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Analysis of discrete -time queue of packets with G/G/1 Queueing model

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Abstract
In an attempt to provide a simple and computationally efficient method for the analysis of manufacturing systems, we have adopted the discrete-time analysis (DTA) approach to problems arising out of manufacturing. The subject of our study is the discrete -time G/G/1 queuing system with infinite waiting room. The time interval between the consecutive arrivals of customers is described in terms of a discrete probability mass function (p.m.f) $a(k)$: $a(k)$ is the probability of having an interval of an integer number of k time units between the arrival of customer number n and customer number $n+1$. The service time of customer n is given in terms of a discrete (p.m.f)$b(k)$. The inter-arrival time and the service times are independent and identically distributed (i.i.d) random variables. The customers are served in the order of their arrival First-Come-First Served (FCFS). An explicit formulae are obtained for the distribution of the occupancy, just before and after the departure epoch by using discrete-time analysis. Time is assumption into equal intervals called slots. Further, some special cases of the model, numerical results, have been carried out in the form of a table and graphs.

Keywords: Batch service**,** Discrete-time queue, Departure epochs, Markov Chain, Packets, Slot, System occupancy and inter departure time distribution.

Introduction
In the study of discrete-time queue of packets by using the bulk service rule (L,K) and the distribution of system occupancy just before and after the departure epochs following discrete-time analysis. Time is assumed to be divided into equal intervals called slots. In the study, many computer and communication systems such as information in binary encoded form in the Asynchronous Transfer Mode (ATM) networks will help cut cost of transportation of packets arriving from a manufacturing source, and wait for a transportation in a transit location.

The packets arrive one by one inter-arrival time following geometric distribution, where in a slot is ‘p’ , in the probability of successfully arrival and unsuccessfully arrival is$'\mathrm{q}'$. If an arriving packet finds the server busy, it joins the queue in first in and first out (FIFO) order. A server transports packets in batches of the minimum number L and maximum K. The service time following common geometric distribution with successful probability ‘p_1' and unsuccessfully $'\mathrm{q}_1'$. Recently, queues with bulk service rule have attracted considerable interest and have been widely used in the performance analysis of communication. The studies on queue with bulk service rule can go back to; Harris (1970) has studied a bulk arrival queue with state dependent service rate. In this paper, he has found the steady-state probability and the expected queue length using Imbedded Markov Chain approach. For more detailed information on continuous queue with accessible batches could be referred to in the Klenrock (1975) [5]. Neuts (1981) [9] used the concept of Morkov Chains to analyze queue which have a matrix geometric probability vector.

Chaudhry and Templeton (1983) [3] have presented extensive discussions of bulk service systems that operate according to a rule admitting non-accessible batch. Mathias (1998) [7] studies a inter-departure time distribution for batches and studies correlation between inter-departure times and batch sizes. Mathias and Alexander (1998) [8] provide an discrete-time analysis in the performance evaluation of manufacturing systems. Baburaj (2000) [1] has considered a single service queueing system with a single and batch service. Sivasamy (1990) [10], Sivasamy and Elangovan (2003) [11] on the other hand, the batch, which entries service queue with accessible and non-accessible batches. Sivasamy and Pukazhenthi (2009) [12] have carried out and analyzed the discrete time bulk service for the accessible batch with the arrivals time geometrical distribution and services time negative binomial distribution. Vijaya Laxmi Pikala *et al.,* (2013) [15] have studied the discrete- time renewal input bulk service queue with changeover time. Banerjee *et al.,* (2014) [2] have analysis the finite-buffer discrete -time batch service queue with batch size dependent service. Pukazhenthi and Ezhilvanan (2014) [13] discussed the analyzed of the discrete time queue length distribution with a bulk service rule (L,K). Pukazhenthi and Ezhilvanan (2015) [14] recently used the analysis of discrete time queues with single server using correlated times.

This paper focuses on the analysis of discrete time queue of packets. Customers arrive according to a geometrical arrival process with parameter p, where $\mathrm{p}(0<p<1)$ is the probability that an arrival occurs in a slot and the service time is assumed to follow geometrical probability law, probability p_1. Using the embedded Markov Chain (MC) technique, a Transition Probability Matrix (TPM) at the embedded point has been developed to obtain the steady state distributions of the number in the queue at pre-arrival epochs. Sensitivity analysis has been presented in the form of a table and two graphs.

