International Journal of Multidisciplinary Research and Development
Online ISSN: 2349-4182 Print ISSN: 2349-5979
www.allsubjectjournal.com
Volume 2; Issue 12; December 2015; Page No. 364-371

Estimation of expected time of personnel to leave the organisation – A model with two thresholds

[1] N Vijayasankar, [2] S Vijay Amirtharaj, [3] R Sathiyamoorthi

[1] Assistant Professor, Department of Statistics, Annamalai University, Annamalainagar, Tamilnadu, India.

[2] Research Scholar, Department of Statistics, Annamalai University, Annamalainagar, Tamilnadu, India.

[3] Professor and Head (Rtd.), Department of Statistics, Annamalai University, Annamalainagar, Tamilnadu, India.

Abstract
In any organisation or industry the manpower is an important factor of production. The exit of personnel due to unsatisfactory decisions or packages by the management is very much possible. The exit of an individual from the organisation may occur due to any one of the two reasons or both. The propensity to leave is of cumulative character at the successive decision epochs. As and when the total propensity to leave crosses a level called random threshold level the person leaves the organisation. Also if the intensity of emotional feeling due to unsatisfactory decisions crosses the maximum threshold level the person quits the organisation. Under the assumption that the inter arrival times between decision epochs are correlated exchangeable, exponential random variables the expected time of a person to leave the organisation is estimated. Numerical illustration is also given.

Keywords: propensity to leave, emotional threshold and correlated inter arrival times.

Introduction

The use of manpower planning is very much essential in many organization or industry. Without proper level or magnitude of manpower availability and also the quality of manpower available, the targets cannot be achieved. For this reason only the Human Resource Management is considered to be a vital branch of Management Science. The planning of the organization for future expansion and targets is very much based on the manpower planning.

In any organization or industrial unit frequent decisions are taken regarding promotion, pay revision and work schedule. On such occasions the attrition or leaving of personnel cannot be avoided because the decision taken by the management may affect the individuals in their propensity to leave the organization as well as the emotional feelings. So the exit of personnel takes place on such decision occasions. In the previous chapter a stochastic model to estimate the expected time for a person to leave the organization, either due to the cumulative propensity to leave crosses the threshold level or the emotion crosses the emotional threshold. It is assumed that the inter arrival times between the decision epochs are i.i.d random variables. In this chapter it is assumed that the inter arrival times between decision epochs are random variables, which are exponentially distributed constantly correlated and exchangeable random variables.

The concept of exchangeable, exponential distributed and constantly correlated random variables has been discussed by Gurland (1955) [1]. The expected time to leave the organization and its variance are derived, using the concept of shock model and cumulative damage process which has been discussed by Esary, Proschan and Marshall (1973) [2]. The concept of constantly correlated random variables for inter arrival times in Shock models has been discussed by Sathiyamoorthi (1979) [3]. The concept attrition especially in software organisations and causes for the same has been discussed in detail by Gurumoorthy (2010) [4].

Assumptions

* Decisions regarding work schedules, pay revisions, promotions are taken by the management at random time points.
* At each decision epoch there is some random amount of contribution to the propensity of an individual to leave the organisation and it is of cumulative nature.
* When the total amount of cumulative propensity crosses a particular level called the threshold the person leaves the organisation.
* At each decision epoch there is an upsurge of emotional feeling of the individual worker. When its level is more than the threshold level of emotion at any decision epoch the person leaves the organisation.
* The inter arrival times between decision epochs are constantly correlated, exponentially distributed exchangeable random variables.

Notations

X_i: The random amount of propensity to leave on the i^{th} decision epoch,
i = 1,2,3,..…,k with probability density function q(.) with cumulative distribution function Q(.).
Y_i: The random amount of emotional feeling on the i^{th} decision epoch,
i = 1,2,3,……,k with probability density function p(.) and cumulative distribution function P(.).

