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Double vertex max-fuzzy graph and complete double vertex max-fuzzy graph

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Abstract
In this paper, we have introduced the concept of double vertex max-fuzzy graph and complete double vertex max-fuzzy graph of a fuzzy graph which are analogous to the concepts double vertex graph and complete double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these operations. We have obtained the degrees of vertices in the double vertex max-fuzzy graph and the complete double vertex max-fuzzy graph of a fuzzy graph. Also we have provided some examples and illustrations.

Keywords: Effective Fuzzy Graph, Connected Fuzzy Graph, Complete Fuzzy Graph, Double Vertex Max-fuzzy graph of a Fuzzy Graph, Complete Double Vertex Max-fuzzy graph of a Fuzzy Graph.

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Later on, Bhattacharya [1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson.J.N. And Peng.C.S. [2]. the conjunction of two fuzzy graphs was defined by Nagoor Gani.A and Radha.K. [3]. we defined the direct sum [4], the strong product [6], the lexicographic products [8], maximal product [9] of two fuzzy graphs, and the double vertex graphs of a fuzzy graph [11] and studied the properties of these operations. In this paper, we have introduced the concept of double vertex max-fuzzy graph and complete double vertex max-fuzzy graph of a fuzzy graph which are analogous to the concept double vertex graph and complete double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these operations. We have obtained the degrees of vertices in the double vertex max-fuzzy graph and the complete double vertex max-fuzzy graph of a fuzzy graph. Also we have provided some examples and illustrations.

First let us recall some preliminary definitions that can be found in [1, 13].

A fuzzy graph G is a pair of functions (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G :(σ, μ) is denoted by G* :(V, E) where $E \subseteq V\times V$. A fuzzy graph G :(σ, μ) with underlying crisp graph G* :(V, E) is called a connected fuzzy graph if for all u, v∈V there exists at least one non-zero path between u and v.

A fuzzy graph G:(σ, μ) is called an effective fuzzy graph if $\mu(u\,v) = \sigma(u) \wedge \sigma(v)$ for all uv∈E and a complete fuzzy graph if $\mu(u\,v) = \sigma(u) \wedge \sigma(v)$ for all u, v∈V. Therefore G is complete if and only if G is effective and G* is complete.

The degree and total degree of a vertex u of a fuzzy graph G:(σ, μ) with underlying crisp graph G*:(V,E) are defined by $d_G(u) = \sum_{uv\in E} \mu(uv)$ and $td_G(u) = \sum_{uv\in E} \mu(uv) + \sigma(u)$ respectively.

Let G = (V, E) be a graph with order n ≥ 2. The double vertex graph, denoted by U_2 (G), is the graph whose vertex set consists of all nC_2 unordered pairs of V such that two vertices {x, y} and {u, v} are adjacent if and only if $|\{x, y\}\cap\{u, v\}| = 1$ and if x = u, then y and v are adjacent in G.

Let G = (V, E) be a graph with order n ≥ 2. The complete double vertex graph, denoted by $CU_2(G)$, is the graph whose vertex set consists of all $(n+1)C_2$ unordered pairs of V, that is, it contains all the vertices of $U_2(G)$ and all 2-element multisets of the form {a, a}, such that two vertices {x, y} and {u, v} are adjacent if and only if $|\{x, y\}\cap\{u, v\}| = 1$ and if x = u, then y and v are adjacent in G.

Notation

In a fuzzy graph G :(σ,μ) with underlying crisp graph G*:(V, E), the relation σ ≥ μ means that σ(u) ≥ μ (uv) for every u∈V and for every uv∈E.

2. Double Vertex Max-Fuzzy Graph of a Fuzzy Graph

2.1. Definition

Let G :(σ,μ) be a fuzzy graph with underlying crisp graph G*:(V, E) of order n ≥ 2. Define $\check{D}(G):(\sigma_{d\vee}, \mu_{d\vee})$ on $U_2(G^*):(V_d, E_d)$ whose vertex set V_d consists of all nC_2 unordered pairs of V such that two vertices {x,y} and {u,v} are adjacent if and only if $|\{x,y\}\cap\{u,v\}|=1$ and if x = u then y and v are adjacent in G such that,

$\sigma_{d\vee}(\{u_i, u_j\}) = \sigma(u_i) \vee \sigma(u_j)$, for all $\{u_i, u_j\} \in V_{d\vee}$ and $\mu_{d\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma(u_1) \vee \mu(u_2u_3),\ \forall \{u_1, u_2\}\{u_1, u_3\} \in E_d$

Now $\sigma(u_1) \vee \mu(u_2 u_3) \leq \sigma(u_1) \vee [\sigma(u_2) \wedge \sigma(u_3)]$
$= [\sigma(u_1) \vee \sigma(u_2)] \wedge [\sigma(u_1) \vee \sigma(u_3)]$
$= \sigma_{d\vee}(u_1, u_2) \wedge \sigma_{d\vee}(u_1, u_3)$.