This paper is organized as follows: In section 2, the notation and the concepts of the model in time domain are presented. This section contains the methodology to compute the distribution of waiting times of a discrete-time G/G/1 system. In section 3, the joint distribution of system occupancy at departure epochs are given. In section 4, the inter-departure time and number of packets in a batch have been discussed. In section 5, the performance measure of the cost function for certain probability pre-assigned values of the parameters are studied. A brief conclusion is presented in section 6.

2. The Model Description

2.1. Arrival Distribution

Assume that before and after customer's arrival follow the geometric distribution, with the queue of packets in an infinite capacity, on the bulk server rule of L size minimum and K size of maximum. The packets are independent and identically distribution (i.i.d) the random variables.

2.1.1. The mean number of customers in the inter-arrival time distribution

Let a_k be the number of customers that arrive during slot k. In this model we assume that the inter - arrival "A_n" of customers are independent and geometric distribution with probability mass function (p.m.f). $\{ a(k) = Pr\,(A_n = k): k = 0,1,2 \ldots. \}$, i.e.

$$a(k) = q^{k-1}\,p: k = 0,1,2 \ldots. \quad \ldots (1)$$

The probability generating function (p.g.f)

$$A(z) = \sum_{k=0}^{\infty} a(k)\,z^k \quad = \sum_{k=0}^{\infty} pq^k q^{-1}\,z^k \quad = \frac{p}{q}\sum_{k=0}^{\infty}(qz)^k \quad = \frac{p}{q}[1-qz]^{-1}$$

$$A(z) = \frac{p}{q(1-qz)}$$

The first order derivative,

$$A'(z) = \frac{p}{q}(1-qz)^{-1} \quad = \frac{p}{(1-qz)^2}$$

The second order derivative,

$$A''^{(z)} = p(1-qz)^{-2} \quad = \frac{2pq}{(1-qz)^3}$$

By putting z =1 we get

$$A'(1) = \frac{p}{(1-qz)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$A''(1) = \frac{2pq}{(1-q)^3} = \frac{2pq}{p^3} = \frac{2q}{p^2}$$

Hence,

$$E(A) = A'(1) = \frac{1}{p} \quad \ldots (2)$$

2.1.2. The variance of number of customers in the inter-arrival distribution

From the previous calculation

$$V(A) = A''(1) + A'(1) - [A'(1)]^2$$

$$= \frac{2q}{p^2} + \frac{1}{p} - \left(\frac{1}{p}\right)^2 \quad = \frac{q+1}{p^2} - \frac{1}{p^2}$$

Hence,

$$V(A) = \frac{q}{p^2} \quad \ldots (3)$$

2.2. Service Distribution

The service time follows geometric distribution, and services of packets are independent and identically distributed the random variables.

2.2.1. The mean number of customers in the service time distribution

The service time for the n^{th} batch is of length B_n, where the B_n are random variables follows geometric distribution with (p. m. f). $\{ b(k) = Pr\,(B_n = k): k = 0,1,2 \ldots. \}$, service times are independent of the number of slots in the batch.

$$B(k) = (1-p_1)^{k-1}\,p_1:\; k = 0,1,2 \ldots. \quad \ldots (4)$$

And the corresponding (p.g.f),

$$B(z) = \sum_{k=0}^{\infty} b(k)\,z^k \quad = \sum_{k=0}^{\infty}(1-p_1)^{k-1}\,p_1\,z^k \quad = \frac{p_1}{q_1}\sum_{k=0}^{\infty}(q_1 z)^k$$

$$B(z) = \frac{p_1}{q_1(1 - q_1 z)}$$

The first order derivatives,

$$B'(z) = \frac{p_1}{q_1}(1 - q_1 z)^{-1}$$

And second order derivatives,

$$B''(z) \quad = p_1(1 - q_1 z)^{-2}$$

By putting z =1 we get

$$B'(1) = \frac{p_1}{(1 - q_1 z)^2} = \frac{p_1}{{p_1}^2} = \frac{1}{p_1}$$

$$B''(1) = \frac{2p_1 q_1}{(1 - q_1)^3} = \frac{2p_1 q_1}{{p_1}^3} = \frac{2q_1}{{p_1}^2}$$