Z_1: The threshold level of propensity to leave, a random variable with probability density function m(.) and cumulative distribution function M(.).
Z_2: The threshold level of emotion feeling, a random variable with probability density function $n(.)$ and cumulative distribution function N(.).
U_i: The inter arrival times between successive decision epochs, which are i.i.d with probability density function f(.) and cumulative distribution function F (.).
F_k(.): Cumulative distribution function of the partial sum of $X_1+X_2+X_3+\ldots+X_k$.
R: Correlation between any Ui, Uj, $i \neq j$.
T: Time to leave the organization.
L(t): 1 – S(t) where S(t) = P(T > t).
b: a(1-R).
***l**(s):** Laplace transform of *l*(t).

Results
The survivor function is defined as S(t) = P[T > t] = P [The cumulative level of propensity to leave the organisation due 'k' decisions at 'k' epochs not greater than the threshold level and maximum level of emotion does not cross the corresponding threshold].
Therefore
S(t) = Pr [that there exactly 'k' decision & epochs in (0, t)]

$$S(t) = P\left[\sum_{i=1}^{k} x_i < z_1 \cap \max(y_1, y_2, \ldots y_k) < z_2\right]$$
$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \times P\left[\sum_{i=1}^{k} x_i < z_1\right] P\left[\sum_{i=1}^{k} \max(y_1, y_2, \ldots y_k) < z_2\right]$$
$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \times P\left[\sum_{i=1}^{k} x_i < z_1\right] \bigcap_{i=1}^{k} P[y_i < z_2]$$

Hence the expression for S(t) is given by
$$S(t) = \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \left[\int_0^{\infty} q_k(x)\overline{M(x)}\, dx\right]\left[\int_0^{\infty} p_k(y)\overline{N(y)}\, dy\right] \ldots\ldots (4.1)$$
let us assume that $Z_1 \sim \exp(\theta)$ *and hence* $M(x) = 1 - e^{-\theta x}, \overline{M(x)} = e^{-\theta x}$
$Z_2 \sim \exp(\alpha)$ *and so* $\overline{N(y)} = e^{-\lambda y}$
$$\therefore\ S(t) = \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \left[\int_0^{\infty} q_k(x)e^{-\theta x}\, dx\right]\left[\int_0^{\infty} p_k(y)e^{-\lambda y}\, dy\right] \ldots\ldots (4.2)$$
$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\ [q_k{}^*(\theta)p_k{}^*(\lambda)]$$
$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\ [q^*(\theta)p^*(\lambda)]^k$$
$$= [F_0(t) - F_1(t)](1) + [F_1(t) - F_2(t)][q^*(\theta)p^*(\lambda)] + [F_1(t) - F_2(t)][q^*(\theta)p^*(\lambda)]^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$= 1 - [1 - q^*(\theta)p^*(\lambda)]F_1(t) + [1 - q^*(\theta)p^*(\lambda)]F_2(t)q^*(\theta)p^*(\lambda)$$
$$= 1 - [1 - q^*(\theta)p^*(\lambda)]\sum_{k=0}^{\infty} F_k(t)[q^*(\theta)p^*(\lambda)]^{k-1}$$

Now $L(t)$ *is defined as* $P[T > t]$
Therefore
$$L(t) = 1 - S(t)$$
$$= [1 - q^*(\theta)p^*(\lambda)]\sum_{k=0}^{\infty} F_k(t)[q^*(\theta)p^*(\lambda)]^{k-1} \ldots\ldots (4.3)$$
Taking Laplace transform of both sides $L(t)$ *we have*
$$l^*(s) = [1 - q^*(\theta)p^*(\lambda)]\sum_{k=0}^{\infty}[f^*(s)]^k[q^*(\theta)p^*(\lambda)]^{k-1} \ldots\ldots\ldots (4.4)$$
we assume that $f(.) \sim \exp(\mu)$ *and* $f^*(s) = \dfrac{\mu}{\mu + s}$

$$q(.)\sim \exp(\varphi) and\ q^*(\theta) = \frac{\varphi}{\varphi+\theta}$$
$$p(.)\sim \exp(\gamma) and\ p^*(\lambda) = \frac{\gamma}{\gamma+\lambda}$$

