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Behaviour analysis of a utensil industry

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Abstract
In this paper availability analysis of a utensil industry using RPGT is discussed. In which two units can work in reduced capacity and third unit have sub units in series. The distribution of the failure times and repair times are exponential and general. There is single repair facility catering to needs of all the units when need arises. Failures and repairs are assumed to be independent of each other. Repair is considered to be perfect. The system is down if any of units fails completely. Expressions for mean time to system failure, availability & busy period of the server are derived under steady state conditions using RPGT.

Keywords: Reliability, Availability, Primary Circuit, Secondary Circuit, Tertiary Circuit, Base-State, Regenerative Point Graphical Technique (RPGT), MTSF, Busy period of the server, Expected number of Server's visits

1. Introduction

In this paper the reliability model for availability analysis of utensils manufacturing industry is developed. The industry is divided into three sub systems i.e. cutting system 'B' in which sheets are cut into circular sheets as per the requirements. Pressing system (C) which converts the cut sheets into the shape of utensil. Spinning and Buffing system (D) which give utensils the final shape and polishes them. Steel sheets are supplied from separate plants. Sub system C have sub units in series hence fails directly from full capacity to complete failure, sub system C and D can fail from full capacity to complete failure only through partial failures. When anyone of C or D or both units are working in reduced state then the whole system works in reduced state.

Availability analysis of various process industries and software has been carried out by a number of researchers Cao, Jinhva and Wu, Yanhong [1] evaluated Reliability Analysis of A Two-unit Cold Standby System With Replaceable Repair facility, Chander, S. and Bansal, R.K. [2] Discussed Profit analysis of single-unit reliability models with repair at different failure modes, Malik S. C., Chand, P. and Singh, J. [3] presented A model for availability analysis of distributed software/ hardware systems. Gupta, V. K. Singh J., & Vanita [4] Discussed the New Concept of Base-State in the Reliability Analysis, Das [5] and Fukuta [6] discussed about the reliability, Osaki [7] discussed about redundant system and Chung [8] discussed about the repairable and non-repairable system, Gupta V.K [9] and Jindal [10] discussed the RPGT technique for reliability analysis. Malik Navneet & Goel Pardeep [11] discussed the availability analysis of single module. In this paper the behavioral and availability analysis of two module software system which can work in reduced capacity is done. The mean time to system failure, availability and other key parameters of the software are evaluated using the Regenerative Point Graphical Technique (RPGT).

There is a single repairman for all sub systems. Repairs are perfect. The system is down if anyone of the unit fails. The distributions of the failure times are exponential and are different for both type of failure. They are also assumed to be independent of each other. The repair times are general. The system is discussed for steady state conditions. Using the *Regenerative Point Graphical Technique* (RPGT) the following system characteristics have been evaluated to study the system performance.

1. Mean Time To Software Failure (MTSF).
2. Total fraction of time for which the software is available.
3. The busy period of the server doing any given job.
4. The number of the server's visits.

Tables and graphs are prepared to represent the behaviour of the model.

2. Assumptions and Notations

The following assumptions and notations/symbols are used:

1. The System consists of three non-identical units 'B', 'C' & 'D' in which 'B' cannot work in reduced state and the other two units 'C' & 'D' can work in reduced state.
2. The unit 'C' & 'D' can fail partially and hence can be in up-state, partially failed state (reduced state) or totally failed state. The system can work with reduced capacity in a partially failed state.
3. There is a single repair facility catering to the needs of both the units as and when need arises.
4. The distribution of the failure times & the repair times are exponential and general respectively and also different for operating and standby unit. They are also assumed to be independent of each other.
5. Repairs are perfect i.e. the repair facility never does any damage to the units.
6. A Repaired unit works like a new one.
7. The system is down if any one of the units fails completely.
8. Nothing can fail further when the system is in failed state
9. The system is discussed for steady state conditions.
10. Priority in repair will be given to the main unit B.

pr /pf : Probability/transition probability factor.

$q_{i,j}$ (t): Probability density function (p. d. f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$p_{i,j}$: Steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j}=q^{*}_{i,j}(0)$; where* denotes Laplace transformation.

$\overline{cycle}$: A circuit formed through un-failed states.

k-cycle : A circuit (may be formed through regenerative or non-regenerative/failed states) whose terminals are at the regenerative state k.

k-$\overline{cycle}$: A circuit (may be formed through only un-failed regenerative/ non- regenerative states) whose terminals are at the regenerative state k.

$\left(i\xrightarrow{Sr}j\right)$: *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi\xrightarrow{sff}i\right)$: A directed simple failure free path from ξ -state to i-state.

$V_{k,k}$: *pf* of the state k reachable from the terminal state k of the k-cycle.

$V_{\overline{k,k}}$: *pf* of the state k reachable from the terminal state k of the k-$\overline{cycle}$.

$R_i(t)$: Reliability of the system at time t, given that the system entered the un-failed regenerative state i at t=0.

$A_i(t)$: Probability that the system is available in up-state at time t, given that the system entered regenerative state i at t=0.

