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Behaviour analysis of a soap industry

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Abstract
This paper discuss availability analysis of soap industry using RPGT called Regenerative Point Graphical Technique. The reliability model for availability of soap industry with redundancy is developed which consists of three non-identical units in which main unit can work in reduced state after partial failure. The main unit can fail partially and hence can be in up-state, partially failed state or totally failed state. The system can work with reduced capacity in a partially failed state. The repair of the unit, treatment of the server and switch devices are considered as perfect. Using the RPGT the Mean time to system failure, Total fraction of time for which the system is available, the busy period of the server, the Number of server's visits has been evaluated to study the system performance followed by illustrations special cases, Tables and Graphs.

Keywords: Reliability, Availability, Primary Circuit, Secondary Circuit, Tertiary Circuit, Base-State, Regenerative Point Graphical Technique (RPGT), Busy Period of the Server, MTSF, Expected number of server's visits

1. Introduction

In this paper the reliability model for availability analysis of soap industry is developed in which main unit can work in reduced state after partial failure. In soap industry, first process boiling in which Fats and alkali are melted in a kettle, which is a steel tank that can stand three stories high and hold several thousand pounds. After boiling, the mass thickens as the fat reacts with the alkali, producing soap and glycer in. The common industrial process of purifying soap involves removal of sodium chloride, sodium hydroxide, and glycerol (Glycerine). These components are removed by boiling the crude soap curds in water and re-precipitating the soap with salt. After this the water is then removed from the soap. The now dry soap (approximately 6-12% moisture) is then compacted into small pellets. These pellets are now ready for soap finishing. The finishing process converts raw soap pellets into a saleable product, usually bars. Soap pellets are combined with fragrances and other materials and blended to homogeneity in an amalgamator (mixer). The mass is then discharged from the mixer into a refiner which, by means of an auger, forces the soap through a fine wire screen. From the refiner the soap passes over a roller mill (French milling or hard milling) in a manner similar to calendering paper or plastic or to making chocolate liquor. The soap is then passed through one or more additional refiners to further plasticize the soap mass. Immediately before extrusion it passes through a vacuum chamber to remove any entrapped air. It is then extruded into a long log or blank, cut to convenient lengths, passed through a metal detector and then stamped into shape in refrigerated tools. The pressed bars are then packaged in many ways. Using the RPGT the following system characteristics have been evaluated to study the system performance. Mean Time to System Failure (MTSF), Total fraction of time for which the system is available, the busy period of the server, the number of server's visits. Gupta [1] defined different types of circuits/cycles like primary, secondary, tertiary circuits etc. which are located in the transition diagram of the system and introduced the concept of a base-state of the system for determining the key parameters more quickly and easily while using RPGT. Using RPGT Jindal [2] discussed behaviour and availability analysis of industrial systems. Gupta & Singh [3, 4] presented a new approach for availability analysis, behaviour and profit analysis of process industries. Goel & Singh [5] discussed availability analysis of stand by Complex system having imperfect switch. Chander, S., & Bansal, R.K. [6] discussed the Profit Analysis of a Single Unit Reliability Models with Repair at Different Failure Modes. Gupta, P., Singh, J. & Singh, I.P. [7] discussed the Availability Prediction for Neat Soap Production System in a Soap Plant. Gupta, P., Singh, J. & Singh, I.P [8] discussed the Availability Analysis of Soap

Cakes Production. Gupta, V.K. & Singh, J. [9] discussed the Behaviour and Profit Analysis of a Soap Industry Govil Jindal, Pardeep Goel, V.K Gupta, Jai Singh [10] discussed the Availability and Behavioural Analysis of a Single Unit Redundant system having Imperfect Switch.

2. Assumptions and Notations

The following assumptions and notations/symbols are used:

1) The System consists of three non-identical units 'A', 'B' & 'C' in which 'A' can work in reduced state after partial failure.
2) The unit 'A' can fail partially and hence can be in up-state, partially failed state (reduced state) or totally failed state. The system can work with reduced capacity in a partially failed state.
3) There is a single repair facility catering to the needs of three units as and when need arises.
4) The distribution of the failure & repair times are exponential and general respectively and also different for three units. They are also assumed to be independent of each other.
5) Repairs are perfect i.e. the Repair facility never does any damage to the units.
6) A Repaired unit works like a new one.
7) The system is down if any one of the unit fails completely.
8) Nothing can fail further when the system is in failed state.
9) The system is discussed for steady state conditions.
10) Units 'B' and 'C' have subunits in series.

pr /pf: Probability/transition probability factor.

$q_{i,j}$ (t): probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$p_{i,j}$: Steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j} = q^*_{i,j}(0)$ where * denotes Laplace transformation.

$\overline{cycle}$: A circuit formed through un-failed states.

K-cycle: A circuit (may be formed through regenerative or non-regenerative/failed states) whose terminals are at the regenerative state k.

K-$\overline{cycle}$: A circuit (may be formed through only un-failed regenerative /. non- regenerative states) whose terminals are at the regenerative state k.

$\left(i \xrightarrow{S_r} j\right)$: *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$: A directed simple failure free path from ξ -state to i-state.