Hence $\mu_{d\vee}(\{u_1, u_2\}\{u_1, u_3\}) \leq \sigma_{d\vee}(\{u_1, u_2\}) \wedge \sigma_{d\vee}(\{u_1, u_3\})$. Therefore $\check{D}(G) :(\sigma d_{\vee}, \mu_{d\vee})$ is a fuzzy graph. This is called the double vertex max-fuzzy graph of the fuzzy graph G.

2.2. Remark

If $G:(\sigma,\mu)$ has 'n' non zero vertices and 'm' non zero edges, then the double vertex max-fuzzy graph $\check{D}(G):(\sigma_{d\vee}, \mu_{d\vee})$ has $n(n-1)/2$ non zero vertices and $m(n-2)$ non zero edges.

2.3. Example

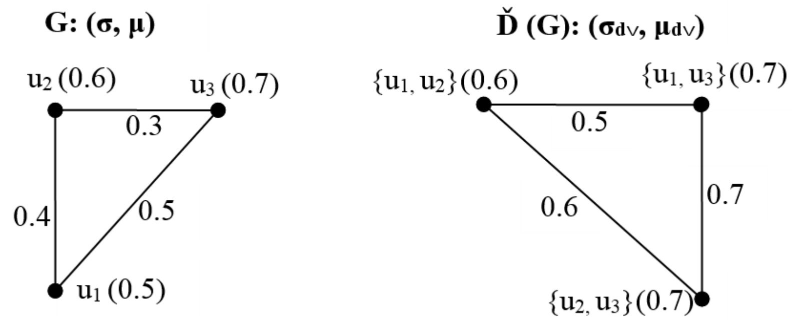

Fig 1

2.4. Theorem

The double vertex max-fuzzy graph of an effective fuzzy graph is an effective fuzzy graph.

Proof

Let $G :(\sigma,\mu)$ be an effective fuzzy graph. Then $\mu(u_1u_2) = \sigma(u_1) \wedge \sigma(u_2)$ for all $u_1u_2 \in E$.
Let $\check{D}(G) :(\sigma d_{\vee}, \mu_{d\vee})$ be the double vertex max-fuzzy graph of G defied on $U_2(G^*) :(V_d, E_d)$. Then proceeding as in the definition, $\sigma_{d\vee}(\{u_i,u_j\})=\sigma(u_i)\vee\sigma(u_j)$, for all $\{u_i,u_j\} \in V_d$ and

$$\mu_{d\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma(u_1) \vee \mu(u_2u_3),\ \forall \{u_1, u_2\}\{u_1, u_3\} \in E_d$$

Now $\sigma(u_1) \vee \mu(u_2 u_3) = \sigma(u_1) \vee [\sigma(u_2) \wedge \sigma(u_3)] = [\sigma(u_1) \vee \sigma(u_2)] \wedge [\sigma(u_1) \vee \sigma(u_3)]$
$= \sigma_{d\vee}(\{u_1, u_2\}) \wedge \sigma_{d\vee}(\{u_1, u_3\})$.

Hence $\mu_{d\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma_{d\vee}(\{u_1, u_2\}) \wedge \sigma_{d\vee}(\{u_1, u_3\})$. This is true for all $\{u_1, u_2\}\{u_1,u_3\} \in E_d$ and therefore $\check{D}(G):(\sigma_{d\vee}, \mu_{d\vee})$ is an effective fuzzy graph.

2.5. Example

The following Figure-2 illustrates the theorem2.3.

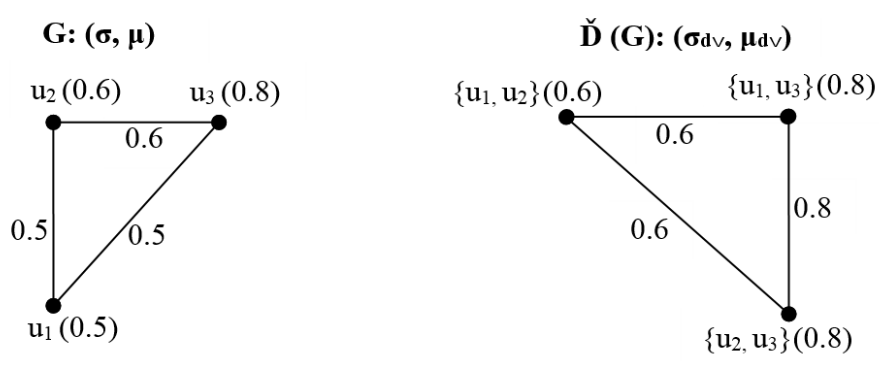

Fig 2.

2.6. Theorem

The double vertex max-fuzzy graph of a connected fuzzy graph is a connected fuzzy graph.