$B_i(t)$: Probability that the server is busy doing a particular job at epoch t, given that the system entered regenerative state i at t=0.

$V_i(t)$: The expected number of visits of the server for a given job in (0,t], given that the system entered regenerative state i at t=0.

$W_i(t)$: Probability that the server is busy doing a particular job at epoch t without transiting to any other regenerative state 'i' through one or more non- regenerative states, given that the system entered the regenerative state 'i' at t=0.

μ_i : Mean sojourn time spent in state i, before visiting any other states;

$\mu_i = \int_0^\infty R_i(t)dt$.

μ_i^1 : The total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

η_i : Expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

f_j : Fuzziness measure of the j-state.

λ_1: Constant failure rate of unit C from full capacity to partial capacity state.

λ_2: Constant failure rate of unit C from reduced capacity to complete failure.

λ_3: Constant failure rate of unit D from full capacity to partial capacity state

λ_4: Constant failure rate of unit D from reduced capacity to complete failure.

λ_5: Constant failure rate of unit B to complete failure.

$G_1(t)/G_2(t)/g_1(t)/g_2(t)$: Probability density function/cumulative distribution function of the repair-time of the unit C from partial & complete failure.

$G_3(t)/G_4(t)/g_3(t)/g_4(t)$: Probability density function/cumulative distribution function of the repair-time of the unit D from partial & complete failure.

$G_5(t)/g_5(t)$: Probability density function/cumulative distribution function of the repair-time of the unit B from complete failure.

B/b: Unit B in operative state/ failed state.

C/$\underline{C}$/c: Main unit C in the operative state/partial failed state/failed state

D/$\underline{D}$/d: Main unit D in the operative state/partial failed state/failed state

3. Transition Diagram of the System

Following the above assumptions and notations, the transition diagram of the systems are shown in fig.1

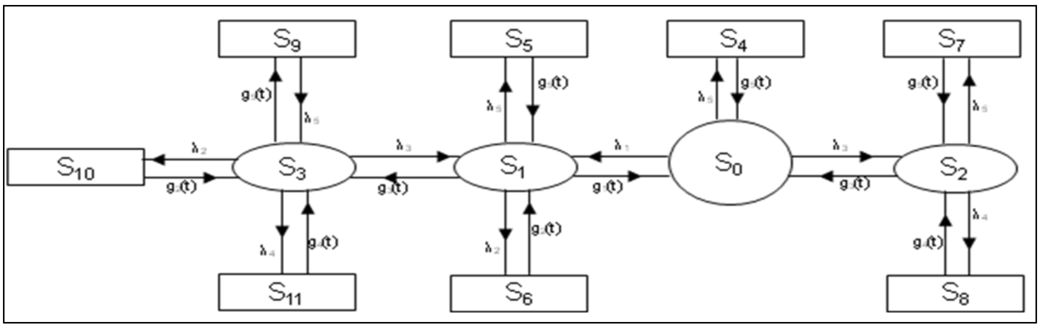
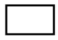
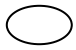


Fig 1:

The system can be in any of the following states with respect to the above symbols.

S_0 = BCD	S_1= B$\underline{C}$D
S_2 = BCD	S_3 = B$\underline{CD}$
S_4 = bCD	S_5 = b$\underline{C}$D
S_6 =BcD	S_7 =bC$\underline{D}$
S_8 = BCd	S_9 =b$\underline{CD}$
S_{10} = BcD	S_{11} =B$\underline{C}$d

States S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10} & S_{11} are regenerative states.

State	Symbol
Regenerative state/point	•
Up-state:	
Failed state:	
Degenerated/Reduced state	

4 Evaluation of Parameters of the System

4.1 Analysis of System

The key parameters (under steady state conditions) of the system are evaluated by determining a 'base-state' and applying RPGT. The MTSF is determined w. r. t the initial state'0' and the other parameters are obtained by using base-state.

4.1.1 Determination of base-state

From the Transition diagram (fig.1), The various Primary, Secondary, Tertiary Circuits w.r.t simple path at base-state and various paths from state 'i' to reachable state 'j' at all vertices are shown in Table-1 & Table-2.