$V_{k,k}$: *pf* of the state k reachable from the terminal state k of the k-cycle.

$V_{\overline{k,k}}$: *pf* of the state k reachable from the terminal state k of the k-$\overline{cycle}$.

R_i(t): Reliability of the system at time t, given that the system entered the un-failed regenerative state i at t=0.

A_i(t): Probability that the system is available in up-state at time t, given that the system entered regenerative state i at t=0.

B_i(t): Probability that the server is busy doing a particular job at epoch t, given that the system entered regenerative state i at t=0.

V_i(t): The expected number of visits of the server for a given job in (0,t], given that the system entered regenerative state i at t=0.

W_i(t): Probability that the server is busy doing a particular job at epoch t without transiting to any other regenerative state 'i' through one or more non- regenerative states, given that the system entered the regenerative state 'i' at t=0.

μ_i : Mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt.$$

μ_i^1 : The total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

η_i : Expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

f_j : Fuzziness measure of the j-state.

λ/λ_1: Constant failure rate of the main operative unit/the redundant unit.

G(t)/g(t): Probability density function/cumulative distribution function of the repair-time of the operative unit.

H(t)/h(t): Probability density function/cumulative distribution function of the repair-time of the redundant unit.

A/ Ā/a: Main unit in the operative state/ partial failed state/failed state

B/b: Redundant unit in operative state/ failed state.

C/c: Redundant unit in operative state/ failed state.

The system can be in any of the following states with respect to the above symbols.

S_0 = ABC	S_1= ĀBC
S_2 = aBC	S_3 = AbC
S_4 =ABc	S_5 = ĀbC
S_6 = ĀBc	

States S_0, S_1, S_2, S_3, S_4, S_5 & S_6 are regenerative states.

Transition Diagram of the System

Following the above assumptions and notations, the transition diagram of the system is shown in fig.1

State	Symbol
Regenerative state/point	•
Up-state:	○
Failed state:	□
Degenerated/Reduced state	⬭

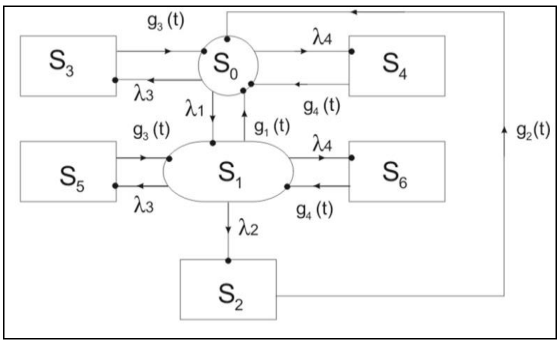

Fig 1:

4 Evaluation of Parameters of the System

4.1 Analysis of System

The key parameters (under steady state conditions) of the system are evaluated by determining a ‘base-state’ and applying RPGT. The MTSF is determined w. r. t the initial state’0’ and the other parameters are obtained by using base-state.

4.1.1 Determination of base-state

From the Transition diagram (fig.1), The Primary, Secondary, Tertiary Circuits at all vertices are shown in Table-1.

Table 1: Paths from State ’i’ to the Reachable State ‘j’:P0

	0	**1**	**2**	**3**	**4**	**5**	**6**
0	{0,1,0} {0,1,2,0} {0,3,0} {0,4,0}	{0,1}	{0,1,2}	{0,3}	{0,4}	{0,1,5}	{0,1,6}
1	{1,0} {1,2,0}	{1,2,0,1} {1,0,1} {1,5,1},{1,6,1}	{1,2}	{1,0,3} {1,2,0,3}	{1,0,4} {1,2,0,4}	{1,5}	[1,6}
2	{2,0}	{2,0,1}	{2,0,1,2}	{2,0,3}	{2,0,4}	{2,0,1,5}	{2,0,1,6}
3	{3,0}	{3,0,1}	{3,0,1,2}	{3,0,3}	{3,0,4}	{3,0,1,5}	{3,0,1,6}
4	{4,0}	{4,0,1}	{4,0,1,2}	{4,0,3}	{4,0,4}	{4,0,1,5}	{4,0,1,6}
5	{5,1,0} {5,1,2,0}	{5,1}	{5,1,2}	{5,1,2,0,3} {5,1,0,3}	{5,1,0,4} {5,1,2,0,4}	{5,1,5}	{5,1,6}
6	{6,1,0} {6,1,2,0}	{6,1}	{6,1,2}	{6,1,2,0,3}	{6,1,0,4} {6,1,2,0,4}	{6,1,5}	{6,1,6}

Table 2: Primary, Secondary, Tertiary Circuits at a Vertex ‘j’