Hence,

$$E(A) = B'(1) = \frac{1}{p_1} \qquad \dots(5)$$

2.2.2. The variance of number of customers in the service time distribution

$$V(B) = B''(1) + B'(1) - \left(B'(1)\right)^2$$

$$= \frac{2q_1}{{p_1}^2} + \frac{1}{p_1} - \left(\frac{1}{p_1}\right)^2$$

Hence,

$$V(B) = \frac{q_1}{{p_1}^2} \qquad \dots(6)$$

2.2.3. Queueing system Utilization

Utilization is the proportion of the system resources which is used by the traffic which arrives at it (Geometric arrivals and single Geometric server), then it is given by the mean arrival rate over the mean service rate that is,

$$\rho \ = \frac{E(B)}{KE(A)} \qquad \dots(7)$$

Where E (A) is mean arrival and E (B) is mean service rate more generally.

$$\rho \ = \frac{\frac{1}{p_1}}{K\frac{1}{p}}$$

Where k is the number of servers, such as in an M/M/k Queue.

$$\rho \ = \frac{1}{p_1}\frac{p}{K}$$

$$\rho \ = \frac{p}{Kp_1} < 1 \qquad \dots(8)$$

Which is less than 1.

3. The Joint Distribution of System Occupancy at Departure Epochs

Let $X_n \in \{0,1,2, \dots, \infty\}$ be the number of packets accumulated in the system queue and service just after the server has left with n^{th}batch. The steady state distribution $\left\{x(k) = \lim_{n\to\infty} x_n(k) = \Pr(X_n = k) : k = 0,1,2, \dots \infty\right\}$ of system occupancy at departure epochs is derived using the embedded Markov Chain technique. Let V_n be random variable denoting the number of packets that reach the system during the n^{th} service. Then the joint distribution $\{v_n(k): k = 0,1,2, \dots\}$ of V_n can be derived as follows.

$$v(k) = \sum_{m=k+1}^{\infty} \binom{m-1}{k} \{p^k q^{m-1-k}\}\{p_1 {q_1}^{m-1}\}; k = 0,1, \dots \qquad \dots(9)$$

Putting $\mathrm{r = m - 1 - k}$, so that $\mathrm{m - 1 = \ r \ + \ k}$. Further on the limit $\mathrm{m = k + 1, r = 0}$ and $\mathrm{m = \infty}$ implies$\mathrm{r = \infty}$. Then the above expression, will be

$$v(k) = \sum_{m=k+1}^{\infty} \binom{m-1}{k} \{p^k (1-p)^{m-1-k}\}\{p_1(1-p_1)^{m-1}\}$$

$$v(k) = \sum_{r=0}^{\infty} \binom{m-1}{k} \{p^k (1-p)^r\}\{p_1(1-p_1)^r(1-p_1)^k\}$$

$$v(k) = \{p^k p_1(1-p_1)^k\} \sum_{r=0}^{\infty} \binom{m-1}{k} \{p_1(1-p)^r(1-p_1)^r\}$$

$$= \frac{p^k p_1 (1-p_1)^k}{\left(1-(1-p)(1-p_1)\right)^{k+1}} \quad where \sum_{r=o}^{\infty} \binom{r+k}{k} \; d^r = 1\!\!\big/\!(1-d)^{k+1}$$
$$= \frac{p^k p_1 (1-p_1)^k}{\left(1-\left((1-p)(1-p_1)\right)\right)^{k+1}} \qquad = \frac{p^k p_1 (1-p_1)^k}{\left(1-(1-p_1-p+pp_1)\right)^{k+1}}$$
$$v(k) = \frac{p^k p_1 (1-p_1)^k}{(p_1+p-pp_1)^{k+1}}$$
$$v(k) = \left(\frac{p_1}{p_1+p-pp_1}\right) \quad \left(1-\frac{p_1}{p_1+p-pp_1}\right)^k$$
For $k = 0,1,2, \ldots \infty$
$$i.e, \qquad v(k) = U(1-U)^k \qquad \ldots (10)$$
$$where\; U = \frac{p_1}{p+p_1-pp_1}$$
The sequence $\{X_n \in S\}$ form a Markov Chain (MC) on the finite state space $S = \{0,1,2, \ldots, \infty\}$ with unit step Transition Probability Matrix (TPM) $P = (p_{ij})$ where

$$p_{ij} = \begin{cases} \sum_{r=0}^{K-L} v(r) & ; 0 \le i \le L-1 \; and\; j = 0 \\ v(K-L+j) & ; 0 \le i \le L-1 \; and\; j \ge 1 \\ \sum_{r=0}^{K-i} v(r) & ; L \le i \le K \; and\; j = 0 \\ v(K-i+j) & ; L \le i \le K \; and\; j \ge 1 \\ v(j+K-i) & ; i \ge (K+1) and\; j \ge (i-K) \\ 0 & ; otherwise \end{cases} \qquad \ldots (11)$$