Now it is assumed that the inter arrival times between decision epochs are constantly correlated, exchangeable, exponentially distributed sum of random variables. According to Gurland (1955) the expression for $f^*(s)$ is given as

$$f^*(s) = \left[\frac{1}{(1+bs)^k}\cdot\frac{1}{\left(1+\frac{kRbs}{(1-R)(1+bs)}\right)}\right]$$

substituting this in equation (4.4) we get

$$l^*(s) = [1-q^*(\theta)p^*(\lambda)]\sum_{k=1}^{\infty}\left[\frac{1}{(1+bs)^k}\cdot\frac{1}{\left(1+\frac{kRbs}{(1-R)(1+bs)}\right)}\right][q^*(\theta)p^*(\lambda)]^{k-1} \ldots (4.5)$$

$$E(T) = \left.\frac{-dl^*(s)}{ds}\right|_{s=0}$$
$$= [1-q^*(\theta)p^*(\lambda)][q^*(\theta)p^*(\lambda)]^{k-1}\sum_{k=1}^{\infty}\frac{1}{(1+bs)^k}\cdot\left[\frac{(1-R)(1+bs)}{(1-R)(1+bs)+kRbs}\right] \ldots\ldots (4.6)$$

$$\frac{dl^*(s)}{ds} = (1-R)\{(1+bs)^{1-k}(-1)[(1-R)(1+bs)+KRbs]^{-1-1}[(1-R)\mathrm{b}+\mathrm{kRb}] + [(1-R)\mathrm{b}+\mathrm{kRb}]^{-1}(1-\mathrm{k})(1+\mathrm{bs})^{1-\mathrm{k}-1}\mathrm{b}\}\big|_{s=0} \ldots\ldots\ldots (4.7)$$
$$= (1-R)(1)^{1-k}[1-R]^{-2}[(1-R)\mathrm{b}+\mathrm{kRb}] + [(1-R)(1)]^{-1}(1-\mathrm{k})(1)^{-k}\mathrm{b}$$
$$= -(1-R)[1-R]^{-2}[b-Rb+kRb] + [1-R]^{-2}[1-R](b-bk)$$
$$= -(1-R)[1-R]^{-2}[b-Rb+kRb+b-bk-Rb+Rbk]$$
$$\left.\frac{dl^*(s)}{ds}\right|_{s=0} = \frac{-bk}{(1-R)}\ On\ simplification$$
$$\left.\frac{-dl^*(s)}{ds}\right|_{s=0} = \frac{bk}{(1-R)} \ldots\ldots (4.8)$$

Substitute equation (4.8) in equation (4.6)

$$1-q^*(\theta)p^*(\lambda) = 1-\frac{\varphi}{\varphi+\theta}\cdot\frac{\gamma}{\gamma+\lambda}$$
$$1-q^*(\theta)p^*(\lambda) = 1-\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}$$
$$1-q^*(\theta)p^*(\lambda) = 1-\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}$$
$$1-q^*(\theta)p^*(\lambda) = \frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda} \ldots\ldots\ldots (4.9)$$
$$[q^*(\theta)p^*(\lambda)]^{k-1} = \left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k-1} \ldots\ldots\ldots (4.10)$$

Substitute equation (4.8), equation (4.9) and equation (4.10) in equation (4.6) we get

$$\frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k-1}\sum_{k=1}^{\infty}\frac{bk}{(1-R)}$$
$$= \frac{b(\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}\sum_{k=1}^{\infty}k\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k-1}$$
$$= \frac{b(\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}\left[1+2\left(\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right)+3\left(\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right)+\cdots.\right]$$
$$= \frac{b(\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}\left[1-\left(\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right)\right]^{-2}$$
$$= \frac{b(\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}\left[1-\left(\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\right)\right]^{-2}$$
$$= \frac{b(\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}\left[\frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\right]^{-2}\ On\ simplification \ldots. (4.11)$$

$$E(T)=\frac{b(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)}{(1-R)(\varphi\lambda+\gamma\theta+\theta\lambda)}\ \ldots..(4.12)$$