Paths from State 'i' to the Reachable State 'j':P0

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8	j=9	j=10	j=11
0	(0,1,0) (0,2,0) (0,4,0)	(0,1)	(0,2)	(0,1,3)	(0,4)	(0,1,5)	(0,1,6)	(0,2,7)	(0,2,8)	(0,1,3,9)	(0,1,3,10)	(0,1,3,11)
1	(1,0)	(1,0,1) (1,5,1) (1,6,1)	(1,0,2)	(1,3)	(1,0,4)	(1,5)	(1,6)	(1,0,2,7)	(1,0,2,8)	(1,3,9)	(1,3,10)	(1,3,11)
2	(2,0)	(2,0,1)	(2,0,2)	(2,0,1,3)	(2,0,4)	(2,0,1,5)	(2,0,1,6)	(2,7)	(2,8)	(2,0,1,3,9)	(2,0,1,3,10)	(2,0,1,3,11)
3	(3,1,0)	(3,1)	(3,1,0,2)	(3,1,3) (3,9,3) (3,10,3) (3,11,3)	(3,10,4)	(3,1,5)	(3,1,6)	(3,1,0,2,7)	(3,1,0,2,8)	(3,9)	(3,10)	(3,11)
4	(4,0)	(4,0,1)	(4,0,2)	(4,0,1,3)	(4,0,4)	(4,0,1,5)	(4,0,1,6)	(4,0,2,7)	(4,0,2,8)	(4,0,1,3,9)	(4,0,1,3,10)	(4,0,1,3,11)
5	(5,1,0)	(5,1)	(5,1,0,2)	(5,1,3)	(5,1,0,4)	(5,1,5)	(5,1,6)	(5,1,0,2,7)	(5,1,0,2,8)	(5,1,0,2,9)	(5,1,3,10)	(5,1,3,11)
6	(6,1,0)	(6,1)	(6,1,0,2)	(6,1,3)	(6,1,0,4)	(6,1,5)	(6,1,6)	(6,1,0,2,7)	(6,1,0,2,8)	(6,1,3,9)	(6,1,3,10)	(6,1,3,11)
7	(7,2,0)	(7,2,0,1)	(7,2)	(7,2,0,1,3)	(7,2,0,4)	(7,2,0,1,5)	(7,2,0,1,6)	(7,2,7)	(7,2,8)	(7,2,0,1,3,9)	(7,2,0,1,3,10)	(7,2,0,1,3,11)
8	(8,2,0)	(8,2,0,1)	(8,2)	(8,2,0,1,3)	(8,2,0,4)	(8,2,0,1,5)	(8,2,0,1,6)	(8,2,7)	(8,2,8)	(8,2,0,1,3,9)	(8,2,0,1,3,10)	(8,2,0,1,3,11)
9	(9,3,1,0)	(9,3,1)	(9,3,1,0,2)	(9,3)	(9,3,1,0,4)	(9,3,1,5)	(9,3,1,6)	(9,3,1,0,2,7)	(9,3,1,0,2,8)	(9,3,9)	(9,3,10)	(9,3,11)
10	(10,3,10)	(10,3,1)	(10,3,1,0,2)	(10,3)	(10,3,1,0,4)	(10,3,1,5)	(10,3,1,6)	(10,3,1,0,2,7)	(10,3,1,0,2,8)	(10,3,9)	(10,3,10)	(10,3,11)
11	(11,3,1,0)	(11,3,1)	(11,3,1,0,2)	(11,3)	(11,3,1,0,4)	(11,3,1,5)	(11,3,1,6)	(11,3,1,0,2,7)	(11,3,1,0,2,8)	(11,3,9)	(11,3,10)	(11,3,11)

Table 2: Primary, Secondary, Tertiary Circuits at a Vertex.

	Primary (CL1)	Secondary (CL2)	Tertiary (CL3)
0	(0,1,0) (0,2,0) (0,4,0)	(1,5,1) (1,6,1) NIL NIL	NIL NIL NIL NIL
1	(1,0,1) (1,5,1) (1,6,1)	(0,2,0) (0,4,0) NIL NIL	NIL NIL NIL NIL
2	(2,0,2)	(0,1,0) (0,4,0)	(1,5,1),(1,6,1) NIL
3	(3,1,3) (3,9,3) (3,10,3) (3,11,3)	(1,0,1) (1,5,1) (1,6,1) NIL NIL	(0,2,0),(0,4,0) NIL NIL NIL NIL

		NIL	NIL
4	(4,0,4)	(0,1,0) (0,2,0)	(1,5,1),(1,6,1) NIL
5	(5,1,5)	(1,0,1) (1,6,1)	(0,2,0),(0,4,0) NIL
6	(6,1,6)	(1,0,1) (1,5,1)	(0,2,0),(0,4,0) NIL
7	(7,2,7)	(2,0,2)	(0,1,0),(0,4,0)
8	(8,2,8)	(2,0,2)	(0,1,0),(0,4,0)
9	(9,3,9)	(3,1,3) (3,10,3) (3,11,3)	(1,0,1),(1,5,1),(1,6,1) NIL NIL
10	(10,3,10)	(3,1,3) (3,9,3) (3,11,3)	(1,0,1),(1,5,1),(1,6,1) NIL NIL
11	(11,3,11)	(3,1,3) (3,9,3) (3,11,3)	(1,0,1),(1,5,1),(1,6,1) NIL NIL

In the Transition diagram of fig. 1,there are three three, one, four, one, one, one, one, one, one, one and one primary circuits are at vertices 0,1,2,3,4,5,6,7,8,9,10 & 11 respectively two, two, two, three, two, two, two, one, one, three, three and three secondary circuits are at vertices 0,1,2,3,4,5,6,7,8,9,10 & 11 respectively and zero, zero, two, two, two, two, two, two, two, three, three and three tertiary circuits are at vertices 0,1,2,3,4,5,6,7,8,9,10 & 11 respectively. As there is largest number of primary circuits at the vertex '3'with lesser number of secondary & tertiary circuits, therefore '3' is a base-state.