Proof
Let G: (σ,μ) be a connected fuzzy graph. Then $\mu^{\infty}(u_1u_2) > 0$ for all $u_1u_2 \in E$.
Let Ď (G): ($\sigma_{d\vee}$, $\mu_{d\vee}$) be the double vertex max-fuzzy graph of G defined on U_2 (G*): (V_d, E_d).
Consider any two vertices {u, v} and {u, w} in Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$).
Since G is connected, there is a path between v and w, say $v = v_0 v_1 \ldots\ldots v_m = w$.
Then {u, v} {u, v_1} {u, v_2}……. {U, w} is a path in Ď (G) and hence $\mu_{d\vee}^{\infty}(\{u, v\}\{u, w\}) > 0$ for all $\{u, v\}\{u, w\} \in E_d$. Hence Ď (G) is connected.

2.7. Example
The following Figure-2 illustrates the theorem2.5.

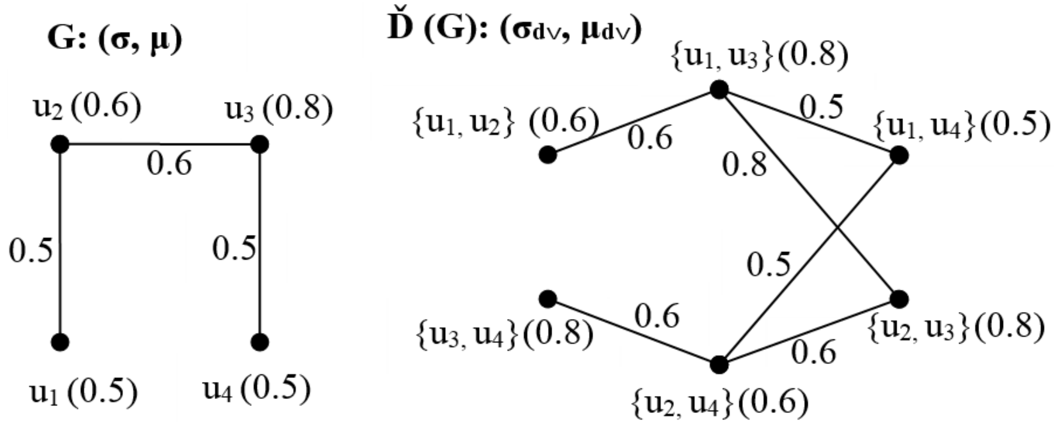

Fig 3.

2.8. Remark
The double vertex max-fuzzy graph of a complete fuzzy graph need not be a complete fuzzy graph. Consider the following fuzzy graph G:(σ,μ) and its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) where G:(σ,μ) is a complete fuzzy graph and Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) is not a complete one.

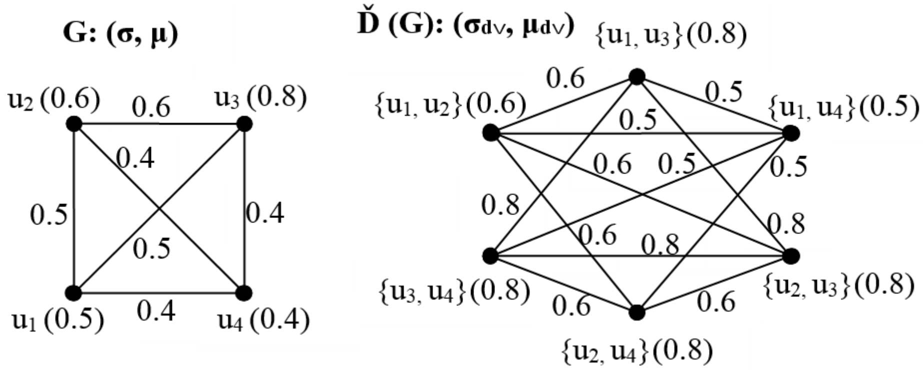

Fig 4.

2.9. Remark
If G:(σ,μ) is a complete fuzzy graph with two vertices then double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) is a fuzzy graph with only one vertex.
If G:(σ,μ) is a complete fuzzy graph with three vertices then double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) is also a complete fuzzy graph with the same number of vertices and edges.
Since every complete fuzzy graph is an effective fuzzy graph, from theorem2.4, the double vertex max-fuzzy graph of a complete fuzzy graph is an effective fuzzy graph.