From equation (4.7)

$$\frac{dl^*(s)}{ds}=(1-R)\{-(1+bs)^{1-k}[(1-R)(1+bs)+kRbs]^{-2}[(1-R)(b)+kRb]$$
$$+[(1-R)(1+bs)+kRbs]^{-1-1+1}(b-bk)(1+bs)^{-k}\}$$
$$=(1-R)\{-(1+bs)^{1-k}[(1-R)(1+bs)+kRbs]^{-2}[(1-R)(b)+kRb]$$
$$+[(1-R)(1+bs)+kRbs]^{-2}[(1-R)(1+bs)+kRbs](b-bk)(1+bs)^{-k}\}$$
$$=(1-R)\{(1+bs)^{-k}[(1-R)(1+bs)+kRbs]^{-2}[-(1+bs)(b-Rb+kRb)]+[(1-R)(1+bs)+kRbs](b-bk)\}$$
$$=(1-R)\{(1+bs)^{-k}[(1-R)(1+bs)+kRbs]^{-2}[(-1-bs)(b-Rb+kRb)]+[1+bs-R-Rbs+kRbs](b-bk)\}$$
$$=(1-R)\{(1+bs)^{-k}[(1-R)(1+bs)+kRbs]^{-2}[-bk-b^2sk+Rb^2ks-k^2b^2Rs]\}$$
$$=(1-R)\{(1+bs)^{-k}[(1-R)(1+bs)+kRbs]^{-2}[-bk(1+bs)+Rb^2ks(1-k)]\}\ \ldots\ldots(4.13)$$

Differentiate equation (4.13) with respect to 's', we have

$$\left.\frac{d^2l^*(s)}{ds^2}\right|_{s=0}=b(1-R)\{-k(1+bs)^{-k-1}(b)[(1-R)(1+bs)+kRbs]^{-2}[-k(1+bs)+Rkbs(1-k)]+(1+bs)^{-k}[(1-R)(1+bs)+kRbs]^{-2}[-k(b)+bRk(1-k)]+(1+bs)^{-k}[-k(1+bs)+Rkbs(1-k)](-2)[(1-R)(1+bs)+kRbs]^{-3}[(1-R)(b)+kRb]\}$$
$$=b(1-R)\{k^2b[(1-R)^{-2-1+1}+[1-R]^{-2-1+1}[-kb+bRk-bRk^2]+2k[1-R]^{-3}[b-Rb+kRb]\}$$
$$=b(1-R)[1-R]^{-3}\{k^2b[1-R]+[1-R][-kb+bRk-bRk^2]+2k[b-Rb+kRb]$$
$$=\frac{b}{(1-R)^2}[kb-bR^2k+bR^2k^2+k^2b]$$
$$=\frac{b}{(1-R)^2}[kb(1-R^2)+bk^2(1+R^2)]$$
$$=\frac{b^2}{(1-R)^2}[k(1-R^2)+k^2(1+R^2)]\ \ldots\ldots.(4.14)$$

Substitute equation (4.14) in equation (4.6)

$$=\frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\sum_{k=1}^{\infty}\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k-1}\frac{b^2}{(1-R)^2}[k(1-R^2)+k^2(1+R^2)]$$
$$=\frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}.\frac{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma}.\frac{b^2}{(1-R)^2}\sum_{k=1}^{\infty}\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}[k(1-R^2)+k^2(1+R^2)]$$
$$=\frac{b^2(\varphi\lambda+\gamma\theta+\theta\lambda)}{\varphi\gamma(1-R)^2}\left\{\sum_{k=1}^{\infty}\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}k(1-R^2)+\sum_{k=1}^{\infty}\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}k^2(1+R^2)\right\}$$
$$=\frac{b^2(\varphi\lambda+\gamma\theta+\theta\lambda)}{\varphi\gamma(1-R)^2}\left\{(1-R^2)\sum_{k=1}^{\infty}k\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}+(1+R^2)\sum_{k=1}^{\infty}k^2\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}\right\}\ \ldots\ldots(4.15)$$