Table 3: Primary, Secondary, Tertiary Circuits w. r. t Simple Paths (Base-State'3')

Vertex j	$(3 \xrightarrow{S_R}$ **j):P0**	(P1)	(P2)	(P3)
0	$(3 \xrightarrow{S_1} 0)$:(3,1,0)	(0,2,0) (0,4,0) (1,5,1) (1,6,1)	NIL NIL NIL NIL	NIL NIL NIL NIL
1	$(3 \xrightarrow{S_1} 1)$:(3,1)	(1,0,1) (1,5,1) (1,6,1)	(0,2,0) (0,4,0) NIL NIL	NIL NIL NIL NIL
2	$(3 \xrightarrow{S_1} 2)$:(3,1,0,2)	(0,4,0) (1,5,1) (1,6,1)	NIL NIL NIL	NIL NIL NIL
4	$(3 \xrightarrow{S_1} 4)$: (3,1,0,4)	(0,2,0) (1,5,1) (1,6,1)	NIL NIL NIL	NIL NIL NIL
5	$(3 \xrightarrow{S_1} 5)$: (3,1,5)	(1,0,1) (1,6,1)	(0,2,0) (0,4,0) NIL	NIL NIL NIL
6	$(3 \xrightarrow{S_1} 6)$: (3,1,6)	(1,0,1) (1,5,1)	(0,2,0) (0,4,0) NIL	NIL NIL NIL
7	$(3 \xrightarrow{S_1} 7)$: (3,1,0,2,7)	(0,4,0) (1,5,1) (1,6,1)	NIL NIL NIL	NIL NIL NIL
8	$(3 \xrightarrow{S_1} 8)$: (3,1,0,2,8)	(0,4,0) (1,5,1) (1,6,1)	NIL NIL NIL	NIL NIL NIL
9	$(3 \xrightarrow{S_1} 9)$: (3,9)	NIL	NIL	NIL
10	$(3 \xrightarrow{S_1} 10)$: (3,10)	NIL	NIL	NIL
11	$(3 \xrightarrow{S_1} 11)$: (3,11)	NIL	NIL	NIL

4.1.2 Transition Probabilities and the mean sojourn times

Table 4:

$q_{i,j}(t)$	$p_{i,j} = q^{*}_{i,j}(0)$
$q_{0,1}= \lambda_1 e^{-(\lambda 1+ \lambda 3+ \lambda 5)}$ $q_{0,2}= \lambda_3 e^{-(\lambda 1+ \lambda 3+ \lambda 5)}$ $q_{0,4}= \lambda_5 e^{-(\lambda 1+ \lambda 3+ \lambda 5)}$	$p_{0,1}= \lambda_1/ \lambda_1+ \lambda_3+ \lambda_5$ $p_{0,2}= \lambda_3/ \lambda_1+ \lambda_3+ \lambda_5$ $p_{0,4}= \lambda_5/ \lambda_1+ \lambda_3+ \lambda_5$
$q_{1,0}= g_1(t)e^{-(\lambda 2+ \lambda 3+ \lambda 5)}$ $q_{1,6}= \lambda_2 e^{-(\lambda 2+ \lambda 3+ \lambda 5)}\bar{G}_1(t)$ $q_{1,3}= \lambda_3 e^{-(\lambda 2+ \lambda 3+ \lambda 5)} \bar{G}_1(t)$ $q_{1,5}= \lambda_5 e^{-(\lambda 2+ \lambda 3+ \lambda 5)} \bar{G}_1(t)$	$p_{1,0}= g_1^{*}(\lambda_1+ \lambda_3+ \lambda_5)$ $p_{1,6}= \lambda_2/ \lambda_1+ \lambda_3+ \lambda_5[1- g_1^{*}(\lambda_1+ \lambda_3+ \lambda_5)]$ $p_{1,3}= \lambda_3/ \lambda_1+ \lambda_3+ \lambda_5[1- g_1^{*}(\lambda_1+ \lambda_3+ \lambda_5)]$ $p_{1,5}= \lambda_5/ \lambda_1+ \lambda_3+ \lambda_5[1- g_1^{*}(\lambda_1+ \lambda_3+ \lambda_5)]$
$q_{2,0}= g_3(t)e^{-(\lambda 4+ \lambda 5)}$ $q_{2,8}= \lambda_4 e^{-(\lambda 4+ \lambda 5)} \bar{G}_3(t)$ $q_{2,7}= \lambda_5 e^{-(\lambda 4+ \lambda 5)} \bar{G}_3(t)$ $q_{3,1}= g_3(t)e^{-(\lambda 2+ \lambda 4+ \lambda 5)}$ $q_{3,10}= \lambda_2 e^{-(\lambda 2+ \lambda 4+ \lambda 5)}\bar{G}_3(t)$ $q_{3,11}= \lambda_4 e^{-(\lambda 2+ \lambda 4+ \lambda 5)} \bar{G}_3(t)$ $q_{3,9}= \lambda_5 e^{-(\lambda 2+ \lambda 4+ \lambda 5)} \bar{G}_3(t)$	$p_{2,0}= g_3^{*}(\lambda_4+\lambda_5)$ $p_{2,8}= \lambda_4/ \lambda_4+\lambda_5[1- g_1^{*}(\lambda_4+\lambda_5)]$ $p_{2,7}= \lambda_5/ \lambda_5+\lambda_5[1- g_1^{*}(\lambda_4+\lambda_5)]$ $p_{3,1}= g_1^{*}(\lambda_2+ \lambda_4+ \lambda_5)$ $p_{3,10}= \lambda_2/ \lambda_2+ \lambda_4+ \lambda_5[1- g_1^{*}(\lambda_2+ \lambda_4+ \lambda_5)]$ $p_{3,11}= \lambda_4/ \lambda_2+ \lambda_4+ \lambda_5[1- g_1^{*}(\lambda_2+ \lambda_4+ \lambda_5)]$ $p_{3,9}= \lambda_5/ \lambda_2+ \lambda_4+ \lambda_5[1- g_1^{*}(\lambda_2+ \lambda_4+ \lambda_5)]$
$q_{4,0}= g_5(t)$	$p_{4,0}= g_5^{*}(0)=1$
$q_{5,1}= g_5(t)$	$p_{4,0}= g_5^{*}(0)=1$
$q_{6,1}= g_2(t)$	$p_{4,0}= g_5^{*}(0)=1$
$q_{7,2}= g_5(t)$	$p_{4,0}= g_5^{*}(0)=1$
$q_{8,2}= g_4(t)$	$p_{4,0}= g_4^{*}(0)=1$
$q_{9,3}= g_5(t)$	$p_{4,0}= g_5^{*}(0)=1$
$q_{10,3}= g_2(t)$	$p_{4,0}= g_2^{*}(0)=1$
$q_{11,3}= g_4(t)$	$p_{4,0}= g_4^{*}(0)=1$