Vertex ‘j’	**Primary(CL1)**	**Secondary(CL2)**	**Tertiary(CL3)**
0	{0,1,0} {0,3,0} {0,4,0} {0,1,2,0}	{1,5,1} {1,6,1} NIL NIL NIL	NIL NIL NIL NIL
1	{1,0,1} {1,5,1} {1,6,1} {1,2,0,1}	{0,3,0} {0,4,0} NIL NIL {0,3,0} {0,4,0}	NIL NIL NIL NIL NIL NIL
2	{2,0,1,2}	{0,1,0} {0,3,0} {0,4,0} {1,5,1} {1,6,1} {1,0,1}	{1,5,1} {1,6,1} NIL NIL NIL NIL {0,3,0} {0,4,0}
3	{3,0,3}	{0,1,0} {0,1,2,0} {0,4,0}	{1,5,1} {1,6,1} {1,5,1} {1,6,1} NIL
4	{4,0,4}	{0,1,0} {0,1,2,0} {0,3,0}	{1,5,1} {1,6,1} {1,5,1} {1,6,1} NIL
5	{5,1,5}	{1,2,0,1} {1,0,1} {1,6,1}	{0,3,0} {0,4,0} {0,3,0} {0,4,0} NIL
6	{6,1,6}	{1,5,1} {1,0,1} {1,2,0,1}	NIL {0,3,0} {0,4,0} {0,3,0} {0,4,0}

In the Transition diagram of fig.1, there are four, four, one, one, one, one & one primary circuits are at vertices 0,1,2,3,4,5& 6 respectively and four, four, six, three, three, three & three secondary circuits are at vertices 0,1,2,3,4,5 & 6 respectively and zero, zero, four, four, four, four & four tertiary circuits are at vertices 0,1,2,3,4,5 & 6 respectively. Since there are largest no. of primary circuits at the vertex '0'with less no. of secondary & tertiary circuits. Therefore '0' is a base-state.

Table 3: Primary, Secondary, Tertiary Circuits W.R.T Simple Paths (Base-State'0')

Vertex	$(0 \xrightarrow{S_R} j)$:P0	(P1)	(P2)
1	$(0 \xrightarrow{S_1} 1)$:{0,1}	{1,5,1} {1,6,1}	NIL NIL
2	$(0 \xrightarrow{S_1} 2)$:{0,1,2}	{1,5,1} {1,6,1}	NIL NIL
3	$(0 \xrightarrow{S_1} 3)$:{0,3}	NIL	NIL
4	$(0 \xrightarrow{S_1} 4)$:{0,4}	NIL	NIL
5	$(0 \xrightarrow{S_1} 5)$: {0,1,5}	{1,5,1} {1,6,1}	NIL NIL
6	$(0 \xrightarrow{S_1} 6)$: {0,1,6}	{1,5,1} {1,6,1}	NIL NIL

4.1.2 Transition Probabilities and the Mean Sojourn Times

Transition Probabilities

$q_{i,j}(t)$: probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$p_{i,j}$: steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j} = q^{*}_{i,j}(0)$; where * denotes Laplace transformation.

Table 4:

$q_{i,j}(t)$	$p_{i,j}=q_{i,j}^*(0)$
$q_{0,1}= \lambda_1 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$ $q_{0,3}= \lambda_3 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$ $q_{0,4}= \lambda_4 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$	$p_{0,1}= \lambda_1/ \lambda_1+\lambda_3+\lambda_4$ $p_{0,3}= \lambda_3/ \lambda_1+\lambda_3+\lambda_4$ $p_{0,4}= \lambda_4/ \lambda_1+\lambda_3+\lambda_4$
$q_{1,0}=g_1(t)e^{-(\lambda_2+\lambda_3+\lambda_4)t}$ $q_{1,2}= \lambda_2 e^{-(\lambda_2+\lambda_3+\lambda_4)t}\bar{G}_1(t)$ $q_{1,5}= \lambda_3 e^{-(\lambda_2+\lambda_3+\lambda_4)t}\bar{G}_1(t)$ $q_{1,6}= \lambda_4 e^{-(\lambda_2+\lambda_3+\lambda_4)t}\bar{G}_1(t)$	$p_{1,0}=g_1^*(\lambda_2+\lambda_3+\lambda_4)$ $p_{1,2}= \lambda_2[1-g_1^*(\lambda_2+\lambda_3+\lambda_4)]/ \lambda_2+\lambda_3+\lambda_4$ $p_{1,5}= \lambda_3[1-g_1^*(\lambda_2+\lambda_3+\lambda_4)]/ \lambda_2+\lambda_3+\lambda_4$ $p_{1,6}= \lambda_4[1-g_1^*(\lambda_2+\lambda_3+\lambda_4)]/ \lambda_2+\lambda_3+\lambda_4$
$q_{2,0}(t)=g_2(t)$	$p_{2,0}=g_2^*(0)=1$
$q_{3,0}(t)=g_3(t)$	$p_{3,0}=g_3^*(0)=1$
$q_{4,0}(t)=g_4(t)$	$p_{4,0}=g_4^*(0)=1$
$q_{5,1}(t)=g_3(t)$	$p_{5,1}=g_3^*(0)=1$
$q_{6,1}(t)=g_4(t)$	$p_{6,1}=g_4^*(0)=1$

It can be easily verified that $p_{0,1}+p_{0,3}+p_{0,4}=1$, $p_{1,0}+p_{1,2}+p_{1,5}+p_{1,6}=1$ $p_{2,0}=1$, $p_{3,0}=1$, $p_{4,0}=1$, $p_{5,1}=1$, $p_{6,1}=1$

