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On algebraic properties of neutrosophic rough set relations

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Abstract
The main objective of this study is to introduce a new hybrid intelligent structure called rough neutrosophic sets on the Cartesian product of two universe sets and the concept application of the same.

Keywords: rough set, solitary set, relative set, accuracy, neutrosophic rough set, multi criteria decision making

1. Introduction

The rough sets theory introduced by Pawlak [10, 11] is an excellent mathematical tool for the analysis of uncertain, inconsistency and vague description of objects. The rough sets philosophy is based on the assumption that every object of the universe is associated with a certain amount of information. This information in the form of data is expressed by means of some attributes used for object description. Objects having the same description are indiscernible with respect to the available information. The indiscernibility relation thus generated constitutes a mathematical basis of the rough sets theory; it induces a partition of the universe into blocks of indiscernible objects, which can be used to build knowledge about a real or abstract world. The basic idea of rough set is based upon the approximation of sets by pair of sets known as lower approximation and upper approximation. Here, the lower and upper approximation operators are based on equivalence relation. However, in many real life problems, rough set model cannot be applied due to the restrictive condition of requirement of equivalence relation. To this end, rough set is generalized to fuzzy environment such as fuzzy rough set, and rough fuzzy set [7].

Neutrosophic sets and rough sets are two different topics, none conflicts the other. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness. By combining the Neutrosophic sets and rough sets the rough sets in neutrosophic approximation space [2] and Neutrosophic neutrosophic rough sets [4] were introduced Multi criterion decision making (MCDM) is a process in which decision makers evaluate each alternative according to multiple criteria. Many representative methods are introduced to solve MCDM problem in business and industry areas. However, a drawback of these approaches is that they mostly consider the decision making with certain information of the weights and decision values. This makes them much less useful when managing uncertain information. To this end, multi criteria fuzzy decision making has been studied in [7, 9]. Several attempts have already been made to use the rough set theory to decision support. But, in many real life problems, an information system establishes relation between two universal sets. Multi criterion decision making on such information system is very challenging. This paper discusses how neutrosophic rough set on two universal sets can be employed on MCDM problems for taking decisions.

2. Preliminaries

Definition 2.1 [13]

A Neutrosophic set A on the universe of discourse X is defined as A = $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$, Where $T, I, F: X \to]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$ ..

Definition 2.2 [12]

Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets

$A_1(X) = \{x: [x]_R \subseteq X\}$

$A_2(X) = \{x: [x]_R \cap X \neq \emptyset\}$

are called lower approximation and upper approximation respectively of X and the pair S= (U, R) is called approximation space. The equivalence relation R is called indiscernibility relation. The pair A(X) = ($A_1(X)$, $A_2(X)$) is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.3 [4]

Let U be a non-empty universe of discourse. For an arbitrary fuzzy neutrosophic relation R over $U \times U$ the pair (U, R) is called fuzzy neutrosophic approximation space. For any $A \in FN(U)$, we define the upper and lower approximation with respect to (U,R), denoted by $\overline{R}$ and $\underline{R}$ respectively.

$$\overline{R}(A) = \{< x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) > / x \in U\}$$
$$\underline{R}(A) = \{< x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) > x \in U\}$$

$$T_{\overline{R}(A)}(x) = \bigvee_{y\in U}[T_R(x,y) \wedge T_A(y)]$$
$$I_{\overline{R}(A)}(x) = \bigvee_{y\in U}[I_R(x,y) \wedge I_A(y)]$$
$$F_{\overline{R}(A)}(x) = \bigwedge_{y\in U}[F_R(x,y) \wedge T_A(y)]$$
$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in U}[F_R(x,y) \wedge T_A(y)]$$
$$I_{\underline{R}(A)}(x) = \bigwedge_{y\in U}[1 - I_R(x,y) \wedge I_A(y)]$$
$$F_{\underline{R}(A)}(x) = \bigvee_{y\in U}[T_R(x,y) \wedge F_A(y)]$$

The pair ($\underline{R}$,$\overline{R}$) is fuzzy neutrosophic rough set of A with respect to (U,R) and $\overline{R}, \underline{R}$:FN(U)→FN(U) are refered to as upper and lower Fuzzy neutrosophic rough approximation operators respectively.