Let

$$\sum_{k=1}^{\infty}k\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}=\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]+2\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^2+3\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^3+\cdots$$
$$=\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\left[1+2\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]+3\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^2+\cdots\right]$$
$$=\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\left[1-\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{-2}$$
$$=\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\left[\frac{\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\right]^{-2}\text{ On simplification}$$
$$=\frac{\varphi\gamma[\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda]}{(\varphi\lambda+\gamma\theta+\theta\lambda)^2}\ \ldots\ldots..(4.16)$$

$$\sum_{k=1}^{\infty}k^2\left[\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^{k}=\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\frac{\left[1+\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]}{\left[1-\frac{\varphi\gamma}{(\varphi+\theta)(\gamma+\lambda)}\right]^3}$$
$$=\frac{\varphi\gamma}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\frac{\left[=\frac{\varphi\gamma+\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}{\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda}\right]}{\frac{(\varphi\lambda+\gamma\theta+\theta\lambda)^3}{(\varphi\gamma+\varphi\lambda+\gamma\theta+\theta\lambda)^3}}$$

$$= \frac{\varphi\gamma[2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(\varphi\lambda + \gamma\theta + \theta\lambda)^3} \quad \dots\dots (4.17)$$

Substitute equation (4.16) and equation (4.17) in equation (4.15)

$$= \frac{b^2(\varphi\lambda + \gamma\theta + \theta\lambda)}{\varphi\gamma(1-R)^2}\left\{(1-R^2)\frac{\varphi\gamma[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(\varphi\lambda + \gamma\theta + \theta\lambda)^2} + (1+R^2)\frac{\varphi\gamma[2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(\varphi\lambda + \gamma\theta + \theta\lambda)^3}\right\}$$

$$= \frac{b^2}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)}\left\{(1-R^2)[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda] + (1+R^2)\frac{[2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(\varphi\lambda + \gamma\theta + \theta\lambda)}\right\}$$

$$= \frac{b^2}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)}$$
$$\left\{\frac{(1-R^2)[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda](\varphi\lambda + \gamma\theta + \theta\lambda) + [2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(\varphi\lambda + \gamma\theta + \theta\lambda)}\right\}$$

$$= \frac{b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2}\{(1-R^2)[\varphi\lambda + \gamma\theta + \theta\lambda] + (1+R^2)[2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]\}$$

$$= \frac{b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2}\{[\varphi\lambda + \gamma\theta + \theta\lambda - R^2\varphi\lambda - R^2\gamma\theta - R^2\theta\lambda + 2R^2\varphi\gamma + R^2\varphi\lambda + R^2\gamma\theta + R^2\theta\lambda + 2\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]\}$$

$$= \frac{b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2}[2R^2\varphi\gamma + 2\varphi\gamma + 2\varphi\lambda + 2\gamma\theta + 2\theta\lambda]$$

$$E(T^2) = \frac{2b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][R^2\varphi\gamma + \varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2} \quad \dots\dots (4.18)$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{2b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][R^2\varphi\gamma + \varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2} - \left[\frac{b(\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda)}{(1-R)(\varphi\lambda + \gamma\theta + \theta\lambda)}\right]^2$$

$$= \frac{2b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda][R^2\varphi\gamma + \varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2} - \frac{b^2(\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda)^2}{(1-R)^{2s}(\varphi\lambda + \gamma\theta + \theta\lambda)^2}$$

$$= \frac{b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]^2[2R^2\varphi\gamma + 2\varphi\gamma + 2\varphi\lambda + 2\gamma\theta + 2\theta\lambda - \varphi\gamma - \varphi\lambda - \gamma\theta - \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2}$$

$$= \frac{b^2[\varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]^2[2R^2\varphi\gamma + \varphi\gamma + \varphi\lambda + \gamma\theta + \theta\lambda]}{(1-R)^2(\varphi\lambda + \gamma\theta + \theta\lambda)^2} \quad \dots\dots..(4.19)$$

Numerical Illustration.