3. Degree Of A Vertex In The Double Vertex Max-Fuzzy Graph

3.1. Theorem
If G:(σ,μ) is a fuzzy graph such that σ ≥ μ, then the degree of a vertex in the double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) is given by,

$$d_{\check{D}(G)}(\{u_i, u_j\}) = \begin{cases} \left[d_{G^*}(u_j) - 1\right]\sigma(u_i) + \left[d_{G^*}(u_i) - 1\right]\sigma(u_j) & \text{if } u_iu_j \in E \\ d_{G^*}(u_j)\,\sigma(u_i) + d_{G^*}(u_i)\sigma(u_j), & \text{if } u_iu_j \notin E \end{cases}$$

Proof:
Let G:(σ,μ) be a fuzzy graph such that σ ≥ μ. This implies that σ ∨ μ = σ. Then the degree of any vertex {u, v} ∈ V_d is given by,

$$d_{\check{D}(G)}(\{u_i,u_j\}) = \sum_{\{u_i,u_j\}\{u_k,u_\ell\}\in E_d} \mu_{d\vee}(\{u_i,u_j\}\{u_k,u_\ell\})$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i)\vee\mu(u_ju_\ell) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)\vee\mu(u_iu_k)$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j) \ldots\ldots\ldots\ldots \quad (3.1)$$

By definition, when $u_i = u_j$, $u_l \neq u_i$ and when $u_j = u_l$, $u_k \neq u_j$. Therefore when $u_iu_j \in E$, (3.1) becomes,

$$d_{\check{D}(G)}(\{u_i,u_j\}) = \left[d_{G^*}(u_j)-1\right]\sigma(u_i) + \left[d_{G^*}(u_i)-1\right]\sigma(u_j)$$

When $u_iu_j \notin E$, (3.1) becomes, $d_{\check{D}(G)}(\{u_i,u_j\}) = d_{G^*}(u_j)\sigma(u_i) + d_{G^*}(u_i)\sigma(u_j)$

3.2. Example
Consider the following fuzzy graph G:(σ,μ) in which σ ≥ μ and its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$).

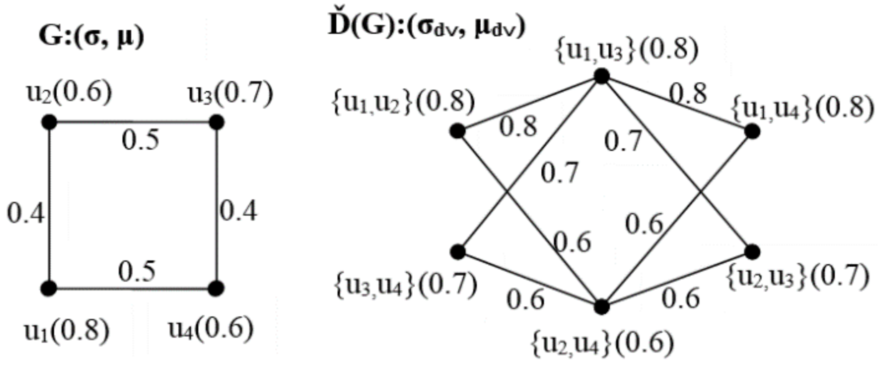

Fig 5.

Now,

$$d_{G^*}(u_2)\sigma(u_1) + d_{G^*}(u_1)\sigma(u_2) - (\sigma(u_1)+\sigma(u_2)) = 2(0.8)+2(0.6)-(0.8+0.6) = 1.4 = d_{\check{D}(G)}(\{u_1,u_2\})$$

$$d_{G^*}(u_3)\sigma(u_2) + d_{G^*}(u_2)\sigma(u_3) - (\sigma(u_2)+\sigma(u_3)) = 2(0.6)+2(0.7)-(0.6+0.7) = 1.3 = d_{\check{D}(G)}(\{u_2,u_3\})$$

$$d_{G^*}(u_3)\sigma(u_1) + d_{G^*}(u_1)\sigma(u_3) = 2(0.8)+2(0.7) = 3.0 = d_{\check{D}(G)}(\{u_1,u_3\})$$

3.3. Remark
From the above example 3.3, it is clear that if a fuzzy graph G:(σ,μ) is regular then its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) need not be regular.

3.4. Remark
The total degree of any vertex in the double vertex max-fuzzy graph of a fuzzy graph G:(σ,μ) is given by,

$$td_{\check{D}(G)}(\{u_i,u_j\}) = \sum_{\{u_i,u_j\}\{u_k,u_\ell\}\in E_d} \mu_{d\vee}(\{u_i,u_j\}\{u_k,u_\ell\}) + \sigma_{d\vee}(\{u_i,u_j\})$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i)\vee\mu(u_ju_\ell) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)\vee\mu(u_iu_k) + (\sigma(u_i)\vee\sigma(u_j))$$

3.5. Theorem
If G:(σ,μ) be a fuzzy graph such that σ ≥ μ, then the total degree of a vertex in the double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) is given by,

$$td_{\check{D}(G)}(\{u_i,u_j\}) = d_{G^*}(u_j)\sigma(u_i) + d_{G^*}(u_i)\sigma(u_j) - (\sigma(u_i)\wedge\sigma(u_j))$$