3. Neutrosophic Rough Set on Two Universal Sets

Now, we present the definitions, notations and results of neutrosophic rough set on two universal We define the basic concepts leading to neutrosophic rough set on two universal sets in which we denote for truth function T_{R_N} , indeterminacy I_{R_N} and falsity function F_{R_N} for non membership functions that are associated with an neutrosophic rough set on two universal sets.

Definition 3.1 [11]

Let *U* and *V* be two non empty universal sets. An neutrosophic relation R_N from $U \to V$ is an neutrosophic set of (*U* x *V*) characterized by the truth value function T_{R_N} , indeterminacy function and falsity function F_{R_N} where

$$R_N = \{\langle (x,y), T_{R_N}(x,y), I_{R_N}(x,y), F_{R_N}(x,y), \rangle \mid x \in U, y \in V\} \text{ with}$$
$$0 \le T_{R_N}(x,y) + I_{R_N}(x,y) + F_{R_N}(x,y) \le 3 \text{ for every } (x,y) \in U \times V.$$

Definition 3.2 [13]

Let U and V be two non empty universal sets and R_N is a neutrosophic relation from U to V. If for $x \in U$, $T_{R_N}(x,y) = 0$, $I_{R_N}(x,y) = 0$ and $F_{R_N}(x,y) = 1$ for all $y \in$ V, then *x* is said to be a solitary element with respect to R_N . The set of all solitary elements with respect to the relation R_N is called the solitary set *S*. That is,

$$S = \{x \mid x \in U, T_{R_N}(x,y) = 0, I_{R_N}(x,y) = 0, F_{R_N}(x,y) = 1, \ \forall\, y \in V$$

Definition 3.3 [13]

Let *U* and *V* be two non empty universal sets and R_N is a neutrosophic relation from *U* to *V*. Therefore, (U,V,R_N) is called a neutrosophic approximation space. For $Y \in N(V)$ an neutrosophic rough set is a pair ($\underline{R_N Y}$, $\overline{R_N Y}$) of neutrosophic set on *U* such that for every $x \in$ U.

$$\underline{R_N}(Y) = \{\langle x, T_{\underline{R_N}(Y)}(x), I_{\underline{R_N}(Y)}(x), F_{\underline{R_N}(Y)}(x), \rangle \mid x \in U\}\} \quad (11)$$

$$\overline{R_N}(Y) = \{\langle x, T_{\overline{R_N}(Y)}(x), I_{\overline{R_N}(Y)}(x), F_{\overline{R_N}(Y)}(x) \rangle \mid x \in U\} \quad (12)$$

Where

$$T_{\overline{R}(A)}(x) = \bigvee_{y\in V}[T_R(x,y) \wedge T_A(y)]$$
$$I_{\overline{R}(A)}(x) = \bigvee_{y\in V}[I_R(x,y) \wedge I_A(y)]$$
$$F_{\overline{R}(A)}(x) = \bigwedge_{y\in V}[F_R(x,y) \wedge T_A(y)]$$
$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in V}[F_R(x,y) \wedge T_A(y)]$$
$$I_{\underline{R}(A)}(x) = \bigwedge_{y\in V}[1 - I_R(x,y) \wedge I_A(y)]$$
$$F_{\underline{R}(A)}(x) = \bigvee_{y\in V}[T_R(x,y) \wedge F_A(y)]$$

The pair ($\underline{R}_N(Y)$, $\overline{R}_N(Y)$) is called the neutrosophic rough set of Y with respect to (U,V,R_N) where $\underline{R}_N(Y)$, $\overline{R}_N(Y)$: *N(U)* $\rightarrow$ *N(V)* are referred as lower and upper neutrosophic rough approximation operators on two universal sets.