E(T) and V(T) due to the changes in the different parameters associated with the distribution of the random variables in the model is explained by taking a numerical example.

Table 1.1: Variation in E(T) and V(T) for Changes in θ

$\lambda = 0.5$, b = 0.6, $\varphi = 0.5$, $\gamma = 0.4$, R = 0.1		
θ	E(T)	V(T)
1	0.783	0.614
2	0.732	0.536
3	0.712	0.507
4	0.701	0.492
5	0.695	0.483

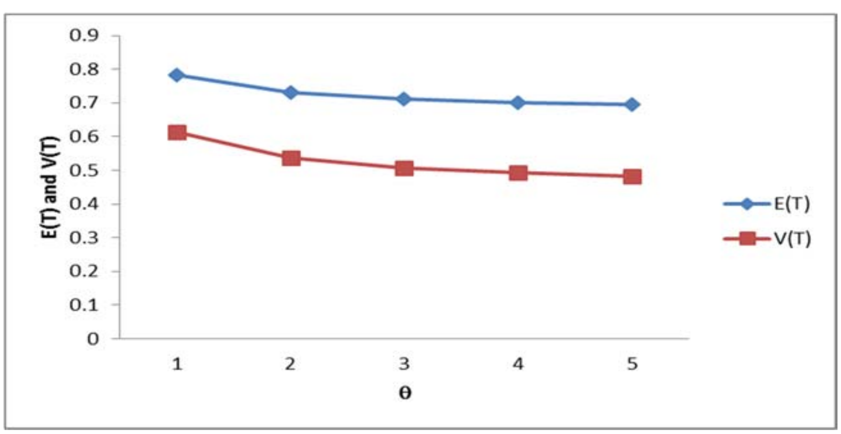

Fig 1.1: Variation in E(T) and V(T) for Changes in θ

Table 1.2: Variation in E(T) and V(T) for Changes in λ

θ = 0.8, b = 0.6, φ = 0.5 γ = 0.4, R = 0.1		
λ	E(T)	V(T)
0.2	0.897	0.808
0.4	0.825	0.684
0.6	0.788	0.623
0.8	0.765	0.586
1.0	0.749	0.562

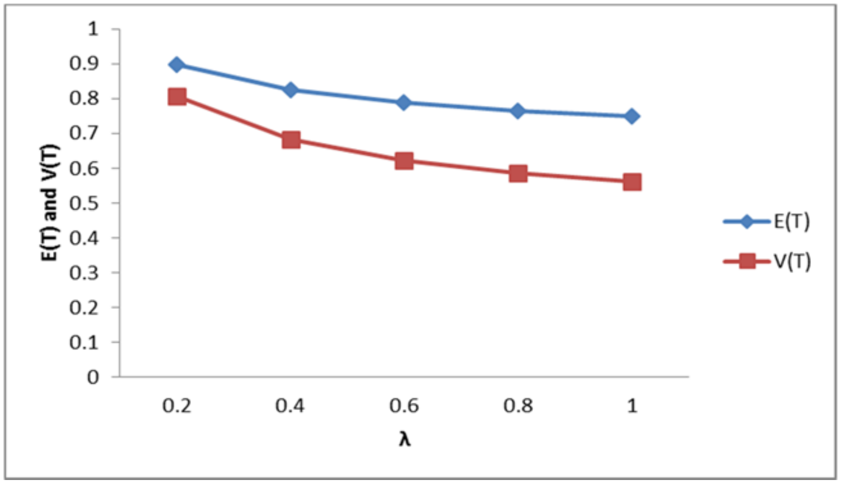

Fig 1.2: Variation in E(T) and V(T) for Changes in λ

Table 1.3: Variation in E(T) and V(T) for Changes in φ

γ = 0.4, θ = 0.8, λ = 0.5, R = 0.1, b = 0.6		
φ	E(T)	V(T)
0.3	0.723	0.524
0.6	0.824	0.681
0.9	0.872	0.764
1.2	0.909	0.831
1.5	0.939	0.886

