāsa*, *Sautrāmaṇī*, *Aśvamedha* etc. This Śulvasūtra employs eleven technical terms which signifies cultivation and advanced knowledge of practical geometry in architecture. It also presupposes knowledge of arithmetic and algebra, for which formulas are given which use square root and decimals.

3.3 Kātyāyana Śulvasūtra

The Kātyāyana Śulvasūtra is divided into six chapters in 101 sūtras^[9]. The KSS begins with practical directions for describing a circle with a pole and a rope which equals the pole in length, and then instructs the way to determine north, south east and west. The purpose is to building hearths for the three different fires in three different shapes –circular, rectangular and semicircular. Measuring these hearths, their exact situation, and the distance between them is the main subject of the text. It deals how a square turn into an oblong and an oblong into a square, a circle in to a square in same area and a square in to a circle in same area, a triangle in to a rectangle in same area. Then come altars for the different sacrifices; these are described with the bricks, their exact measurements, the order of piling them in the various prescribed shapes and sizes, as well as the exact angles and directions at which the sacrificial articles are to be placed.

3.4 Mānava Śulvasūtra

The Mānava Śulvasūtra is divided into three parts. The Mānava Śulvasūtra like all the Śulvasūtras, contained approximate constructions of circles from rectangles, and squares from circles, which can be thought of as giving approximate values of π (π), various geometrical measurements, concept of *saviṣeṣa*, various types of bricks. It describe the construction rules of varies *vedī*, *maṇḍapa*, *citi-Cārakyavedī*, *Pāśukivedī*, *Paitṛkivedī*, $7\frac{1}{2}$ square *puruṣa* *Rathackraciti* etc. Its subject meters also same in others Śulvasūtras.

4. Units of Linear dimensions

The units like *aṅgulam*, *padam*, *prakramam*, *prādeśam* and many others, had a long tradition and used earlier in the *Saṁhitas* and *Brāhmanic* literature in the same sense as those in the Śulvasūtras. The terms reveal that these were coined from body measures being commonly used in daily life, and became quite popular in social life. Although they are derived from the basic unit “*Puruṣa*”, [size of the *Yajamāna*] standardization measures are also taken. An *aṅgulam* is a standard unit of measurement; it is equal to fourteen^[10] grains of the *aṅu* and it is equal to thirty-four *tilam* (sesame) grains

⁶ BSS-I.48, “*dirghacaturasrayākṣanayārajjuh pārśvamānī tiryānmānī ca yat prthagbhute kurutastadubhayaṁ karoti*”

⁷ ĀSS-I.8, “*dirghasyākṣanayārajjuh pārśvamānī tiryānmānī ca yat prthagbhute kurutastadubhayaṁ karoti*”

⁸ ĀSS-I.12, “*pramāṇam trīyena vardhayettaceturthenātmacatustrimśonen sa viṣeṣaḥ*”

⁹ Scholars are of different opinion regarding the chapters and numbers of sūtras of the KSS.

¹⁰ “*Caturdaśāṅvaḥ*”, BSS. I.4.

put together with their broad sides. The units of measurement process are shown below:-

4.1 Baudhāyana Śulvasūtra (BŚS)

1 aṅgulaṃ = 14 aṅṇu	(BŚS.1.4) = 1.9 cm. ¹¹¹
1 aṅgulaṃ = 34 tilam ¹¹²	(BŚS.1.5) = 1.9 cm.
1kṣudrapadam = 10 aṅgulaṃ	(BŚS.1.6) = 19 cm.
1prādeśam = 12 aṅgulaṃ	(BŚS.1.7) = 22.8 cm.
1 pṛtha = 13 aṅgulaṃ	(BŚS.1.8) = 24.7 cm.
1 padaṃ = 15 aṅgulaṃ	(BŚS.1.9) = 28.5 cm.
1 īṣā = 188 aṅgulaṃ	(BŚS.1.10) = 357.2 cm
1akṣam = 104 aṅgulaṃ	(BŚS.1.11) = 197.6 cm
1yugaṃ = 86 aṅgulaṃ	(BŚS.1.12) = 163.4 cm
1jānu = 32 aṅgulaṃ	(BŚS.1.13) = 60.8 cm.
1śamyā = 1 bāhu = 36 aṅgulaṃ	(BŚS.1.14) = 68.4 cm.
1prakramam = 2 padaṃ = 30 aṅgulaṃ	(BŚS.1.15) = 57.0 cm.
1aratni = 2 prādeś = 24 aṅgulaṃ	(BŚS.1.16) = 45.6 cm.
1puruṣa = 5 aratni = 120 aṅgulaṃ	(BŚS.1.19) = 228 cm.
1vyāma = 5 aratni = 120 aṅgulaṃ	(BŚS.1.20) = 228 cm.
1vyāyamam = 4 aratni = 96 aṅgulaṃ	(BŚS.1.29) = 182.4 cm
1virāt = 10 padaṃ = 150 aṅgulaṃ	(BŚS.1.78) = 285.0 cm