Proof

Let G:(σ,μ) be a fuzzy graph such that $\sigma \geq \mu$. This implies that $\sigma \vee \mu = \sigma$. Then the degree of any vertex $\{u, v\} \in V_d$ is given by,

$$td_{\check{D}(G)}(\{u_i,u_j\}) = \sum_{\{u_i,u_j\}\{u_k,u_\ell\}\in E_d} \mu_{d\vee}(\{u_i,u_j\}\{u_k,u_\ell\}) + \sigma_{d\vee}(\{u_i,u_j\})$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i)\vee\mu(u_ju_\ell) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)\vee\mu(u_iu_k) + \sigma_{d\vee}(\{u_i,u_j\})$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j) + \sigma_{d\vee}(\{u_i,u_j\})$$

$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i) + \sigma(u_i) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j) + \sigma(u_j) - (\sigma(u_i)+\sigma(u_j)) + \sigma_{d\vee}(\{u_i,u_j\})$$

$$= d_{G*}(u_j)\sigma(u_i) + d_{G*}(u_i)\sigma(u_j) - (\sigma(u_i)+\sigma(u_j)) + (\sigma(u_i)\vee\sigma(u_j))$$

$$= d_{G*}(u_j)\sigma(u_i) + d_{G*}(u_i)\sigma(u_j) - [(\sigma(u_i)+\sigma(u_j)) - (\sigma(u_i)\vee\sigma(u_j))]$$

$$= d_{G*}(u_j)\sigma(u_i) + d_{G*}(u_i)\sigma(u_j) - (\sigma(u_i)\wedge\sigma(u_j))$$

3.6. Remark

In the above theorem 3.5 if $u_iu_j \notin E$ then the total degree of the vertex $\{u_i, u_j\}$ in the double vertex max-fuzzy graph **Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$)** is given by,

$$td_{\check{D}(G)}(\{u_i,u_j\}) = d_{G*}(u_j)\sigma(u_i) + d_{G*}(u_i)\sigma(u_j) + (\sigma(u_i)\vee\sigma(u_j))$$

3.7. Example

Consider the fuzzy graph G:(σ,μ) in which $\sigma \geq \mu$ and its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) given in example 3.2. Now,

$$td_{\check{D}(G)}(\{u_1,u_2\}) = 2(0.8)+2(0.6)-(0.8+0.6)+(0.8\vee 0.6) = 2.2$$

$$td_{\check{D}(G)}(\{u_2,u_3\}) = 2(0.6)+2(0.7)-(0.6+0.7)+(0.6\vee 0.7) = 2.0$$

$$td_{\check{D}(G)}(\{u_1,u_3\}) = d_{G*}(u_3)\sigma(u_1) + d_{G*}(u_1)\sigma(u_3) + (\sigma(u_1)\vee\sigma(u_3)) = 2(0.8)+2(0.7)+(0.8\vee 0.7) = 3.8$$

3.8. Remark

If a fuzzy graph G:(σ,μ) is totally regular then its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) need not be totally regular. Consider the following fuzzy graph G:(σ,μ) which is totally regular and its double vertex max-fuzzy graph Ď(G):($\sigma_{d\vee}$, $\mu_{d\vee}$) which is not totally regular.

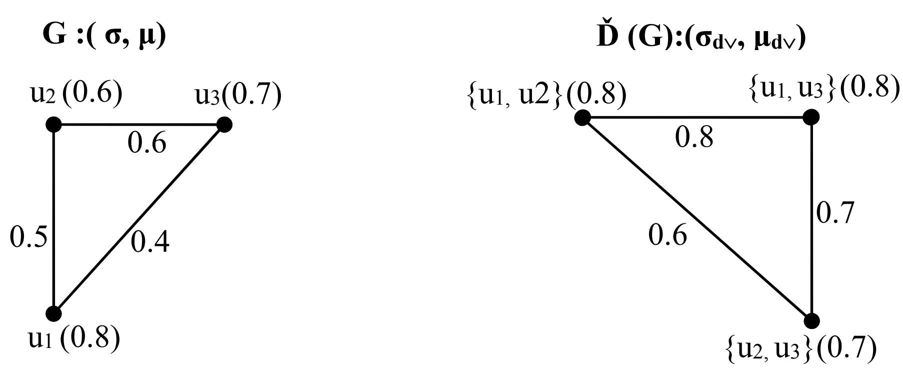

Fig 6

4. Complete Double Vertex Max-Fuzzy Graph Of A Fuzzy Graph

4.1. Definition

Let G:(σ,μ) be a fuzzy graph with underlying crisp graph G*:(V, E) of order $n \geq 2$. Define CĎ(G):($\sigma_{cd\vee}$, $\mu_{cd\vee}$) on $CU_2(G^*)$:(V_{cd}, E_{cd}) whose vertex set V_{cd} consists of all $(n+1)C_2$ unordered pairs of vertices in V(duplicates allowed). That is, it contains all the vertices of U2(G) and all 2-element multi-sets of the form {a, a}. Again two vertices {x,y} and {u,v} are adjacent if and only if $|\{x,y\}\cap\{u,v\}|=1$ and if x = u then y and v are adjacent in G such that,