The Transition Probability Matrix (TPM) $P = [p_{ij}]$. Let the steady state probability vector of the TPMP be $X_n = [x_0, x_1, x_2, \ldots]$.So that
$$X_n = X_n P,\; and\; X_n e = 1 \qquad \ldots (12)$$
Where $e = (1,1,1, \ldots 1)$denotes a unit column vector of unities. A number of numerical methods could be suggested to solve the system of equations (12). Latouche and Ramasamy (1999) have developed more efficient algorithm to solve the equation (12).

3.1 Distribution of before departure epochs

Let Y_n be the random variable denote system occupancy immediately after the beginning of the n^{th} service period.
$$Y_n = \begin{cases} X_{n-1} & if X_{n-1} > L \\ L & if X_{n-1} \le L \end{cases} \qquad \ldots (13)$$
As the respected distribution of Y_n is $\{y_n(k) = \Pr(Y_n = k) : k = L, L+1, L+2, \ldots\}$could be seen that $\left\{y(k) = \lim_{n\to\infty} Y_n(k) = \Pr(Y_n = k) :\; k = 0,1,2, \ldots \;\; \infty\right\}$ which is
$$Y_n = \begin{cases} X(i); & if X_{n-1} = k > L \\ \sum_{i=0}^{L} X(i); & if X_{n-1} and\; k = L \end{cases} \qquad \ldots (14)$$
Let $G_n$random variable denotes system occupancy just before the n^{th} batch service epoch. It is observed that $\left\{g(k) = \lim_{n\to\infty} g_n(k) = \Pr(G_n = k) :\; k = 0,1,2, \ldots \;\; \infty\right\}$ and it is
$$g(k) = y(k)^* v(k); \qquad K \ge L \qquad \ldots (15)$$
Where $y(k)^* v(k)$ acts as the usual convolution operator distribution y (k) and v (k). It is unnecessary to remark that distribution $\{x(k)\}$ and $\{g(k)\}$ is related to each other as follows:
$$X(i) = \begin{cases} g(i+k) & if\; i > L \\ \sum_{j=L}^{K} g(i) & if\; i = o \end{cases} \qquad \ldots (16)$$

4. Inter-Depature Time and Number of Packets in A Batch

Let D_b random variable denote the time number of slots between the departures of two consecutive batches of packets. Given that at least L packets in the system at a departure epoch the time B of the batch being served. If there are not enough packets to make up a batch i.e. X<L, the inter-departure time consist of the inter - arrival time of the (L-X) packets needed to make up a batch and service time of that batch. Hence the distribution $\{d_b(k) = \Pr(D_b = k)\}$ of inter-departure time D_b is

$$d_b(k) = b(k)\sum_{i=L}^{\infty} g(i) + \sum_{i=0}^{L-1}\left[a^{*(L-i)} * b(k)\right]x(i)\,;\, k \geq 1 \qquad \dots(17)$$

Where * stands for convolution operation and a* denotes I-fold convolution of a (k) with itself. Let S decodes the random variable of the number of lots in a batch beginning of a batch service and $\{s(k) = \Pr(S = k) : k = L, L+1, L+2, \ \dots K\}$

$$S(k) = \begin{cases} Y(L) = \sum_{i=0}^{L} x(i) & if \ \ k = L \\ y(k) = x(k) & if \ \ L < K < k \\ \sum_{i=K}^{\infty} y(k) = \sum_{i=k}^{\infty} x(i) & if \ k = K \end{cases} \qquad \dots(18)$$

The joint distribution $\{h(k,j) = \Pr(D_b = k, S = j) : k \geq 1, L \leq j \leq K\}$ of D_s and S_2 follows

$$h(k,j) = \partial(K,j)\left\{b(k)\ \sum_{i=k}^{\infty} x(i)\right\} + \partial(L,j) + \sum_{i=0}^{L-1}\left[a^{*(L-i)} * b(k)\right]x(i)\left\{1 - \partial(L,j) - \partial(K,j)\right\}s(j)b(k) \qquad \dots(19)$$

Where $\partial(i,j) = 1$ when $i = j$ and 0 otherwise.