4.2 Mānava Śulvasūtra (MŚS)

1 īṣā = 188 aṅgulaṃ	(MŚS. 10.1.2.1) = 357.2 cm
1 akṣam = 104 aṅgulaṃ	(MŚS. 10.1.2.1) = 197.6 cm
1 yugaṃ = 86 aṅgulaṃ	(MŚS. 10.1.2.1) = 163.4 cm
1 sarṣapaṃ = 6 keśh	(MŚS. 10.1.4.3)
1 yaba = 6 sarṣapaṃ	(MŚS. 10.1.4.3) = 0.32 cm.
1 aṅgulaṃ = 6 yaba	(MŚS. 10.1.4.4) = 1.9 cm.
1 prādeśam = 10 aṅgulaṃ	(MŚS. 10.1.4.4) = 19 cm.
1 vitasti = 12 aṅgulaṃ	(MŚS. 10.1.4.4) = 22.8 cm.
1 aratni = 2 vitasti = 24 aṅgulaṃ	(MŚS. 10.1.4.4) = 45.6 cm.
1 vyāyamam = 4 aratni = 96 aṅgulaṃ	(MŚS.10.1.4.4) = 182.4 cm.
1 puruṣa = 5 aratni = 120 aṅgulaṃ	(MŚS. 10.1.4.5) = 228 cm.
1 arva = 12 aṅgulaṃ	(MŚS. 10.3.1.3) = 11.4 cm.
1 bāhu = 36 aṅgulaṃ	(MŚS. 10.3.1.9) = 68.4 cm.
1 kṛṣṇāla = 3 yaba	(MŚS. 10.1.4.6)
1 niṣka = 4 kṛṣṇāla	(MŚS. 10.1.4.6)

4.3 Āpastamba Śulvasūtra (ĀŚS)

1 prakramam = 2 /3 padaṃ = 30 /45 aṅgulaṃ (ĀŚS. 6.2.) = 57.0/85 cm	
1akṣam = 104 aṅgulaṃ	(ĀŚS. 6.13) = 197.6 cm.
1 īṣā = 188 aṅgulaṃ	(ĀŚS. 6.13) = 357.2 cm.
1 yugaṃ = 86 aṅgulaṃ	(ĀŚS. 6.13) = 163.4 cm.
1 śamyā = 36 aṅgulaṃ	(ĀŚS. 6.22) = 68.4 cm.
1 padaṃ = 15 aṅgulaṃ	(ĀŚS. 6.23) = 28.5 cm.
1 aṅṇkaṃ = 30 aṅgulaṃ	(ĀŚS. 11.4) = 57.0 cm.
1 aratni = 24 aṅgulaṃ	(ĀŚS. 11.5, 15.12) = 45.6 cm.
1 pūrbasthi = 20 aṅgulaṃ	(ĀŚS. 11.6) = 38.0 cm.
1 puruṣa = 5 aratni = 120 aṅgulaṃ	(ĀŚS. 15.10) = 228 cm.
1 vyāyamam = 4 aratni = 96 aṅgulaṃ	(ĀŚS. 15.11) = 182.4 cm.
1 prādeśam = 1/2 aratni = 12 aṅgulaṃ	(ĀŚS. 15.13) = 22.8 cm.
1jānu = 32 aṅgulaṃ	(ĀŚS. 10.18) = 60.8 cm.
1 nābhi = 64 aṅgulaṃ	(ĀŚS. 10.19) = 121.6 cm.
1 āsya = 96 aṅgulaṃ	(ĀŚS. 10.20) = 182.4 cm.

4.4 Kātyayana Śulvasūtra (KŚS)

1 īṣā = 188 aṅgulaṃ	(KŚS. 2.2) = 357.2 cm.
1 akṣam = 104 aṅgulaṃ	(KŚS. 2.3) = 197.6 cm.
1 yugam = 86 aṅgulaṃ	(KŚS. 2.4) = 163.4 cm.

1 śamyā = 32 aṅgulaṃ	(KŚS. 2.5) = 60.8 cm.
1 puruṣa = 5 aratni = 120 aṅgulaṃ	(KŚS. 5.9) = 228 cm.
= 10 vitasti	(KŚS. 5.9)
1 padaṃ = 12 aṅgulaṃ	(KŚS. 5.9) = 22.8 cm.
1 aratni = 24 aṅgulaṃ	(KŚS. 5.9) = 45.6 cm.
1 vitasti = 1 padaṃ = 12 aṅgulaṃ	(KŚS. 5.9) = 24.7 cm.

However, the units of *padam*, *prakramam* and *śamyā* have differences in their actual measurement in different Śulvasūtras. The conversion into the centimetres (cm.) has been done as per current adopted measurement units but which was not taken under consideration in the days of Śulvasūtras. The Śulvasūtras are the ancient most work of geometry. This paper has presented a brief outline of the Śulvasūtras. The science of geometry originated in India with a fully development and it can be said that India is the first country where the science of geometry originated and started its journey.

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¹¹ Cāra Śulbasūtra, Dr. Raghunath Purusottam Kulakarni, M.S.R.V.V.P.

¹² Tilam arranged side by side, or the width of the middlemost of the middle finger of a man of medium size may be taken to be equal to an aṅgulaṃ.