Definition 3.4

Let (U,V,R) be fuzzy approximation space over two universes and $A \in N(U)$ .Then the (α,β) -level set of lower and upper approximations of A are defined as follows:

$$\underline{R}(A)^{(\alpha,\beta,\gamma)} = \{y \in V / T_{\underline{R}(A)}(y) \geq \alpha, I_{\underline{R}(A)}(y) \geq \beta, F_{\underline{R}(A)}(y) \leq \gamma\},$$

$$\overline{R}(A)^{(\alpha,\beta,\gamma)} = \{y \in V / T_{\overline{R}(A)}(y) \geq \alpha, I_{\overline{R}(A)}(y) \geq \beta, F_{\overline{R}(A)}(y) \leq \gamma\}$$

Where

$0 \leq \alpha + \beta + \gamma \leq 3$ and $0 \leq \alpha, \beta, \gamma \leq 1$.

4. Algebraic Properties

In this section, we discuss the algebraic properties of neutrosophic rough set on two universal sets through solitary set. These are interesting and valuable in the study of neutrosophic rough sets on two universal sets and are useful in finding knowledge from the information system that establishes relation between two universes.

Proposition 4.1

Let U and V be two universal sets. Let R_N be an neutrosophic relation from U to V and further let S be the solitary set with respect to R_N . Then for *X, Y* $\in$ *N(V),* the following properties holds:

a) $\underline{R_N}$ *(V) = U and* $\overline{R_N}$ (ϕ) = ϕ

b) If $X \subseteq Y$*, then* $\underline{R_N}$ *(X)* $\subseteq$ $\underline{R_N}$ *(Y) and* $\overline{R_N}$ *(X)* $\subseteq$ $\overline{R_N}$ *(Y)*

c) $\underline{R_N}$ *(X) =* $\overline{R_N}$ *(X'))' and* $\overline{R_N}$ *(X)=* $\underline{R_N}$ *(X'))'*

d) $\underline{R_N}\phi \supseteq$ *S* and $R_N V \subseteq$ *S', where S' denotes the complement of S in U.*

e) (e) For any given index set *J*, $X_i \in N(V)$, $\underline{R_N}(\bigcup_{i \in J} X_i) \supseteq \bigcup_{i \in J} \underline{R_N} X_i$ *and* $\overline{R_N}(\bigcap_{i \in J} X_i) \subseteq \bigcap_{i \in J} \overline{R_N} X_i$

f) For any given index set J, $X_i \in N(V)$, $\underline{R_N}(\bigcap_{i \in J} X_i) = \bigcap_{i \in J} \underline{R_N} X_i$ and $\overline{R_N}(\bigcup_{i \in J} X_i) = \bigcup_{i \in J} \overline{R_N} X_i$.

Proof:

(i) First note that *V* is a neutrosophic set satisfying $T_Y(x) = 1$, $I_Y(x) = 0$ *and* $F_Y(x) = 1$ for all $x \in V$. Thus, *V* can be represented as $V = \{\langle x,1,1,0 \rangle \cap x \in V\}$

Now, by definition we have

$$T_{\underline{R}(A)}(x) = \bigwedge_{y \in V} F_{R_N}(x,y) \vee T_Y(y) = 1$$

$$I_{\underline{R}(A)}(x) = \bigwedge_{y \in V} 1 - I_{R_N}(x,y) \vee I_Y(y) = 1$$

$$F_{\underline{R}(A)}(x) = \bigvee_{y \in V} T_{R_N}(x,y) \wedge F_Y(y) = 0$$

Therefore we get,

$$\underline{R_N}(V) = \{\langle x, T_{\underline{R_N}(V)}(x), I_{\underline{R_N}(V)}(x), F_{\underline{R_N}(V)}(x), \rangle \mid x \in U\}\}$$

$$= \{\langle x,1,1,0 \rangle \mid x \in U\}$$

Similarly, ϕ is a neutrosophic set satisfying $T_Y(x) = 0$, $I_Y(x) = 1$ *and* $F_Y(x) = 0$ for all $x \in V$. Thus, ϕ can be represented as $\phi = \{\langle x,0,0,1 \rangle \mid x \in V\}$

Now, by definition we have

$$T_{\overline{R}(A)}(x) = \bigvee_{y \in V} [T_R(x,y) \wedge T_A(y)] = 0$$

$$I_{\overline{R}(A)}(x) = \bigvee_{y\in V}[I_R(x,y) \wedge I_A(y)] = 0$$
$$F_{\overline{R}(A)}(x) = \bigwedge_{y\in V}[F_R(x,y) \wedge T_A(y)]=1$$