$\sigma_{cd\vee}(\{u_i, u_j\}) = \sigma(u_i) \vee \sigma(u_j)$, for all $\{u_i, u_j\} \in V_{cd}$ and

$$\mu_{cd\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma(u_1) \vee \mu(u_2u_3), \ \forall \{u_1, u_2\}\{u_1, u_3\} \in E_{cd}$$

Now $\sigma(u_1) \vee \mu(u_2 u_3) \le \sigma(u_1) \vee [\sigma(u_2) \wedge \sigma(u_3)]$
$= [\sigma(u_1) \vee \sigma(u_2)] \wedge [\sigma(u_1) \vee \sigma(u_3)]$
$= \sigma_{cd\vee}(\{u_1, u_2\}) \wedge \sigma_{cd\vee}(\{u_1, u_3\})$.

Hence $\mu_{cd\vee}(\{u_1, u_2\}\{u_1, u_3\}) \le \sigma_{cd\vee}(\{u_1, u_2\}) \wedge \sigma_{cd\vee}(\{u_1, u_3\})$. Therefore $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ is a fuzzy graph. This is called the complete double vertex max-fuzzy graph of G.

4.2. Remark

If $G:(\sigma,\mu)$ has 'n' non zero vertices and 'm' non zero edges, then the complete double vertex max-fuzzy graph $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ has $n(n + 1)/2$ non zero vertices and 'mn' non zero edges.

4.3. Example

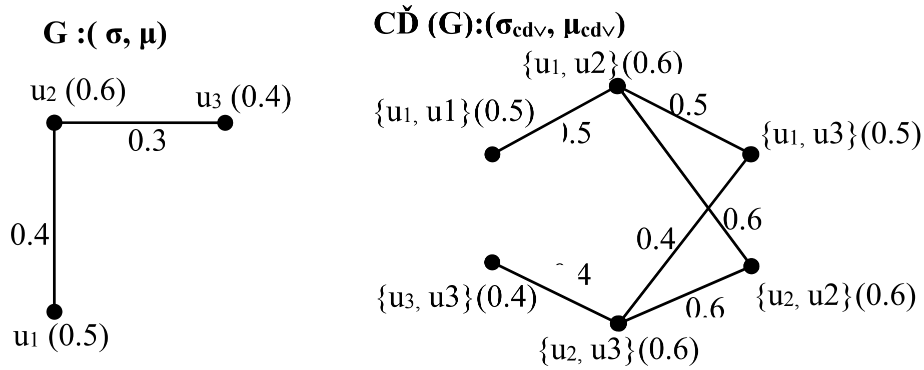

Fig 7.

4.4. Theorem

The complete double vertex max-fuzzy graph of an effective fuzzy graph is an effective fuzzy graph.

Proof

Let $G:(\sigma,\mu)$ be an effective fuzzy graph. Then $\mu(u_1u_2) = \sigma(u_1) \wedge \sigma(u_2)$ for all $u_1u_2 \in E$.
Let $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ be the complete double vertex max-fuzzy graph of G defied on $CU_2(G^*):(V_{cd}, E_{cd})$. Then proceeding as in the definition, $\sigma_{cd\vee}(\{u_i,u_j\})=\sigma(u_i)\vee\sigma(u_j)$, for all $\{u_i,u_j\} \in V_{cd}$ and

$$\mu_{cd\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma(u_1) \vee \mu(u_2u_3), \ \forall \{u_1, u_2\}\{u_1, u_3\} \in E_{cd}$$

Now $\sigma(u_1) \vee \mu(u_2 u_3) = \sigma(u_1) \vee [\sigma(u_2) \wedge \sigma(u_3)] = [\sigma(u_1) \vee \sigma(u_2)] \wedge [\sigma(u_1) \vee \sigma(u_3)]$
$= \sigma_{cd\vee}(\{u_1, u_2\}) \wedge \sigma_{cd\vee}(\{u_1, u_3\})$.
Hence $\mu_{cd\vee}(\{u_1, u_2\}\{u_1, u_3\}) = \sigma_{cd\vee}(\{u_1, u_2\}) \wedge \sigma_{cd\vee}(\{u_1, u_3\})$. This is true for all $\{u_1,u_2\}\{u_1,u_3\} \in E_{cd}$ and therefore $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ is an effective fuzzy graph.

4.5. Example

The following Figure-8 illustrates the theorem4.4.