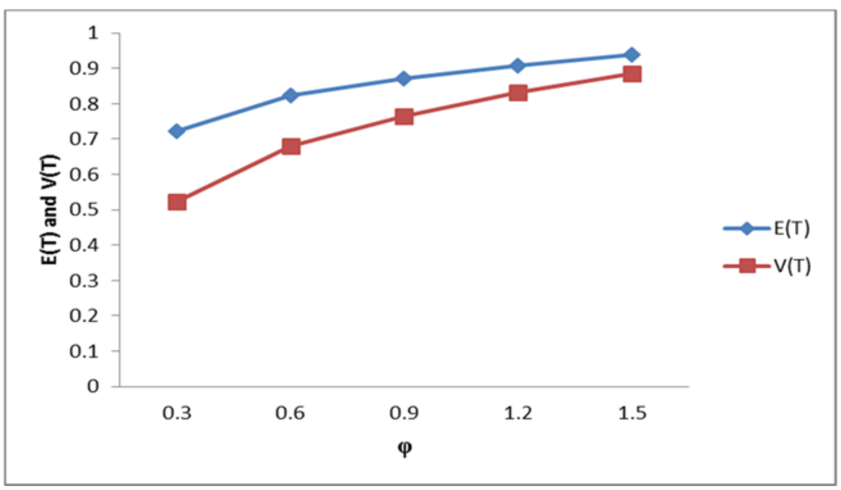

Fig 1.3: Variation in E(T) and V(T) for Changes in φ

Table 1.4: Variation in E(T) and V(T) for Changes in γ

φ = 0.5, θ = 0.8, λ = 0.5, R = 0.1, b = 0.6		
γ	E(T)	V(T)
0.5	0.825	0.684
1	0.897	0.808
1.5	0.937	0.883
2	0.963	0.933
2.5	0.981	0.969

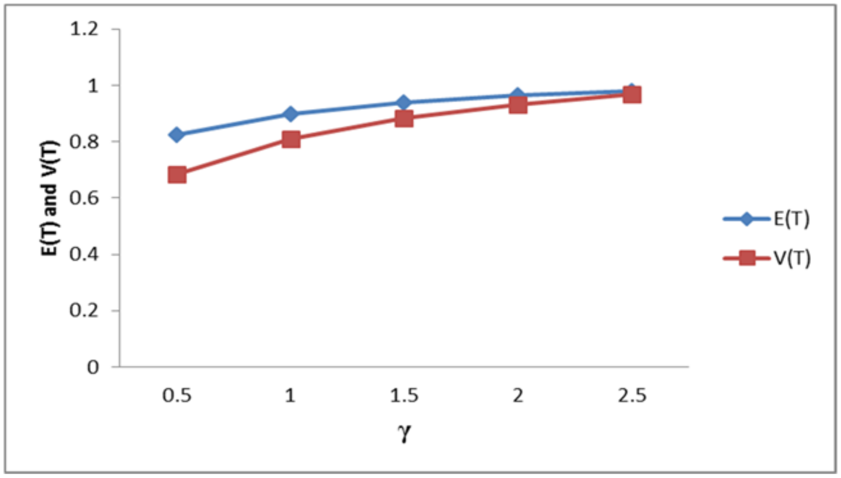

Fig 1.4: Variation in E(T) and V(T) for Changes in γ

Table 1.5: Variation in E(T) and V(T) for Changes in R

φ = 0.5, γ = 0.4, θ = 0.8, λ = 0.5, b = 0.6		
R	E(T)	V(T)
0.1	0.804	0.649
0.2	0.905	0.830
0.3	1.034	1.102
0.4	1.206	1.534
0.5	1.447	2.274