Therefore we get,

$$\overline{R_N}(\phi) = \{\langle x, T_{\overline{R_N}(\phi)}, I_{\overline{R_N}(\phi)}, F_{\overline{R_N}(\phi)}\rangle \mid x \in U\}\}$$

$$= \{\langle x,0,0,1\rangle \mid x \in U\} = \phi .$$

(ii) First note that $X \subseteq Y$ if and only $T_X(x) \le T_Y(x)$, $I_X(x) \le I_Y(x)$ and $F_X(x) \le F_Y(x)$ for all $x \in V$. Therefore, we have

$$T_{\underline{R_N}(X)}(x) = \bigwedge_{y\in V}[F_{R_N}(x,y) \vee T_X(y)] \le \bigwedge_{y\in V}[F_{R_N}(x,y) \vee T_Y(y)] = T_{\underline{R_N}(Y)}(x)$$
$$I_{\underline{R_N}(X)}(x) = \bigwedge_{y\in V}[1 - I_{R_N}(x,y) \vee I_X(y)] \le \bigwedge_{y\in V}[I_{R_N}(x,y) \vee I_Y(y)] = I_{\underline{R_N}(Y)}(x)$$
$$F_{\underline{R_N}(X)}(x) = \bigvee_{y\in V}[T_{R_N}(x,y) \wedge F_X(y)] \ge \bigvee_{y\in V}[T_{R_N}(x,y) \wedge F_Y(y)] = F_{\underline{R_N}(Y)}(x)$$

Therefore, $\underline{R_N}$ *(X)* $\subseteq$ $\underline{R_N}$ *(Y)*

Similarly, we have

$$T_{\overline{R_N}(x)}(x) = \bigvee_{y\in V}[T_{R_N}(x,y) \wedge T_X(y)] \le \bigvee_{y\in V}[I_{R_N}(x,y) \wedge T_Y(y)] = T_{\overline{R_N}(Y)}(x)$$
$$I_{\overline{R_N}(x)}(x) = \bigvee_{y\in V}[I_{R_N}(x,y) \wedge I_X(y)] \le \bigvee_{y\in V}[I_{R_N}(x,y) \wedge I_Y(y)] = I_{\overline{R_N}(Y)}(x)$$
$$F_{\overline{R_N}(x)}(x) = \bigwedge_{y\in V}[F_{R_N}(x,y) \vee F_X(y)] \ge \bigwedge_{y\in V}[F_{R_N}(x,y) \vee F_Y(y)] = F_{\overline{R_N}(Y)}(x)$$

Therefore, $\overline{R_N}$ *(X)* $\subseteq$ $\overline{R_N}$ *(Y).*

(iii) We know that $\overline{R_N}(X') = \{\langle x, T_{\overline{R_N}(X')}(x), I_{\overline{R_N}(X')}(x), F_{\overline{R_N}(X')}(x)\rangle / x \in U\}$, *where*

$$T_{\overline{R_N}(x')}(x) = \bigvee_{y\in V}[T_{R_N}(x,y) \wedge T_{X'}(y)] = \bigvee_{y\in V} T_{R_N}(x,y) \wedge F_Y(y)] = F_{\underline{R_N}(X)}(x)$$
$$I_{\overline{R_N}(x')}(x) = \bigvee_{y\in V}[I_{R_N}(x,y) \wedge I_{X'}(y)] = \bigvee_{y\in V} I_{R_N}(x,y) \wedge 1 - I_Y(y)] = 1 - I_{\underline{R_N}(X)}(x) = I_{\underline{R_N}(X)}(x)$$
$$F_{\overline{R_N}(x)}(x) = [F_{R_N}(x,y) \vee F_{X'}(y)] = [F_{R_N}(x,y) \vee T_X(y)] = T_{\underline{R_N}(X)}(x)$$