Mean Sojourn Times

Table 5:

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_i(t)= e^{-(\lambda_1+\lambda_3+\lambda_4)t}$	$\mu_0=1/ \lambda_1+\lambda_3+\lambda_4$
$R_1(t)= e^{-(\lambda_2+\lambda_3+\lambda_4)t}\bar{G}_1(t)$	$\mu_1=1- g_1^*(\lambda_2+\lambda_3+\lambda_4)/ \lambda_2+\lambda_3+\lambda_4$
$R_2(t)= \bar{G}_2(t)$	$\mu_2= - g_2^{*\prime}(0)$
$R_3(t)= \bar{G}_3(t)$	$\mu_3= - g_3^{*\prime}(0)$
$R_4(t)= \bar{G}_4(t)$	$\mu_4= - g_4^{*\prime}(0)$
$R_5(t)= \bar{G}_3(t)$	$\mu_5= - g_3^{*\prime}(0)$
$R_6(t)= \bar{G}_4(t)$	$\mu_6= - g_4^{*\prime}(0)$

$R_i(t)$: reliability of the system at time t, given that the system in regenerative state i.

μ_i : mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt = R_i^*(0)$$

4.1.3 Evaluation of Parameters

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying *Regenerative Point Graphical Technique (RPGT)* and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state '0' are:

$V_{0,0}$=[(0,1,0)/{1-(1,5,1)}{1-(1,6,1)}+(0,3,0)+(0,4,0)+(0,1,2,0)/{1-(1,5,1)}{1-(1,6,1)}
$=[p_{0,1}p_{1,0}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}+p_{0,3}p_{3,0}+p_{0,4}p_{4,0}+p_{0,1}p_{1,2}p_{2,0}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}$
$V_{0,1}$=[(0,1)/{1-(1,5,1)}{1-(1,6,1)}]
$=[p_{0,1}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$
$V_{0,2}$=[(0,1,2)/ {1-(1,5,1)}{1-(1,6,1)}]
$=[p_{0,1}\, p_{1,2}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$
$V_{0,3}$=[(0,3)]
$= p_{0,3}$
$V_{0,4}$=[(0,4)]
$= p_{0,4}$
$V_{0,5}$=[(0,1,5)/ {1-(1,5,1)}{1-(1,6,1)}]
$=[p_{0,1}\, p_{1,5}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$
$V_{0,6}$=[(0,1,6)/ {1-(1,5,1)}{1-(1,6,1)}]
$=[p_{0,1}\, p_{1,6}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$

(a). MTSF(T_0): From Fig.1, the regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: i = 0,1 For 'ξ' = '0', MTSF is given by

$$\text{MTSF}=\left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} i\right)\right\}\cdot \mu_i}{\Pi_{k_1\neq\xi}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} \xi\right)\right\}}{\Pi_{k_2\neq\xi}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right]$$

MTSF= [(0,0) μ_0+(0,1) μ_1] ÷ [1- {(0,1,0)}]
$= [p_{0,0}\,\mu_0+p_{0,1}\,\mu_1] \div [1- \{(p_{0,1}\, p_{1,0})\}]$
$= [\mu_0+p_{0,1}\,\mu_1] \div [1- \{(p_{0,1}\, p_{1,0})\}]$

(b). Availability of the System (A_0): From Fig.1, the regenerative states, at which the system is available are: j = 0,1 and the regenerative states are i = 0 to 6. For 'ξ' = '0', the total fraction of time for which the system remains available is given by

$$A_0=\left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\} f_j\cdot \mu_j}{\Pi_{k_1\neq\xi}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}\cdot \mu_i^1}{\Pi_{k_2\neq\xi}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right]$$

$$A_0=\left[\sum_j V_{\xi,j}\cdot f_j\cdot \mu_j\right] \div \left[\sum_i V_{\xi,i}\cdot \mu_i^1\right]$$

$A_0 = [V_{0,0}f_0\,\mu_0+ V_{0,1}f_1\,\mu_1] \div [V_{0,0}\,\mu_0^1 V_{0,1}\mu_0^1+ V_{0,2}\,\mu_2^1+ V_{0,3}\,\mu_3^1+ V_{0,4}\mu_4^1+ V_{0,5}\,\mu_5^1+ V_{0,6}\,\mu_6^1]$

Where {f_j=1 for all j}

{$\mu_j^1= \mu_j$ for all j}

(c). Busy period of the Server (B_0): From Fig.1, the regenerative states where Server is busy while doing repairs are: j = 1, 2, 3, 4, 5, 6; and the regenerative states are: i = 0 to 6. For 'ξ' = '0', the total fraction of time for which the Server remains busy is

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}.\eta_j}{\prod_{k_1 \neq \xi}\{1 - V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1 - V_{k_2,k_2}\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j}.\eta_j\right] \div \left[\sum_i V_{\xi,i}.\mu_i^1\right]$$