The expectation of random variable D_b , S and their product D_b , S are given below

$$E(D_b) = E(B) + E(A)\sum_{i=0}^{L-i}(L-i)x(i) \qquad \dots(20)$$

$$E(S) = \sum_{k=L}^{K} ks(k) \qquad \dots(21)$$

$$E(D_bS) = E(S)E(B) + LE(A)\sum_{I=0}^{L-1}(L-i)x(i) \qquad \dots(22)$$

$$r_1 = \frac{E(D_bS) - E(D_b)E(S)}{\sigma_{D_b}\sigma_S} \qquad \dots(23)$$

Similarly Let S_2Random variables denoting the number of lots in a batch beginning of a batch service and $\{s_2(k) = \Pr(S_2 = k) \ : \ \ k = L, L+1, L+2, \ \dots K\}$

$$s_2(j) = \begin{cases} \sum_{j=k}^{\infty} g(j) & j = k \\ g(j) & L \leq j << k \end{cases} \qquad \dots(24)$$

The moment of S_2 can be calculated from (22).By using (16) in one could rewrite (17) in terms of g (.) values as we get:

$$d_b(k) = b(k)\sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}\left[a^{*(L-i)} * b(k)\right]g(i+K) + \left(a^{*L} * b(k)\right)\sum_{j=L}^{K-1} g(j) \qquad \dots(25)$$

We derive joint distribution of D_s and S_2 the usual arguments lead to the following differential defiance equation $\{g(k,j) = \Pr(D_b = k, S_2 = j) : k \geq 1, L \leq j \leq K\}$

$$g(k,j) = \partial(K,j)\left(b(k)\ \sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}\left[a^{*(L-i)} * b(k)\right]g(i+K)\right) + \left(1 - \partial(K,j)\right)\left(a^{*L} * b(k)\right)\, g(j) \qquad \dots(26)$$

Thus

$$E(D_bS_2) = K\left(E(B)\sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}[(L-i)E(A) + E(B)]g(i+k)\right) + \left(LE(A) + E(B)\right)\sum_{j=L}^{K-1} j\, g(j) \qquad \dots(27)$$

As in (17), one can calculate the correlation co-efficient, say r_2 , between D_b and S_2.

$$r_2 = \frac{E(D_bS_2) - E(D_b)E(S_2)}{\sigma_{D_b}\sigma_{S_2}} \qquad \dots(28)$$

5. COST PERFORMANCE MEASURE AND ANALYSIS OF D_b, S

Cost function involving random variables D_b and S_2 can be formulated. Different cost values incurred the coefficient of variation (CV) of S and coefficient of determination (CD) of r_1 can be defined as:

$\$C_1$ = Cost due to Coefficient of Variation $CV(S_1)$ (Cost due to CV (S_2))

$\$C_2$ = Cost due to Coefficient of determination $CD(r_1)$ (Cost due to CV (r_2))

Subject to these costs, the total expected cost (TEC_1) and (TEC_2) of the input parameters $p, p_1, \alpha, L,$ and K of the queuing system under study is given by (D_b S_2) and (D_b S_1) respectively, They are

$TEC_1 = TEC_1(p, p_1, \alpha, L, K)$

$TEC_2 = TEC_2(p, p_1, \alpha, L, K)$

$TEC_1(p, p_1, \alpha, L, K) = C_1 CV(S) + C_2 {r_1}^2$

$TEC_2(p, p_1, \alpha, L, K) = C_1 CV(S_2) + C_2 {r_2}^2$

As TEC_1 and TEC_2 is a convex function, in L increases, CV(S) and $CV(S_2)$ decrease and ${r_1}^2$ and ${r_2}^2$ increase. Since the closed form of expression is not available for each TEC, one can locate minimum in specific cases. An attempt is being made using the above numerical value with C_1 = 175.00, C_2 = 455.00, p=0.910 p_1= 0.440 α =0.413636 and K= 15.