Therefore, we have

$$\overline{R_N}(X') = \{\langle x, T_{\overline{R_N}(X')}(x), I_{\overline{R_N}(X')}(x), F_{\overline{R_N}(X')}(x)\rangle / x \in U\}$$
$$= \{\langle x, F_{\underline{R_N}(X')}(x), 1 - I_{\underline{R_N}(X')}(x), T_{\underline{R_N}(X')}(x)\rangle / x \in U\}$$

indicates that $\left(\overline{R_N}(X')\right)' = \{\langle x, T_{\underline{R_N}(X)}(x), I_{\underline{R_N}(X)}(x), F_{\underline{R_N}(X)}(x), \rangle \mid x \in U$ and consequently $\left(\overline{R_N}(X')\right)' = \underline{R_N}(X)$

$$\underline{R_N}(X') = \{\langle x, T_{\underline{R_N}(X')}(x), I_{\underline{R_N}(X')}(x), F_{\underline{R_N}(X')}(x), \rangle \mid x \in U$$

$$T_{\underline{R_N}(X')}(x) = \bigwedge_{y\in V}[F_{R_N}(x,y) \vee T_{X'}(y)] = \bigwedge_{y\in V}[F_{R_N}(x,y) \vee F_X(y)] = F_{\overline{R_N}(X)}(x)$$
$$I_{\underline{R_N}(X')}(x) = \bigwedge_{y\in V}[1 - I_{R_N}(x,y) \vee I_{X'}(y)] = \bigwedge_{y\in V}[1 - I_{R_N}(x,y) \vee 1 - I_X(y)] = [I_{R_N}(x,y) \wedge I_Y(y)]$$
$$= I_{\overline{R_N}(X)}(x)$$

$$F_{\underline{R_N}(X)}(x) = \bigvee_{y\in V}[T_{R_N}(x,y)\wedge F_{X'}(y)] \geq \bigvee_{y\in V}[T_{R_N}(x,y)\wedge T_Y(y)] = T_{\overline{R_N}(X)}(x).$$

$$\left(\underline{R_N}(X')\right)' = \{\left\langle x, T_{\underline{R_N}(X')}(x), I_{\underline{R_N}(X')}(x), F_{\underline{R_N}(X')}(x), \right\rangle \mid x\in U$$

$$= \{\left\langle x, F_{\underline{R_N}(X)}(x), I_{\underline{R_N}(X)}(x), T_{\underline{R_N}(X)}(x), \right\rangle \mid x\in U$$

It indicates that $\left(\underline{R_N}(X')\right)' = \{\left\langle x, T_{\overline{R_N}(X)}(x), I_{\overline{R_N}(X)}(x), F_{\overline{R_N}(X)}(x)\right\rangle / x\in U\}$ and consequently $\left(\underline{R_N}(X')\right)' = \overline{R_N}(X)$.

(iv) First note that, ϕ is a neutrosophic set satisfying $T_Y(x)=0$, $I_Y(x)=0$ and $F_Y(x)=1$ for all $x\in V$. Thus, ϕ can be represented as

$$\phi = \{\langle x,0,0,1\rangle \mid x\in V\}$$

Also note that, *S* is a solitary set. This indicates that
Therefore, we have for all $x\in S$

$$T_{\underline{R_N(\phi)}}(x) = \bigwedge_{y\in V}[F_{R_N}(x,y)\vee T_\phi(y)] = \bigwedge_{y\in V}[1\vee 0] = 1$$

$$I_{\underline{R_N(\phi)}}(x) = \bigwedge_{y\in V}[1-I_{R_N}(x,y)\vee I_\phi(y)] = \bigwedge_{y\in V}[1\vee 0] = 1$$

$$F_{\underline{R_N(\phi)}}(x) = \bigvee_{y\in V}[T_{R_N}(x,y)\vee F_\phi(y)] = \bigvee_{y\in V}[0\wedge 1] = 0$$

Hence, it is clear that $T_{\underline{R_N(\phi)}}(x)\geq T_{R_N}(x,y)$, $I_{\underline{R_N(\phi)}}(x)\geq I_{R_N}(x,y)$ and $F_{\underline{R_N(\phi)}}(x)\leq F_{R_N}(x,y)$ for $x\in S$.

Therefore, by proposition (ii) we have $\underline{R_N}(\phi)\supseteq S$.

Similarly, by proposition (iii) we have $\overline{R_N}(X) = \left(\underline{R_N}(X')\right)'$.