$B_0 = [V_{0,1}\ \eta_1 + V_{0,2}\ \eta_2 + V_{0,3}\ \eta_3 + V_{0,4}\ \eta_4 + V_{0,5}\ \eta_5 + V_{0,6}\ \eta_6] \div [V_{0,0}\ \mu_0^1 + V_{0,1}\ \mu_1^1 + V_{0,2}\ \mu_2^1 + V_{0,3}\ \mu_3^1 + V_{0,4}\ \mu_4^1 + V_{0,5}\ \mu_5^1 + V_{0,6}\ \mu_6^1]$

Where ($\eta_j = \mu_j$ for all j)
($\mu_j^1 = \mu_j$ for all j)

(d). Expected number of Server's visits (V_0): From Fig.1, the Regenerative States where the Server visits (afresh) for repairs of the system are: j =1, 3, 4; the Regenerative States are: i = 0 to 6. For 'ξ' = '0', the Expected number of Server's Visits per unit time is given by

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq \xi}\{1 - V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1 - V_{k_2,k_2}\}}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}.\mu_i^1\right]$$

$V_0 = [V_{0,1} + V_{0,3} + V_{0,4}] \div [V_{0,0}\ \mu_0^1 + V_{0,1}\ \mu_1^1 + V_{0,2}\ \mu_2^1 + V_{0,3}\ \mu_3^1 + V_{0,4}\ \mu_4^1 + V_{0,5}\ \mu_5^1 + V_{0,6}\ \mu_6^1]$
$= [V_{0,1} + V_{0,3} + V_{0,4}] \div [V_{0,0}\ \mu_0 + V_{0,1}\ \mu_1 + V_{0,2}\ \mu_2 + V_{0,3}\ \mu_3 + V_{0,4}\ \mu_4 + V_{0,5}\ \mu_5 + V_{0,6}\ \mu_6]$

Where ($\mu_j^1 = \mu_j$ for all j)

4.1.4 Profit Function of the System

The Profit analysis of the system can be done by using the profit function:

$$P_0 = C_1.A_0 - C_2.B_0 - C_3.V_0$$

Where,
$C_1 =$ Revenue per unit of time the system is available.
$C_2 =$ Cost per unit time the server remains busy for the repairs.
$C_3 =$ Cost per visit of the server.

5 Particular Case

Let us take;
$g_1(t) = \omega_1 e^{-\omega_1 t}$, $g_2(t) = \omega_2 e^{-\omega_2 t}$, $g_3(t) = \omega_3 e^{-\omega_3 t}$, $g_4(t) = \omega_4 e^{-\omega_4 t}$,
we have,
$p_{0,1} = \lambda_1/ \lambda_1+\lambda_3+ \lambda_4$, $p_{0,3} = \lambda_3/ \lambda_1+\lambda_3+ \lambda_4$, $p_{0,4} = \lambda_4/ \lambda_1+\lambda_3+ \lambda_4$,
$p_{1,0} = \omega_1/ \omega_1+ \lambda_2+\lambda_3+ \lambda_4$, $p_{1,2} = \lambda_2 /\omega_1+ \lambda_2+\lambda_3+ \lambda_4$, $p_{1,5} = \lambda_3 /\omega_1+ \lambda_2+\lambda_3+ \lambda_4$,
$p_{1,6} = \lambda_4/\omega_1+ \lambda_2+\lambda_3+ \lambda_4$, $p_{2,0}=1$, $p_{3,0}=1$, $p_{4,0}=1$, $p_{5,1}=1$, $p_{6,1}=1$
$\mu_0 = 1/\lambda_1+\lambda_3+ \lambda_4$, $\mu_1 = 1/\omega_1+ \lambda_2+\lambda_3+ \lambda_4$, $\mu_2 = 1/\omega_2$, $\mu_3 = 1/\omega_3$, $\mu_4=1/\omega_4$, $\mu_5=1/\omega_3$, $\mu_5=1/\omega_4$