Table 1: Values of TEC for different values L

L	$C_1CV(S)$	$C_2 r_1^2$	TEC1	$C_1CV(S)$	$C_2 r_1^2$	TEC2
1	24.22	0.07	24.289	105.97	0.01	105.979
2	12.02	0.27	12.289	86.09	0.05	86.138
3	07.95	0.59	8.540	72.12	0.16	72.281
4	05.90	1.04	6.946	61.60	0.43	62.033
5	04.67	1.60	6.275	53.22	1.00	54.223
6	03.84	2.25	6.096	46.18	2.14	48.320
7	03.24	2.97	6.209	39.98	4.22	44.201
8	02.77	3.70	6.472	34.26	7.73	41.997
9	02.38	4.40	6.774	28.80	13.06	41.854
10	02.03	5.01	7.041	23.42	20.09	43.512
11	01.70	5.47	7.173	18.08	27.70	45.783
12	01.37	5.77	7.142	12.82	33.26	46.074

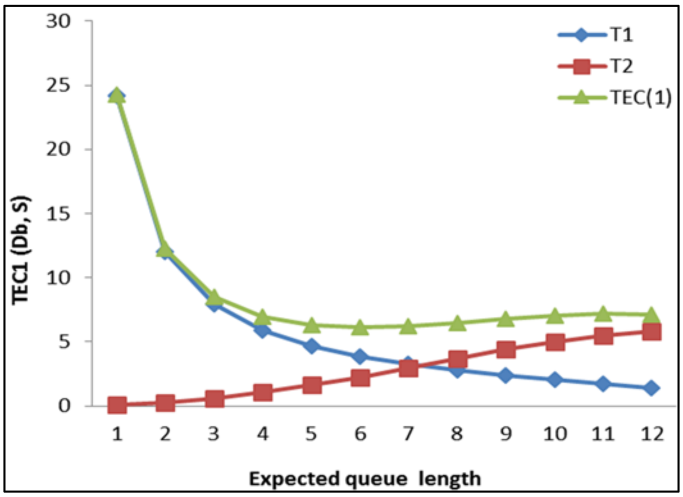

Fig 1: Graph of TEC1 for different values L TEC1 Curve Db and S with C_1 = 175.00, C_2 = 455.00, p = 0.910, p_1 = 0.440 ρ = 0.413636 α = 3 and K= 15.

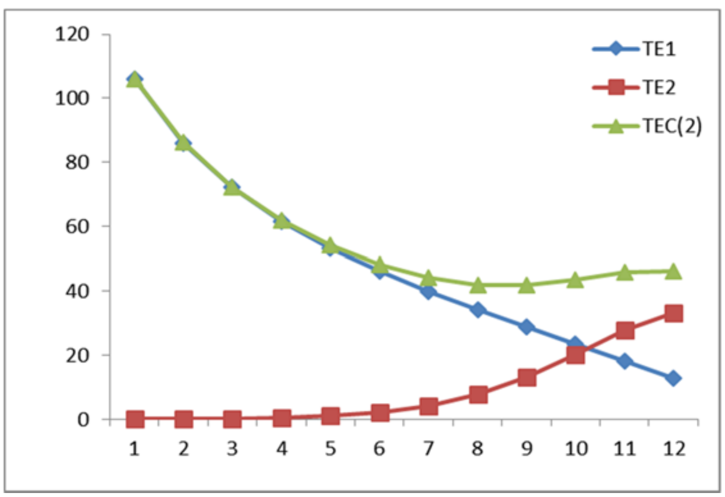

Fig 2: Graph of TEC2 for different values L TEC1 Curve Db and S with C_1 = 175.00, C_2 = 455.00, p = 0.910, p_1 = 0.440 ρ = 0.413636, α = 3 and K= 15.

The corresponding results of TEC_1/TEC_2 are presented in Table-1 where CV= (S.D/Mean) only. It can be noticed that local minimum TEC_1 occurs when L = 6 and TEC_2 does when the local minimum is L = 9. Based on this information regarding local minimum on 'L' ,the facility rendering batch service with the rule (L,K) can call the server (the transportation vehicle from outside) to come to the service point, and thereby saving the amount of transportation cost, which is the primary objective in transportation and optimization problems.

6. Conclusion

In this paper, we investigated the inter-departure time distribution of a batch service process. Using discrete-time analysis, we derived the inter-departure time distribution of batches and of individual lots. We also computed the distribution of the number of lots in a batch and the co-efficient of correlation of batch inter-departure time and batch size. The proposed approach is computationally efficient. Furthermore, this approach allows modelling more complicated system. It will be the object of further research to observe how other batching strategies affect the departure process of a batch server. The cost of performance measure in the form of tables and graphs are reported to demonstrate how the parameters of the model influence the behavior of the system.

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