On taking $x\in V$ we get $\overline{R_N}(V) = \left(\underline{R_N}(V')\right)'$.

But $V'=\phi$.Again by proposition (iv), we have $\underline{R_N}(\phi)\supseteq S$. It implies that $(\underline{R_N}(\phi))'\subseteq S'$.

Therefore, we get $\overline{R_N}(V)\subseteq S'$.

(v) From the properties of union, for any index set J={1,2,3….,n}

$$X_1\subseteq\bigcup_{i\in J}X_i,\ X_2\subseteq\bigcup_{i\in J}X_i, X_3\subseteq\bigcup_{i\in J}X_i, \ldots\ldots\ldots\ldots X_n\subseteq\bigcup_{i\in J}X_i.$$

Therefore, by proposition (ii) we have

$$\underline{R_N}(X_1)\subseteq\underline{R_N}(\bigcup_{i\in J}X_i),\ \underline{R_N}(X_2)\subseteq\underline{R_N}(\bigcup_{i\in J}X_i), \ldots\ldots\ldots \underline{R_N}(X_n)\subseteq\underline{R_N}(\bigcup_{i\in J}X_i)$$

It indicates that,

$$\bigcup_{i\in J}\underline{R_N}(X_i)\subseteq\underline{R_N}(\bigcup_{i\in J}X_i), \text{ ie } \underline{R_N}(\bigcup_{i\in J}X_i)\supseteq\bigcup_{i\in J}\underline{R_N}(X_i).$$

Similarly, for any index set J={1,2,3….,n}

$$\bigcap_{i\in J}X_i\subseteq X_1,\ \bigcap_{i\in J}X_i\subseteq X_2, \bigcap_{i\in J}X_i\subseteq X_3, \ldots\ldots\ldots\ldots \bigcap_{i\in J}X_i\subseteq X_n.$$

Therefore, by proposition (ii) we have

$$\overline{R_N}(\bigcap_{i\in J}X_i)\subseteq\overline{R_N}(X_1),\ \overline{R_N}(\bigcap_{i\in J}X_i)\subseteq\overline{R_N}(X_2), \ldots\ldots\ldots \overline{R_N}(\bigcap_{i\in J}X_i)\subseteq\overline{R_N}(X_n)$$

It indicates that, $\overline{R_N}(\bigcap_{i\in J}X_i)\subseteq\bigcap_{i\in J}\overline{R_N}(X_i)$.

(vi) For any index set *J* ={1, 2, 3,...., *n*}, $X_i\in N(V)$,

$\underline{R_N}(\bigcap_{i\in J}X_i) = \{\left\langle x, T_{\underline{R_N}(\bigcap_{i\in J}X_i)}(x), I_{\underline{R_N}(\bigcap_{i\in J}X_i)}(x), F_{\underline{R_N}(\bigcap_{i\in J}X_i)}(x)\right\rangle / x\in U\}$. But,