By using these results, we get the following:
MTSF (T_0)= $\omega_1+ \lambda_1+ \lambda_2+\lambda_3+ \lambda_4/(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\lambda_1+\lambda_3+ \lambda_4)- \omega_1 \lambda_1$
Availability (A_0)= $[(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\omega_1 \lambda_1+ \lambda_1\lambda_2)+ (\lambda_3+ \lambda_4) (\omega_1+ \lambda_2+ \lambda_4)(\omega_1+ \lambda_2+ \lambda_3)+ \lambda_1(\omega_1+ \lambda_2+\lambda_3+ \lambda_4) (\lambda_1+\lambda_3+ \lambda_4)]\ \omega_2\omega_3\omega_4/ [\{\lambda_1(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\lambda_1+\lambda_3+ \lambda_4)(\omega_3\omega_4\ \lambda_2+ \omega_2\omega_4\ \lambda_3+ \omega_2\omega_3\lambda_4)\}+\{(\omega_4\ \lambda_3+ \omega_3\ \lambda_4) (\lambda_1+\lambda_3+ \lambda_4) (\omega_1+ \lambda_2+ \lambda_4)(\omega_1+ \lambda_2+ \lambda_3) \omega_2\}+\{(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\omega_1 \lambda_1+ \lambda_1\lambda_2)+ (\lambda_3+ \lambda_4) (\omega_1+ \lambda_2+ \lambda_4)(\omega_1+ \lambda_2+ \lambda_3)+ \lambda_1(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\lambda_1+\lambda_3+ \lambda_4)\}\ \omega_2\omega_3\omega_4]$
Busy period of the Server (B_0)= $[\lambda_1\lambda_2(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\lambda_1+\lambda_3+ \lambda_4) \omega_3\omega_4+(\omega_4\ \lambda_1\lambda_3+ \omega_3\ \lambda_1\lambda_4)(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\lambda_1+\lambda_2+ \lambda_4)\omega_2+ \lambda_3(\lambda_1+\lambda_2+ \lambda_4) (\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3) \omega_2\omega_4+ \lambda_4(\lambda_1+\lambda_2+ \lambda_4) (\omega_1+ \lambda_2+ \lambda_4) (\omega_1+ \lambda_2+\lambda_3) \omega_2\omega_1]/[(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\omega_1 \lambda_1+ \lambda_1\lambda_2+ \lambda_1^2+ \lambda_1\lambda_3+ \lambda_1\lambda_4)(\lambda_1+\lambda_2+ \lambda_4) \omega_2\omega_3\omega_4+(\lambda_3+ \lambda_4) (\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3) (\lambda_1+\lambda_2+ \lambda_4) \omega_2\omega_3\omega_4+(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)\{(\lambda_1^2\ \lambda_2+ \lambda_1\ \lambda_2\ \lambda_3+ \lambda_2\lambda_2\ \lambda_4) \omega_3\omega_4+(\omega_2\ \lambda_1\lambda_3+ \omega_1\ \lambda_1\lambda_4) (\lambda_1+\lambda_2+ \lambda_4) \omega_2\}+(\lambda_1+\lambda_2+ \lambda_4) (\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3)(\omega_2\omega_4\ \lambda_3+ \omega_2\omega_3\ \lambda_4)]$
Number of server's visits (V_0)= $[\{\lambda_1(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)^2+ \lambda_3(\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3)+ \lambda_4(\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3)\} (\lambda_1+\lambda_3+ \lambda_4) (\lambda_1+\lambda_2+ \lambda_4) \omega_2\omega_3\omega_4]/[(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)(\omega_1 \lambda_1+ \lambda_1\lambda_2+ \lambda_1^2+ \lambda_1\lambda_3+ \lambda_1\lambda_4)(\lambda_1+\lambda_2+ \lambda_4) \omega_2\omega_3\omega_4+(\lambda_3+ \lambda_4) (\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3) (\lambda_1+\lambda_2+ \lambda_4) \omega_2\omega_3\omega_4+(\omega_1+ \lambda_2+\lambda_3+ \lambda_4)\{ (\lambda_1^2\ \lambda_2+ \lambda_1\lambda_2\ \lambda_3+ \lambda_1\lambda_2\ \lambda_4) \omega_3\omega_4+(\omega_4\ \lambda_1\lambda_3+ \omega_3\ \lambda_1\lambda_4) (\lambda_1+\lambda_2+ \lambda_4) \omega_2\}+(\lambda_1+\lambda_2+ \lambda_4) (\omega_1+ \lambda_2+\lambda_4) (\omega_1+ \lambda_2+\lambda_3)(\lambda_3\omega_2\omega_4+ \lambda_4\omega_2\omega_3)]$

6. Analytical Discussion

The following tables, graphs and conclusions are obtained for:
$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$, $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$

(A) MTSF vs. Repair Rate: The MTSF of the system is calculated for different values of the Failure Rate (λ) by taking λ=0.0005, 0.0006, 0.0007, 0.0008, 0.0009 and 0.0010 and for different values of the Repair Rate (ω) by taking ω=0.80, 0.85, 0.90, 0.95 and 1.00 the data so obtd. are shown in Table 6 and Graphically in fig. 2 .

Table 6:

λ	T_0 (ω=0.80)	T_0 (ω=0.85)	T_0 (ω=0.90)	T_0 (ω=0.95)	T_0 (ω=1.00)
0.0005	1002.5	1065	1127.50	1190	1252
0.0006	802.4	852.7	902.7	952.4	1024.4
0.0007	729.81	775.27	820.72	866.18	911.63
0.0008	617.84	656.30	694.76	733.23	793.84
0.0009	574	609.71	645.42	681.14	740
0.001	502.5	533.75	565	596.25	627.5

Table 6 shows the behaviour of the MTSF (T_0) vs. the repair rate (ω) of the unit of the system for different values of the failure rate (λ). It is concluded that MTSF increases with increase in the values of the repair rate (ω).