It can be easily verified that
$p_{0,1}+ p_{0,2}+p_{0,4}=1$, $p_{5,1}=1$ $p_{1,0}+p_{1,6}+ p_{1,3}+ p_{1,5}=1$, $p_{2,0}+p_{2,8}+ p_{2,7}=1$, $p_{3,1}+p_{3,10}+p_{3,11}+p_{3,9}=1$, $p_{4,0}=1$, $p_{5,1}=1$, $p_{6,1}=1$, $p_{7,2}=1$, $p_{8,2}=1$, $p_{9,3}=1$, $p_{10,3}=1$, $p_{11,3}=1$

4.1.3 Evaluation of Parameters

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying *Regenerative Point Graphical Technique (RPGT)* and using '3' as the base-state of the system as under:
The transition probability factors of all the reachable states from the base state '3' are:

$V_{3,0}$= [(3,1,0)/{1-(1,5,1)}{1-(1,6,1}{1-(0,4,0)}{1-(0,2,0)}]
= $p_{3,1}$ $p_{1,0}$/{1-($p_{1,5}$ $p_{5,1}$)}{1-($p_{1,6}p_{6,1}$)}{1- $p_{0,2}$ $p_{2,0}$}

$V_{3,1}$=[(3,1)/{1-(1,6,1)}{1-(1,5,1)}{1-(1,0,1)/(1-(0,2,0))(1-(0,4,0))}(1-(0,2,0))(1-(0,4,0))
= $p_{3,1}$/{1-($p_{1,6}$ $p_{6,1}$)}{1-($p_{1,5}$ $p_{5,1}$)}{(1- $p_{0,2}$ $p_{2,0}$)(1- $p_{0,4}$ $p_{4,0}$)-($p_{1,0}$ $p_{0,1}$)}

$V_{3,2}$ = [(3,1,0,2)/{1-(1,6,1)}{1-(1,5,1)}{1-(0,4,0)}]
= $p_{3,1}p_{1,0}$ $p_{0,2}$/{1-($p_{1,6}$ $p_{6,1}$)}{1-($p_{1,5}p_{5,1}$)}{1-($p_{0,4}$ $p_{4,0}$)}

$V_{3,3}$=[(3,1,3)/{1-(1,0,1)/(1-(0,2,0))(1-(0,4,0))}(1-(0,2,0))(1-(0,4,0)){1-(1,5,1)}{1-(1,6,1)} +(3,9,3)+(3,10,3)+(3,11,3)
=$p_{3,1}p_{1,3}$/{((1-$p_{0,2}p_{2,0}$)(1-$p_{0,4}$ $p_{4,0}$)-$p_{1,0}p_{2,0}$)(1-$p_{1,5}$ $p_{5,1}$)(1-$p_{1,6}p_{6,1}$)}+{$p_{3,9}$ $p_{9,3}$}+{$p_{3,10}p_{10,3}$}+ {$p_{3,11}p_{11,3}$}

