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On a diophantine problem

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Abstract
A search is made for obtaining three non-zero distinct integers *a,b,c* such that each of the expressions *a+b,b* and *a+2c* is a perfect square.

Keywords: Diophantine problem, Integer triples, System of equations

Introduction

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans)**.** One of the oldest branches of mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on [7, 8]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the third century AD and he was one of the first Mathematicians to introduce symbolism to Algebra. The theory of Diophantine equations is a treasure house in which the search for many hidden relation and properties among numbers form a treasure hunt. In fact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems come from physical problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1, 6]. Also one may refer [9, 14].

In this communication, we attempt for obtaining three non-zero distinct integers a,b,c such that each of the expressions a+b,b,a+2c is a perfect square.

2. Notations Used

- $t_{m,n}$ - Polygonal number of rank n with size m.
- CS_n - Centered Square number of rank n.
- $\Pr_n$ - Pronic number of rank n
- J_n - Jacobsthal number of rank n.
- j_n - Jacobsthal-Lucas number of rank n.
- $CP_{m,n}$ - Centered Pyramidal Number.
- S_r - Star number.
- KY_n - Kynea number of rank n.
- $F_{4,4}^{r}$ - Fourth dimensional Figurate number of rank r.
- G_n - Gnomonic number of rank n.

3. Method of Analysis

Let a,b,c be three non-zero distinct integers such that

$$a+b=p^2 \quad (1)$$

$$a+2c=q^2 \quad (2)$$

$$b=r^2 \quad (3)$$

From (1) and (3), we have

$$a=p^2-r^2 \text{ Such} \quad (4)$$

From (2) and (4), we get

$$c=\frac{q^2+r^2-p^2}{2} \quad (5)$$

Note that c is an integer when

a) p,q,r are all even.

b) Any two of p,q,r are odd and the third one is even.

Choice (a)

p,q,r are all even.

Let $p=2P, q=2Q, r=2R$ (6)

Using (6) in (4), (3) and (5) we have the required values of a,b,c to be

$$a=4(P^2-R^2) \quad (7)$$

$$b=4R^2 \quad (8)$$

$$c=2(Q^2+R^2-P^2) \quad (9)$$

Table 1: Numerical examples

P	Q	R	A	B	C
1	2	3	-32	36	24
7	8	1	192	4	32
6	4	2	128	16	-32

Properties

- $a(1,R-1)+b(R-1)+2c(1,R,R-1)-4CS_R=0.$
- $6[b(R)+2a(P,R)+4t_{4,R}]$ represents a nasty number.
- $2a(Q+1,1)+b(1)+2c(Q+1,Q,1)-8\Pr_Q$ is a perfect square.
- $b(R)+2a(R+1,R)-12t_{4,R}\equiv 16(\bmod 32).$
- $c(1,Q,Q+1)-8t_{3,Q}=0.$
- $a(R+1,R)+b(R)+c(R+1,1,R)-4\Pr_R-(J_1+J_3)=0.$
- $a(P,R)+2c(P,Q^3,R)-4t_{4,Q}CP_{6,Q}=0.$

Choice (b)

Any two of p,q,r are odd and the third one is even.

Case (i):

Let $p=2P+1, q=2Q, r=2R+1$ (10)

Using (10) in (4), (3) and (5) we have

$$a=4(P^2+P-R^2-R)$$

$$b=(2R+1)^2$$

$$c=2(Q^2+R^2+R-P^2-P)$$

Table 2: Numerical examples

P	Q	R	A	b	C
1	3	4	-72	81	54
2	1	3	-24	49	14
3	4	2	24	25	20

Properties

- $a(P,R)+b(R)+2c(P,Q,R)-4t_{4,Q}-4\Pr_R-j_1=0.$
- $b(1)+a(P+2,1)-S_P+2t_{4,P}\equiv 24(\bmod 26)$.
- $a(P,R)+2c(P,2^Q+1,R)-4Ky_Q$ is a cubical integer.
- $b(R)-2c(Q-1,Q,R)+4G_Q\equiv -3(\bmod 4).$
- $2c(R-1,Q,1)-a(R-1,1)-4t_{4,Q}$ is a perfect square.

Case (ii)

Let $p=2P, q=2Q+1, r=2R+1$ (11)

Using (11) in (4), (3) and (5) we get

$$a=4P^2-(2R+1)^2$$

$$b=(2R+1)^2$$

$$c=2(Q^2+Q+R^2+R-P^2)+1$$

Table 3: Numerical examples

P	Q	R	A	b	C
2	1	3	-33	49	21
3	2	4	-45	81	35
5	4	2	75	25	3

Properties

- $a(2^P+1,R)+b(R)-4Ky_P$ is a cubical integer.
- $2c(1,Q,R)-b(R)-S_Q+2t_{4,Q}\equiv -4(\bmod 10)$.
- $a(P,R)+b(R)+2c(P,1,R)-4t_{4,R}-G_R\equiv 1(\bmod 2)$
- $a(P,R)-2c(P,Q,R)-4\Pr_Q-J_1=0.$
- $a(P,1)+b(1)+2c(P,Q+1,1)-t_{10,Q}\equiv 18(\bmod 15)$.

Case (iii)

Let $p=2P+1, q=2Q+1, r=2R$ (12)

Using (12) in (4), (3) and (5) we get

$$a=4(P^2+P-R^2)+1$$

$$b=4R^2$$

$$c=2R^2+2(Q^2+Q-P^2-P)$$

Table 4: Numerical examples

P	Q	R	A	b	C
3	1	2	33	16	-12
1	2	1	77	4	-26
2	3	2	105	16	-28

Properties

- $b(R)-2c(P,P+1,R)-8P$ is a cubical integer.
- $a(P,R)+b(R)-G_P-4t_{4,R}\equiv 0(\bmod 2)$.
- $a(P,R)+2c(P,Q(Q+1),R)-4\{t_{4,Q^2}+t_{4,Q}+\Pr_Q-2CP_{6,Q}\}-1=0.$
- $a(P,2Q)+b(2Q)+2c(p,Q(Q+1),2Q)-12\{2F_{4,5}^Q+F_{4,4}^Q\}+6CP_{6,Q}-10t_{4,Q}-1=0.$

4. Conclusion

In this paper, we have presented three non-zero distinct integers a,b,c such that each of the expressions a+b, b and a+2c is a perfect square. As diophantine problems are rich in variety, one may attempt to find other choices of diophantine problems.

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