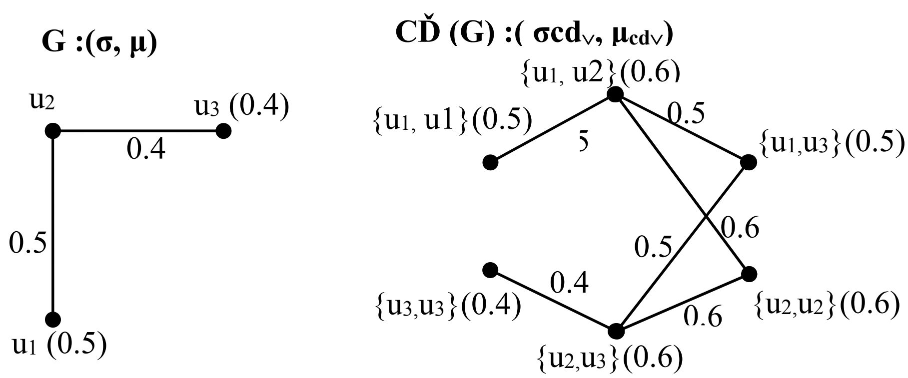

Fig 8.

4.6. Remark
The complete double vertex max-fuzzy graph of a complete fuzzy graph need not be a complete fuzzy graph. Consider the following fuzzy graph G:(σ,μ) and its complete double vertex max-fuzzy graph $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ where G:(σ,μ) is a complete fuzzy graph and $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ is not a complete fuzzy graph.

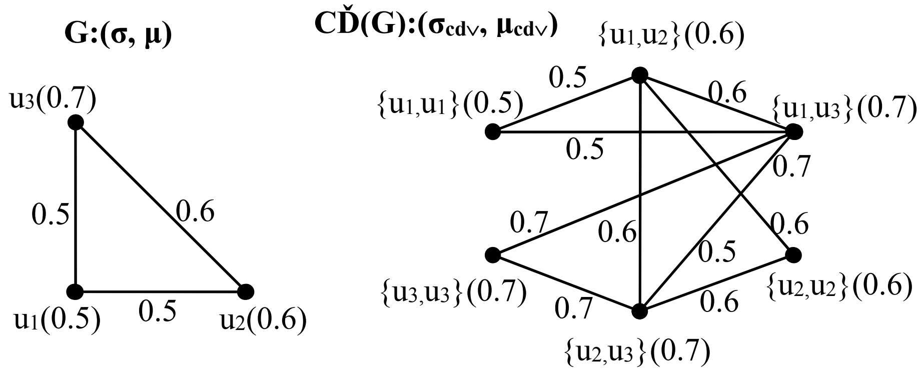

Fig 9.

Since every complete fuzzy graph is an effective fuzzy graph, from theorem4.4, the complete double vertex max-fuzzy graph of a complete fuzzy graph is an effective fuzzy graph.

4.7. Theorem
The complete double vertex max-fuzzy graph of a connected fuzzy graph is a connected fuzzy graph.
Proof: The proof is similar to that of 2.6.

5. Degree Of A Vertex In The Complete Double Vertex Max-Fuzzy Graph Of A Fuzzy Graph
The degree of any vertex in the complete double vertex max-fuzzy graph $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ of a fuzzy graph G:(σ, μ) is given by,
Case (i) if i≠j,

$$d_{C\check{D}(G)}(\{u_i,u_j\}) = \sum_{\{u_i,u_j\}\{u_k,u_\ell\}\in E_{cd}} \mu_{cd\vee}(\{u_i,u_j\}\{u_k,u_\ell\})$$
$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i)\vee\mu(u_ju_\ell) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)\vee\mu(u_iu_k)$$

Case (ii) if i=j,

$$d_{C\check{D}(G)}(\{u_i,u_i\}) = \sum_{\{u_i,u_i\}\{u_i,u_\ell\}\in E_{cd}} \mu_{cd\vee}(\{u_i,u_i\}\{u_i,u_\ell\})$$
$$= \sum_{u_iu_\ell\in E} \sigma(u_i)\vee\mu(u_iu_\ell)$$

5.1. Theorem
If G:(σ,μ) be a fuzzy graph such that σ ≥ μ, then the degree of a vertex in the complete double vertex max-fuzzy graph $C\check{D}(G):(\sigma_{cd\vee}, \mu_{cd\vee})$ is given by,

$$d_{C\check{D}(G)}(\{u_i,u_j\}) = \begin{cases} d_{G^*}(u_j)\sigma(u_i) + d_{G^*}(u_i)\sigma(u_j), & \text{if } i\neq j \\ d_{G^*}(u_i)\,\sigma(u_i), & \text{if } i=j \end{cases}$$

Proof: Let G:(σ,μ) be a fuzzy graph such that σ ≥ μ. This implies that σ ∨ μ = σ. Then to find the degree of any vertex $\{u_i, u_j\} \in V_{cd,}$ we have the following two cases.
Case (i) if i≠j,

$$d_{C\check{D}(G)}(\{u_i,u_j\}) = \sum_{\{u_i,u_j\}\{u_k,u_\ell\}\in E_{cd}} \mu_{cd}(\{u_i,u_j\}\{u_k,u_\ell\})$$
$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i)\vee\mu(u_ju_\ell) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)\vee\mu(u_iu_k)$$
$$= \sum_{u_i=u_k \text{ and } u_ju_\ell\in E} \sigma(u_i) + \sum_{u_iu_k\in E \text{ and } u_j=u_\ell} \sigma(u_j)$$
$$= d_{G^*}(u_j)\sigma(u_i) + d_{G^*}(u_i)\sigma(u_j)$$