$$T_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_{(\bigcap_{i\in J} X_i)}(y)]$$
$$= \bigwedge_{y\in V} [F_{R_N}(x,y) \vee (T_{X_1}(y) \wedge T_{X_2}(y) \wedge T_{X_3}(y) \wedge .... \wedge T_{X_{n-1}}(y) \wedge T_{X_n}(y))]$$
$$= \bigwedge_{y\in V} [(F_{R_N}(x,y) \vee T_{X_1}(y) \wedge (F_{R_N}(x,y) \vee T_{X_2}(y)) \wedge ...... \wedge (F_{R_N}(x,y) \vee T_{X_n}(y))]$$
$$= T_{\underline{R_N}(X_1)}(x) \wedge T_{\underline{R_N}(X_2)}(x) \wedge ..... \wedge T_{\underline{R_N}(X_n)}(x)$$
$$= \text{Min}\{ T_{\underline{R_N}(X_1)}(x) \}$$
$$I_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x) = \bigwedge_{y\in V} [1 - I_{R_N}(x,y) \vee I_{(\bigcap_{i\in J} X_i)}(y)]$$
$$= \bigwedge_{y\in V} [(1 - I_{R_N}(x,y) \vee (I_{X_1}(y) \wedge I_{X_2}(y) \wedge I_{X_3}(y) \wedge .... \wedge I_{X_{n-1}}(y) \wedge I_{X_n}(y))]$$
$$= \bigwedge_{y\in V} [(1 - I_{R_N}(x,y) \vee I_{X_1}(y) \wedge (1 - I_{R_N}(x,y) \vee I_{X_2}(y)) \wedge .... \wedge (1 - I_{R_N}(x,y) \vee I_{X_n}(y))]$$
$$= I_{\underline{R_N}(X_1)}(x) \wedge I_{\underline{R_N}(X_2)}(x) \wedge ..... \wedge I_{\underline{R_N}(X_n)}(x)$$
$$= \text{Min}\{ I_{\underline{R_N}(X_1)}(x) \}$$
$$F_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \wedge F_{(\bigcap_{i\in J} X_i)}(y)]$$
$$= \bigvee_{y\in V} [T_{R_N}(x,y) \wedge (F_{X_1}(y) \wedge F_{X_2}(y) \wedge F_{X_3}(y) \wedge .... \wedge F_{X_{n-1}}(y) \wedge F_{X_n}(y))]$$
$$= \bigvee_{y\in V} [(T_{R_N}(x,y) \wedge F_{X_1}(y) \vee (T_{R_N}(x,y) \wedge F_{X_2}(y)) \vee .... \vee (T_{R_N}(x,y) \wedge F_{X_n}(y))]$$
$$= F_{\underline{R_N}(X_1)}(x) \vee F_{\underline{R_N}(X_2)}(x) \vee ..... \vee F_{\underline{R_N}(X_n)}(x)$$
$$= \text{Max}\{ F_{\underline{R_N}(X_1)}(x) \}$$

Again, for any index set J ={1, 2, 3...., n}, $X_i \in N(V)$,

$$\underline{R_N}(X_1) = \{\left\langle x, T_{\underline{R_N}(X_1)}(x), I_{\underline{R_N}(X_1)}(x), F_{\underline{R_N}(X_1)}(x), \right\rangle | \, x \in U$$
$$\underline{R_N}(X_2) = \{\left\langle x, T_{\underline{R_N}(X_2)}(x), I_{\underline{R_N}(X_2)}(x), F_{\underline{R_N}(X_2)}(x), \right\rangle | \, x \in U$$
$$\underline{R_N}(X_n) = \{\left\langle x, T_{\underline{R_N}(X_n)}(x), I_{\underline{R_N}(X_n)}(x), F_{\underline{R_N}(X_n)}(x), \right\rangle | \, x \in U$$

Therefore, we have

$$\bigcap_{i\in J} \underline{R_N}(X_i) = \{\left\langle x, Min\{T_{\underline{R_N}(X_n)}(x)\}, Min\{I_{\underline{R_N}(X_n)}(x)\}, Max\{F_{\underline{R_N}(X_n)}(x)\} \right\rangle | \, x \in U$$

Hence, it is clear that $\underline{R_N}(\bigcap_{i\in J} X_i) = \bigcap_{i\in J} \underline{R_N}(X_i)$.

Similarly for any index set J ={1, 2, 3,...., n}, $X_i \in N(V)$,

$$\overline{R_N}(\bigcup_{i\in J} X_i) = \{\left\langle x, T_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x), I_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x), F_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x) \right\rangle / \, x \in U\}.$$
$$T_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x) = \text{Max}\{ T_{\overline{R_N}(X_1)}(x) \}$$
$$I_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x) = \text{Max}\{ I_{\overline{R_N}(X_1)}(x) \}$$
$$F_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x) = \text{Min}\{ F_{\underline{R_N}(X_1)}(x) \}$$

Again, for any index set J={1, 2, 3,...., n}, $X_i \in N(V)$,

$$\underline{R_N}(X_1)=\{\left\langle x, T_{\underline{R_N}(X_1)}(x), I_{\underline{R_N}(X_1)}(x), F_{\underline{R_N}(X_1)}(x),\right\rangle \mid x\in U$$

$$\underline{R_N}(X_2)=\{\left\langle x, T_{\underline{R_N}(X_2)}(x), I_{\underline{R_N}(X_2)}(x), F_{\underline{R_N}(X_2)}(x),\right\rangle \mid x\in U$$

$$\underline{R_N}(X_n)=\{\left\langle x, T_{\underline{R_N}(X_n)}(x), I_{\underline{R_N}(X_n)}(x), F_{\underline{R_N}(X_n)}(x),\right\rangle \mid x\in U$$