$V_{3,4}$=[(3,1,0,4)/{1-(1,6,1)}{1-(1,5,1)}{1-(0,2,0)}]
= $p_{3,1}$ $p_{1,0}$ $p_{0,4}$/{1-($p_{1,6}p_{6,1}$)}{1-($p_{1,5}p_{5,1}$)}{1-($p_{0,2}p_{2,0}$)}

$V_{3,5}$=[(3,1,5)/{1-(1,0,1)/{(1-(0,2,0))(1-(0,4,0))}(1-(0,2,0)}(1-(0,4,0)){1-(1,6,1)}
= $p_{3,1}$ $p_{1,5}$/{(1-$p_{0,2}p_{2,0}$)(1- $p_{0,4}$ $p_{4,0}$)- $p_{1,0}p_{0,1}$}{1- $p_{1,6}$ $p_{6,1}$}

$V_{3,6}$=[(3,1,6)/{1-(1,0,1)/{(1-(0,2,0))(1-(0,4,0))}(1-(0,2,0)}(1-(0,4,0)){1-(1,6,1)}
= $p_{3,1}$ $p_{1,6}$/{(1-$p_{0,2}p_{2,0}$)(1- $p_{0,4}$ $p_{4,0}$)- $p_{1,0}p_{0,1}$}{1- $p_{1,6}$ $p_{6,1}$}

$V_{3,7}$=[(3,1,0,2,7)/{1-(1,5,1)}{1-(1,6,1)}{1-(0,4,0)}]
= $p_{3,1}$ $p_{1,0}$ $p_{0,2}$ $p_{2,7}$ /{1-($p_{1,5}p_{5,1}$)}{1-($p_{1,6}p_{6,1}$)}{1-($p_{0,4}p_{4,0}$)}

$V_{3,8}$=[(3,1,0,2,8)/{1-(1,5,1)}{1-(1,6,1)}{1-(0,4,0)}]
= $p_{3,1}p_{1,0}$ $p_{0,2}$ $p_{2,8}$/{1-($p_{1,5}p_{5,1}$)}{1-($p_{1,6}p_{6,1}$)}{1-($p_{0,4}p_{4,0}$)}

$V_{3,9}$=[(3,9)]
= $p_{3,9}$

$V_{3,4}$=[(3,10)]
= $p_{3,10}$
$V_{3,4}$=[(3,11)]
= $p_{3,11}$

(A). MTSF (T_1): From Fig.1, the regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: i = 0,1,2,3. For 'ξ' = '0', MTSF is given by

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} i\right)\right\}.\mu_i}{\Pi_{k_1\neq\xi}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} \xi\right)\right\}}{\Pi_{k_2\neq\xi}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right]$$

MTSF=[(0,0) μ_0+(0,1) μ_1 +(0,2) μ_2+(0,1,3) μ_3] ÷ [1- {(0,1,0)+(0,2,0)}]
=[μ_0+$p_{0,1}$ μ_1+$p_{0,2}\mu_2$+ $p_{0,1}$ $p_{1,3}$ μ_3] ÷ [1- {($p_{0,1}$ $p_{1,0}$)+($p_{0,2}$ $p_{2,0}$)}]

(b). Availability of the system: From Fig.1, the regenerative states, at which the system is available are: j = 0,1,2,3 and the regenerative states are i = 0 to 11. For 'ξ' = '3', the total fraction of time for which the system remains available is given by

$$A_3 = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\} f_j.\mu_j}{\Pi_{k_1\neq\xi}\left\{1-V_{k_1,k_1}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}.\mu_i^1}{\Pi_{k_2\neq\xi}\left\{1-V_{k_2,k_2}\right\}}\right\}\right] = \left[\sum_j V_{\xi,j}.f_j.\mu_j\right] \div \left[\sum_i V_{\xi,i}.\mu_i^1\right]$$

A_3 = [$V_{3,0}f_0$ μ_0+ $V_{3,1}f_1$ μ_1+ $V_{3,2}f_2$ μ_2+ $V_{3,3}f_3\mu_3$] ÷ [$V_{3,0}$ μ_0^1+ $V_{3,1}$ μ_1^1+ $V_{3,2}$ μ_2^1+ $V_{3,3}$ μ_3^1
+ $V_{3,4}$ μ_4^1+ $V_{3,5}$ μ_5^1+ $V_{3,6}$ μ_6^1+ $V_{3,7}$ μ_7^1+ $V_{3,8}$ μ_8^1+ $V_{3,9}$ μ_9^1+ $V_{3,10}$ μ_{10}^1+ $V_{3,11}$ μ_{11}^1]