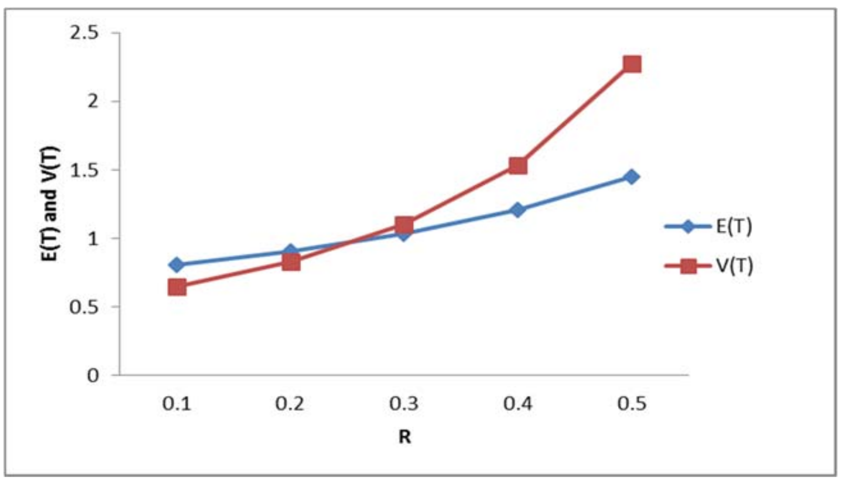

Fig 1.5: Variation in E(T) and V(T) for Changes in R

Conclusion

On the basic of the numerical example worked for the model discussed, the following conclusions are drawn.

1. If θ which is the parameter of the exponential distribution followed by Z_1 denoting the threshold level of propensity to leave increases, E(T) decreases. This is due to the fact that $Z_1 \sim \exp(\theta)$ and $(Z_1) = \frac{1}{\theta}$, when θ increases $\frac{1}{\theta}$ decreases and so the threshold is smaller. Hence E(T) decreases V(T) also shows a decrease. This is indicated in Table 1.1 and Fig (1.1).
2. If λ which is the parameter of the exponential distribution followed by Z_2 which represents the emotional threshold, an increase λ is followed by a decrease in E(T). This is due to the fact that $E(Z_2\ ) = \frac{1}{\lambda}$ decreases as λ increases. Hence E(T) is smaller V(T) also decreases. This is indicated in Table 1.2 and Fig (1.2).
3. The random variable X_i denotes the contribution to the propensity to leave on each decision epoch. It follows exponential distribution with p.d.f q(φ). Hence $E(X_i) = \frac{1}{\varphi}$ so as φ increases $\frac{1}{\varphi}$ decreases. This implies that there is less of contribution to the propensity to leave, so it takes longer time to cross the threshold. Hence E(T) increases V(T) also increases. It is illustrated in Table 1.3 and Fig (1.3).
4. The contribution to emotion is distributed as exp (γ), so $E(Y) = \frac{1}{\gamma}$. As γ increases the average amount of emotion decreases. Hence E(T) shows an increase because emotion is less and it takes longer time to reach the threshold V(T) also increases. it is seen in Table 1.4 and Fig (1.4).

5. One of the assumptions of the model is that the inter arrival times between decision epochs are constantly correlated. If the value of R increases. then it is seen that both E(T) and V(T) increase. The reason for the same is if inter arrival times are shorter then all such intervals are correlated and hence become smaller. Similarly if the inter arrival times are longer they all increase. Therefore the impact of increasing correlation leads to increase of E(T) and V(T).

References

1. Gurland J. Distribution of the maximum of the arithmetic mean of correlated random variables. Ann. Math. Statist, 1955; 26:294-300.
2. Easary JD, Marshall AW, Proschan F. Shock Models and Wear Processes. Annals of Probability, 1973, 627-649.
3. Sathiyamoorthi R. Cumulative damage model with correlated inter arrival times of shocks, IEEE Transaction on Reliability 1979; R-25:23.
4. Gurumoorthy S. The propensity for the software personnel to leave the job in MNC and Indian organisations- Ph.D thesis Management Science, Annamalai university, 2010, 2010.