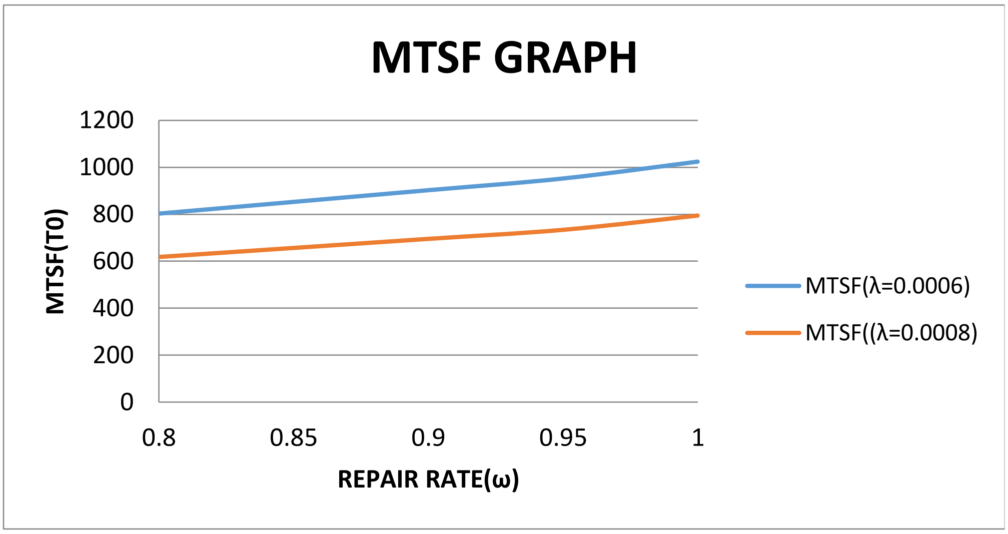

Fig: 2

Further it can be concluded from the fig.3 that values of MTSF (T_0) shows the expected trend for different values of failure rate (λ=0.0006 & 0.0008), as T_0 increases in the values of repair rate(ω).

(B) Availability (A_1) vs. the Repair Rate (ω) & failre rate: The Availability of the system is calculated for different values of the failure rate (λ) by taking λ =0.0005, 0.0006, 0.0007, 0.0008, 0.0009 and 0.001 and for different values of the repair rate (ω) by taking ω =0.80, 0.85, 0.90, 0.95 and 1.0.The data so obtained are shown in table 7 and graphically in fig.3

Table 7:

λ	A_0 (ω=0.80)	A_0 (ω=0.85)	A_0 (ω=0.90)	A_0 (ω=0.95)	A_0 (ω=1.00)
0.0005	0.99874413	0.99883286	0.99888663	0.99894416	0.99900169
0.0006	0.99841479	0.99857967	0.99867059	0.99874185	0.99883277
0.0007	0.99825233	0.99835674	0.99844428	0.99853671	0.99862425
0.0008	0.99800492	0.99812488	0.99822596	0.99831909	0.99842017
0.0009	0.99775714	0.99788715	0.99800686	0.99810756	0.99822727
0.001	0.99750973	0.99765522	0.99778854	0.99790242	0.99802319

Table shows the behaviour of the Availability (A_0) vs. the Repair rate (ω) of the unit of the system for different values of the failure rate (λ).It is concluded that Availability (A_0) increases with increase in the values of the Repair rate (ω) and decreases with the increase in failure rates.

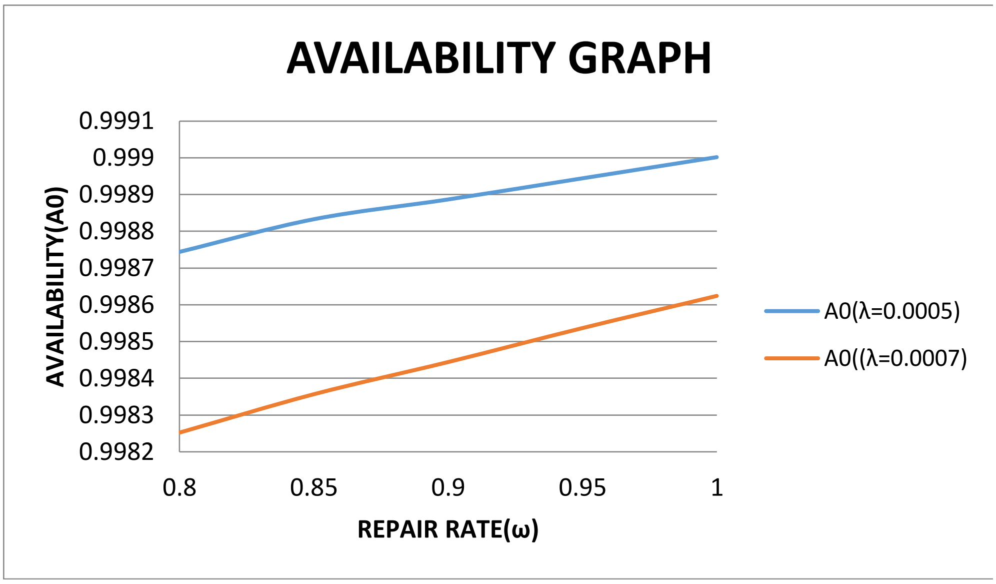

Fig: 3

Further it can be concluded from the fig.3 that values of availability(A_0) shows the expected trend for different values of failure rate(λ=0.0005 & 0.0007), as A_0 increases in the values of repair rate(ω).