Therefore, we have

$$\bigcap_{i\in J}\underline{R_N}(X_i)=\{\left\langle x, Min\{T_{\underline{R_N}(X_n)}(x)\}, Min\{I_{\underline{R_N}(X_n)}(x)\}, Max\{F_{\underline{R_N}(X_n)}(x)\}\right\rangle \mid x\in U$$

Hence, it is clear that $\underline{R_N}(\bigcap_{i\in J} X_i)=\bigcap_{i\in J}\underline{R_N}(X_i)$.

4.2 Approximation of Classification

Definition 4.2.1

Let $F = \{Y_1, Y_2, Y_3, \ldots, Y_n\}$, be a family of non-empty classification of *V* and let R_N be a neutrosophic relation from U $\rightarrow$ *V*. Then the R_N -lower and R_N -upper approximation of the family *F* is given as

$\underline{R_N}$ F = { $\underline{R_N}$ (Y_1), $\underline{R_N}$ (Y_2), $\underline{R_N}$ (Y_3), …, $\underline{R_N}$ (Y_n)} and

$\overline{R_{IF}}$ F = { $\overline{R_N}$ (Y_1), $\overline{R_N}$ (Y_2), $\overline{R_N}$ (Y_3), …, $\overline{R_N}$ (Y_n)} respectively.

4.3 Measures of Uncertainty

This section introduces the concept of measures of uncertainty such as accuracy and quality of approximation employing the neutrosophic relation R_N . We denote the number of objects in a set *V* by card *(V).*

Let F = {Y_1, Y_2,…, Y_n} be a family of non- empty classifications. Then the R_N -lower and R_N -upper approximation of the family F are given as $\underline{R_N}$ F = { $\underline{R_N}$ (Y_1), $\underline{R_N}$ (Y_2), $\underline{R_N}$ (Y_3), …, $\underline{R_N}$ (Y_n)} and $\overline{R_{IF}}$ F = { $\overline{R_N}$ (Y_1), $\overline{R_N}$ (Y_2), $\overline{R_N}$ (Y_3), …, $\overline{R_N}$ (Y_n)} respectively. Now we define accuracy of approximation and quality of approximation of the family F employing the neutrosophic relation R_N as follows:

Definition 4.3.1

The accuracy of approximation of *F* that expresses the percentage of possible correct decisions when classifying objects employing the neutrosophic relation R_N is defined as

$$\alpha_{R_N}(F)=\frac{\sum card\left(\underline{R_N}Y_i\right)}{\sum card\left(\overline{R_N}Y_i\right)} \text{ for } i = 1,2,3,\ldots, n.$$

Definition 4.3.2

The quality of approximation of F that expresses the percentage of objects which can be correctly classified to classes of F by the neutrosophic relation R_N is defined as

$$V_{R_N}(F)=\frac{\sum card\left(\underline{R_N}Y_i\right)}{card(V)} \text{ for } i = 1,2,3,\ldots,n$$

Definition4.3.3

We say that $F = \{Y_1, Y_2, Y_3, \ldots, Y_n\}$ is R_N -definable if and only if $\underline{R_N} = \overline{R_N}$; that is $\overline{R_N}$ (Y_i) = $\overline{R_N}$ (Y_i) for i = 1,2,3…,n.

Definition 4.3.4

Let (U,V,R) be neutrosophic approximation space over two universes and $A\in N(U)$ The roughness measure $\rho_p(\alpha,\beta,\gamma)$ and accuracy $\lambda_p(\alpha,\beta,\gamma)$ for neutrosophic sets A with the parameters α,β,γ in (U,V,R) are defined as follows:

$$\rho_p(\alpha,\beta,\gamma)=1-\frac{\left|\underline{R_N}(A)^{(\alpha,\beta,\gamma)}\right|}{\left|\overline{R_N}(A)^{(\alpha,\beta,\gamma)}\right|}.$$

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