Case (ii) if i=j,

$$d_{C\check{D}(G)}(\{u_i,u_i\}) = \sum_{\{u_i,u_i\}\{u_i,u_\ell\}\in E_{cd}} \mu_{cd\vee}(\{u_i,u_i\}\{u_i,u_\ell\})$$
$$= \sum_{u_i u_\ell \in E} \sigma(u_i)\vee\mu(u_i u_\ell)$$
$$= \sum_{u_i u_\ell \in E} \sigma(u_i)$$
$$= d_{G^*}(u_i)\sigma(u_i)$$

5.2. Example

Consider the fuzzy graph G:(σ,μ) and its complete double vertex max-fuzzy graph CĎ(G):($\sigma_{cd\vee}$, $\mu_{cd\vee}$) given below.

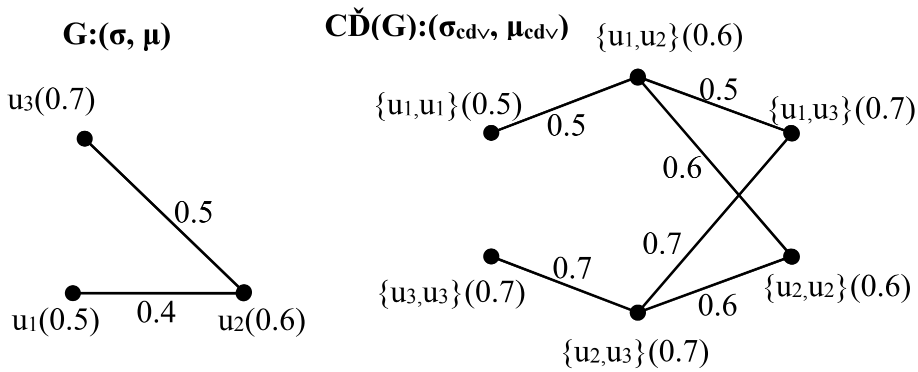

Fig 10.

$$d_{G^*}(u_2)\sigma(u_1) + d_{G^*}(u_1)\sigma(u_2) = 2(0.5) + 1(0.6) = 1.6 = d_{C\check{D}(G)}(\{u_1,u_2\})$$
$$d_{G^*}(u_3)\sigma(u_1) + d_{G^*}(u_1)\sigma(u_3) = 1(0.5) + 1(0.7) = 1.2 = d_{C\check{D}(G)}(\{u_1,u_3\})$$
$$d_{G^*}(u_2)\sigma(u_2) = 2(0.6) = 1.2 = d_{C\check{D}(G)}(\{u_2,u_2\})$$
$$d_{G^*}(u_3)\sigma(u_3) = 1(0.7) = 0.7 = d_{C\check{D}(G)}(\{u_3,u_3\})$$

5.3. Remark

If a fuzzy graph G:(σ,μ) is regular then its complete double vertex max-fuzzy graph CĎ(G):($\sigma_{cd\vee}$, $\mu_{cd\vee}$) need not be regular.

5.4. Remark

Consider the fuzzy graph G:(σ,μ). Then, from the definitions of the double vertex graph D(G):(σ_d, μ_d) of G, the complete double vertex graph CD(G):(σ_{cd},μ_{cd}) of G, the double vertex max-fuzzy graph Ď(G):(σ_{cd}, μ_{cd}) and the complete double vertex max-fuzzy graph CĎ(G):($\sigma_{cd\vee}$, $\mu_{cd\vee}$), it is clear that $\sigma_d \subseteq \sigma_{cd} \subseteq \sigma_{cd\vee}$ and $\mu_d \subseteq \mu_{cd} \subseteq \mu_{cd\vee}$. Hence D(G):($\sigma_d$, μ_d), CD(G):(σ_{cd},μ_{cd}) and Ď(G):(σ_{cd}, μ_{cd}) are fuzzy sub graphs of CĎ(G):($\sigma_{cd\vee}$, $\mu_{cd\vee}$).

6. Conclusion

In this paper, we have introduced the concept of double vertex max-fuzzy graph and complete double vertex max-fuzzy graph of a fuzzy graph which are analogous to the concept double vertex graph and complete double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these operations. We have obtained the degrees of vertices in the double vertex max-fuzzy graph and the complete double vertex max-fuzzy graph of a fuzzy graph. Also we have provided some examples and illustrations. In addition to the existing operations these operations and properties will also be helpful to study large fuzzy graph as a combination of small fuzzy graphs and to derive its properties from those of the small ones.

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