(C) Busy period of Server (B_0) vs. the Repair Rate (ω): The Busy period of Server (B_1) of the system is calculated for different values of the failure rate (λ) by taking λ =0.0005, 0.0006, 0.0007, 0.0008, 0.0009 and 0.001 and for different values of the repair rate (ω) by taking ω =0.80, 0.85, 0.90, 0.95 and 1.0.The data so obtained are shown in table 8 and graphically in fig. 4

Table 8:

λ	B_0(ω=0.80)	B_0(ω=0.85)	B_0(ω=0.90)	B_0(ω=0.95)	B_0(ω=1.0)
0.0005	109.33	111.34	123.71	125.45	164.47
0.0006	94.33	95.75	101.91	103.75	111.77
0.0007	79.79	81.56	87.63	90.54	96.67
0.0008	71.83	73.82	80.61	83.57	91.57
0.0009	66.26	72.53	75.49	78.74	84.49
0.001	56.22	60.13	67.24	71.7	80.7

Table shows the behaviour of theBusy period of Server (B_0) vs. the Repair rate (ω) of the unit of the system for different values of the failure rate (λ).It is concluded that Busy period of Server (B_0) increases with increase in the values of the Repair rate (ω).

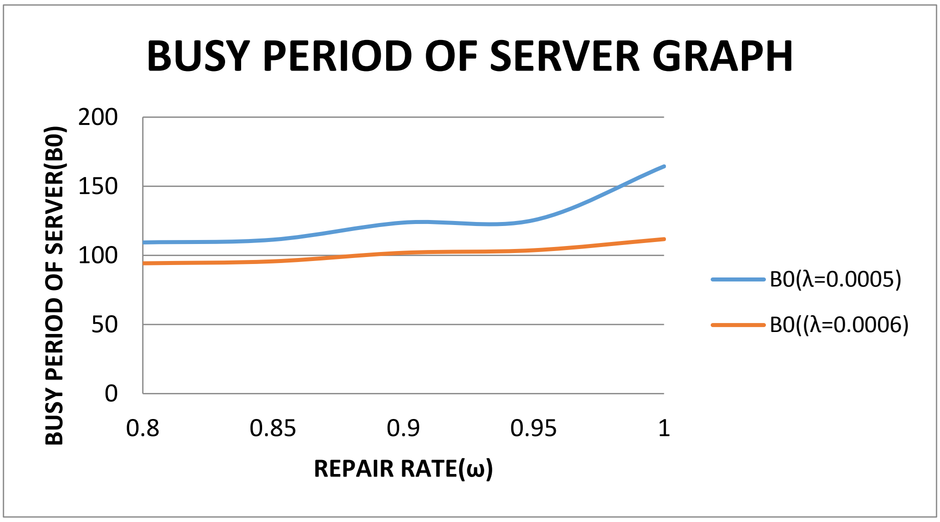

Fig: 4

Further it can be concluded from the fig.4 that the values of Busy period of Server (B_0) shows the expected trend for different values of Failure Rate (λ=0.0006 & 0.0008), as B_0 increases with the increase in the values of Repair Rate(ω).

7. Conclusion

From the Graphs and Tables, we see that as the Repair Rate (ω) increases, Availability of the system increases, which would be. The Regenerative-Point Graphical Technique is useful for the evaluation of the parameters in a simple way, without writing any state equation and without doing any lengthy and cumbersome calculations. This study can be extended to time dependent case. As it is easy for the management to control repair rates in comparison to failure rates, fixing the target of availability repair rates can be determined and managed by having efficient server.

8. References

1. Gupta VK, Singh J, Vanita. The New Concept of Base State in the Reliability Analysis; Journal of Mathematics and Systems Science (JMASS), 2010; 6(2):38-53.
2. Jindal G. Behaviour and Availability Analysis of Some Industrial Systems; Ph.D. Thesis, JJTU (Jhunjhunu). 2012.
3. Gupta VK, Singh J. A New Approach for Availability Analysis; IEEE Transactions on Reliability, Tr 2007-107 Communicated.
4. Gupta VK. Behaviour and Profit Analysis of Some Process Industries'; Ph.D. Thesis, NIT Kurukshetra (India). 2008.
5. Goel P, Singh J. Availability Analysis of a Standby Complex System Having Imperfect Switch; Microelectronics & Reliab., 1995; 35(2):285-288.
6. Chander S, Bansal RK. Profit Analysis of a Single Unit Reliability Models with Repair at Different Failure Modes; Proc. of International Conference on Reliability & Safety Engineering, 2005, 577-588.
7. Gupta P, Singh J. Singh IP. Availability Prediction for Neat Soap Production System in a Soap Plant; National Conference on Mathematical & Statistical Techniques, TIET Patiala. 2001.
8. Gupta P, Singh J, Singh IP. Availability Analysis of Soap Cakes Production System- A Case Study; Proc. National Conference on Emerging Trends in Manufacturing Systems, SLIET, Longowal (Punjab), 2004, 283-295.
9. Gupta VK, Singh J. Behaviour and Profit Analysis of a Soap Industry; JMASS, 2008; 4(1):41-62.
10. Govil Jindal, Pardeep Goel, Gupta VK, Jai Singh. Availability and Behavioural Analysis of a Single Unit Redundant system having Imperfect Switch, 2012; 8(1):110-124, ISSN 0975-5454.