Where {f_j =1 for all j}, {μ_j^1= μ_j for all j}

5 Particular Case
Let us take;
$g_1(t)= \omega_1 e^{-\omega_1 t}$, $g_2(t)= \omega_2 e^{-\omega_2 t}$, $g_3(t)= \omega_3 e^{-\omega_3 t}$, $g_4(t)= \omega_4 e^{-\omega_4 t}$, $g_5(t)= \omega_5 e^{-\omega_5 t}$,
we have
$p_{0,1}= \lambda_1/\lambda_1+ \lambda_3+ \lambda_5$, $p_{0,2}= \lambda_3/\lambda_1+ \lambda_3+ \lambda_5$, $p_{0,2}= \lambda_5/\lambda_1+ \lambda_3+ \lambda_5$, $p_{1,0}=\omega_1/\omega_1+\lambda_2+ \lambda_3+ \lambda_5$, $p_{1,5}= \lambda_5/\omega_1+\lambda_2+ \lambda_3+ \lambda_5$, $p_{1,6}= \lambda_2/\omega_1+\lambda_2+ \lambda_3+ \lambda_5$, $p_{1,3}= \lambda_3/\omega_1+\lambda_2+ \lambda_3+ \lambda_5$, $p_{2,0}= \omega_3/\omega_3+ \lambda_4+ \lambda_5$, $p_{2,7}= \lambda_5/\omega_3+\lambda_4+\lambda_5$, $p_{2,8}= \lambda_4/\omega_3+ \lambda_4+ \lambda_5$, $p_{3,1}=\omega_3/\omega_3+\lambda_2+\lambda_4+ \lambda_5$, $p_{3,9}=\lambda_5/\omega_3+\lambda_2+\lambda_4+\lambda_5$, $p_{3,10}= \lambda_2/\omega_3+\lambda_2+ \lambda_4+ \lambda_5$, $p_{3,11}= \lambda_4/\omega_3+\lambda_2+ \lambda_4+ \lambda_5$, $p_{4,0}=1$, $p_{5,1}=1$, $p_{6,1}=1$, $p_{7,2}=1$, $p_{8,2}=1$, $p_{9,3}=1$, $p_{10,3}=1$, $p_{11,3}=1$
$\mu_0=1/\lambda_1+\lambda_3+\lambda_5$, $\mu_1= 1/\omega_1+\lambda_2+\lambda_3+ \lambda_5$, $\mu_2= 1/\omega_3+\lambda_4+ \lambda_5$, $\mu_3= 1/\omega_3+\lambda_2+\lambda_4+\lambda_5$, $\mu_4=1/\omega_5$, $\mu_5=1/\omega_5$, $\mu_6=1/\omega_2$, $\mu_7=1/\omega_5$, $\mu_8=1/\omega_4$, $\mu_9=1/\omega_5$, $\mu_{10}=1/\omega_2$, $\mu_{11}=1/\omega_4$

6. Analytical Discussion
The following tables, graphs and conclusions are obtained for:
$\lambda_1 = \lambda_2= \lambda_3 = \lambda_4 = \lambda_5 = \lambda$, $\omega_1= \omega_2= \omega_3 = \omega_4 = \omega_5= \omega$

By using these results, we get the following:
MTSF (T_3) = $(\omega+2\lambda)(\omega+4\lambda) + \lambda(\omega+3\lambda)/ 3\lambda(\omega+3\lambda)(\omega+2\lambda) - \omega\lambda(2\omega+5\lambda)$

Availability (A_3) =
$[\{\omega^3(\omega+3\lambda)3\lambda(\omega+2\lambda)^2+\omega^3\lambda(\omega+3\lambda)(3\lambda(\omega+2\lambda)-\omega\lambda)\}+\{(2\omega^2\lambda^2(\omega+3\lambda)+6\lambda^2(\omega+2\lambda)^3$ $(3\lambda(\omega+2\lambda)-\omega\lambda)+2\omega^2\lambda^2(\omega+2\lambda)^2(\omega+3\lambda)\}][(3\lambda(\omega+2\lambda)-\omega\lambda)2\lambda(\omega+3\lambda)-\omega\lambda(3\lambda)]/ 2(\omega+3\lambda)(\omega+2\lambda)^3\omega\lambda[3\lambda(\omega+2\lambda)- \omega\lambda]$

Busy period of the server (B_1) = $\omega_1(\omega_1+\lambda_2) + \lambda_1\lambda_2 / (\omega_1+\lambda_2)^2+ \lambda_1(\omega_1+\lambda_2)$
Expected no. of server's visits (V_1) = $\lambda_1\omega_1+ \lambda_1\lambda_2/ \lambda_1+ \omega_1+ \lambda(\omega_1+\lambda_1+ \lambda_2)$

MTSF vs. Repair Rate
The MTSF of the system is calculated for different values of the Failure Rate (λ) by taking λ=0.0005, 0.0006, 0.0007, 0.0008, 0.0009 and 0.0010 and for different values of the Repair Rate (ω) by taking ω=0.80,0.85,0.90 and 0.95. The data so obtd are shown in Table 6 and Graphically in fig.2

Table 6:

λ	T1 (ω=0.80)	T1 (ω=0.85)	T1 (ω=0.90)	T1 (ω=0.95)	T1 (ω=1.0)
0.0005	1996.2675	1996.4865	1996.6805	1996.8543	1997.0112
0.0006	1662.9380	1663.1562	1663.3502	1663.5235	1663.6803
0.0007	1424.8463	1425.0640	1425.2577	1425.4312	1425.5873
0.0008	1246.2785	1246.4959	1246.6892	1246.8623	1247.0183
0.0009	1107.3932	1107.6102	1107.8031	1107.9760	1108.1317
0.001	996.2855	996.5021	996.6948	996.8673	997.0228

Table 6 shows the behaviour of the MTSF (T_0) vs. the repair rate (ω) of the unit of the system for different values of the failure rate (λ).It is concluded that MTSF increases with increase in the values of the repair rate (ω) and decreases with increase in failure rate.

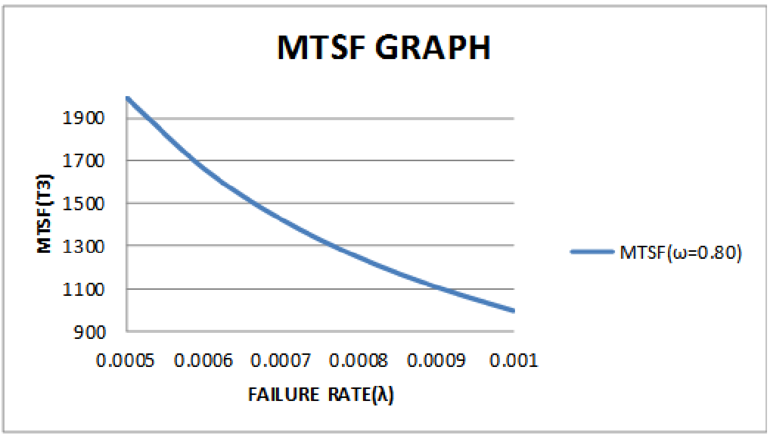


Fig 2:

Further it can be concluded from the fig.2 that the values of MTSF (T_1) shows the expected trend for the values of Repair Rate(ω=0.80), as T_3 decreases with the increase in the values of failure Rate(λ).

The Availability of the system is calculated for different values of the failure rate (λ) by taking λ =0.0005,0.0006,0.0007,0.0008, 0.0009 and 0.001 and for different values of the repair rate(ω) by taking ω =0.80,0.85,0.90,0.95 and 1.0. The data so obtained are shown in table 7 and graphically in fig.3 & fig. 4

(A) Availability (A_1) vs. the Repair Rate (ω) & failre rate:

Table 7:

Λ	A_1 (ω=0.80)	A_1 (ω=0.85)	A_1 (ω=0.90)	A_1 (ω=0.95)	A_1 (ω=1.00)
0.0005	0.99674413	0.99774286	0.99988663	0.99994416	0.99998169
0.0006	0.99541479	0.99664967	0.99867059	0.99974185	0.99987277
0.0007	0.99405233	0.99552674	0.99744428	0.99826671	0.99862425
0.0008	0.99280492	0.99441488	0.99622596	0.99741909	0.99845017
0.0009	0.99105714	0.99346715	0.99500686	0.99611756	0.99722727
0.001	0.98900973	0.99276522	0.99478854	0.99492242	0.99602319

Table shows the behaviour of the Availability (A_1) vs. the Repair rate (ω) of the unit of the system for different values of the failure rate (λ).It is concluded that Availability (A_3) increases with increase in the values of the Repair rate (ω) and decreases with the increase in failure rates.

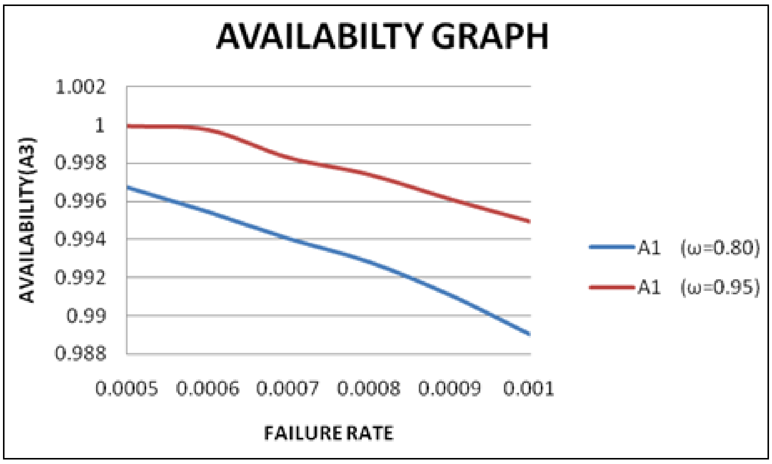


Fig 3:

Further it can be concluded from the fig.3 that values of availability (A_0) shows the expected trend for different values of failure rate(λ=0.0006 & 0.0009), as A_0 increases in the values of repair rate(ω).

5. Conclusion

From the Graphs and Tables, we see that as the Repair Rate (ω) increases, Availability of the system increases, which would be. The Regenerative-Point Graphical Technique is useful for the evaluation of the parameters in a simple way, without writing any state equation and without doing any lengthy and cumbersome calculations. This study can be extended to time dependent case. As it is easy for the management to control repair rates in comparison to failure rates, fixing the target of availability repair rates can be determined and managed by having efficient server.

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