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Content knowledge and pedagogical content knowledge in the ninth grade mathematics textbook of West Bengal board of secondary education

[1]SubirSen,[2]Tuhin Kumar Samanta
[1]Assistant Professor, B. Ed. Department, Kalna College, The University of Burdwan, West Bengal, India.
[2]Associate Professor, Department of Education, the University of Burdwan, West Bengal, India.

Abstract
Present work deals with an analysis of content knowledge and pedagogical content knowledge in ninth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. Concepts which are explored in the algebra together with necessary concepts which should be included are also discussed. An analysis on textual presentation and exercises supplied with necessary recommendations is presented.

Keywords: Mathematics textbook, Algebra, Content knowledge, Pedagogical Content Knowledge, West Bengal Board of Secondary Education.

Introduction
Shulman (1987) defines seven categories to provide a framework for teacher knowledge which are:
1. Content knowledge
2. General pedagogical knowledge eg classroom control, using group work
3. Pedagogical content knowledge
4. Curriculum knowledge
5. Knowledge of learners and their characteristics
6. Knowledge of educational contexts eg schools and the wider community
7. Knowledge of educational ends purposes and values

Shulman (1987) identified seven domains of teacher knowledge, one of which is pedagogical content knowledge. He explained why he identified pedagogical content knowledge as a knowledge domain for teachers as follows:
Pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction.
Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. Shulman claimed that pedagogical content knowledge is a distinct body of knowledge even though knowledge of content and knowledge of pedagogy contribute to it. He also noted that pedagogical content knowledge includes knowledge of learners, knowledge of educational context, and knowledge of instructional materials.
Tamir (1988)[15] made a distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. He claimed that each type of knowledge is composed of four categories-namely, student, curriculum, instruction, and evaluation- but they have different meanings in each domain. He also identified teachers' skills in diagnosing students' conceptual difficulties in a given topic and their knowledge about effective use of instructional tools as subject-matter-specific pedagogical knowledge.
Ball and Bass (2000)[1] identified teachers' knowledge of students' difficulties and appropriate teaching strategies to eliminate those difficulties as part of teachers' pedagogical content knowledge. They defined pedagogical content knowledge as follows:
'Pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy. These bundles offer a crucial resource for teaching mathematics, for they can help the teacher anticipate what students might have trouble learning, and have ready alternative models or explanations to mediate those difficulties (p. 88).'
Wang Wei Sönnerhed (2011)[17] studied algebra textbook for CK and PCK and wrote 'The primary aim of the study is to explore what pedagogical content knowledge regarding solving quadratic equations that is embedded in mathematics textbooks. The secondary aim is to analyze the algebra content related to solving quadratic equations from the perspective of mathematics as a discipline in relation to algebra history. It is about what one can find in the textbook rather than how the textbook is used in the classroom (p-5).'
Sen and Samanta (2015a, 2015b, 2015c)[11-13] analyzed content knowledge and pedagogical content knowledge in sixth, seventh and eighth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. According to them 'It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units discussed before for correlation and evaluation of concepts.'

Methodology
To explore the nature of Content Knowledge (CK) represented in the textbook and expected Pedagogical Content Knowledge (PCK) in the framework, it is necessary to discuss

Van Dormolin's (1986) classification of teaching perspectives and learning perspectives of Schmidt *et al.* (1997)[10].

Based on their classification and the CK-PCK overall framework one may consider the following criteria for analyzing algebra content textual presentation as follows:

1. Consistency and clearness of Mathematical content: A mathematical text should be consistent and clear to the reader. "There must be no errors, either of computation or of logic. Proofs might be incomplete, but not false. Conventions must be used consistently. […] the content must be clear to the intended reader." (Van Dormolen, 1986, p. 151).
2. Mathematical theoretical aspects: This criterion concerns knowledge elements such as mathematical theorems, rules, definitions, methods and conventions. Such mathematical knowledge is called "kernels" (VanDormolen, 1986, p. 146)
3. Mathematical content development and connections: This criterion is based on the classification of Schmidt *et al.* (1997)[10]. By means of this criterion, one may investigate how mathematical content topics relate to each other in the chapter of algebra. The aim is to explore the embedded teaching trajectory related to text.
4. Mathematical representations and applications: This category often reflects different views. A formalistic view regards mathematics as a set of concepts, rules, theorems and structures. Mathematics applications are often regarded as informal view. In an informal view students are encouraged to engage in activities like generalizing, classifying, formalizing, ordering, abstracting, exploring patterns and so on, and new ideas are encouraged (De Lange, 1996;[4]Freudenthal, 1991;[5]Goldin, 2008;[6] Pepin *et al.*, 2001;[9]Van Dormolen, 1986; Vergnaud, 1987)[16].
5. Language use: In which way are mathematical theorems, definitions, and rules explained and illustrated: formally in a mathematical language or pedagogically in combination with everyday language, in order to make sense for a student reader.
6. To analyze different kinds of mathematics exercises, activities and problems as well as tests in the textbook, it is important to analyzing mathematics tasks in the textbooks (Brändström, 2005)[2]. One may consider the following points:

A. Routine exercises refer to the kind of exercises that require students to use newly presented mathematical concepts, rules or algorithmic procedures illustrated in examples, in order to get familiar with the content. This kind of exercises is often at a basic level and requires simple and similar operations or reasoning to those just presented.

B. Exercises that require students to evaluate, analyze and reason mathematically instead of merely computing mechanically (Brändström, 2005)[2]. Such exercises intend to encourage students to understand the integration of mathematics concepts and procedures (Hiebert& Carpenter, 2007;[7]Hiebert&Lefevre, 1986)[8].

C. Exercises that are related to real world contexts. Such exercises are often word problems (or called real world problems) and the pedagogical reason of using them is to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem solving without the practical contact with the real world situation (Chapman, 2006). They reflect the view of mathematics applications in real-life situations (De Lange, 1996;[4]Freudenthal, 1991;[5]Goldin, 2008;[6] Pepin *et al.*, 2001;[9] Van Dormolen, 1986; Vergnaud, 1987)[16].

Results and Discussions

To discuss the algebraic content let us recall the notations

$\mathbb{N}$: The set of natural numbers

$\mathbb{Z}$: The set of integers

$\mathbb{R}$: The set of real numbers

$\mathbb{R}^+$: The set of positive real numbers.

Name of the ninth grade mathematics text book of West Bengal Board of Secondary Education is 'Ganit Prakash (class IX)'.

In this book, content of unit two (pages 21 to 28) is Laws of Indices.

The concepts which are explored in this chapter are

1. Concept of index and base is explored.
2. $x^m \times x^n = x^{m+n}, x \in \mathbb{R},\ m,n \in \mathbb{N}$ is established.
3. $x^m \div x^n = x^{m-n}, x \in \mathbb{R},\ m > n,\ m,n \in \mathbb{N}$ is established.
4. $x^m \div x^n = \frac{1}{x^{n-m}},\ x \in \mathbb{R},\ m < n,\ m,n \in \mathbb{N}$ is established.
5. $(x^m)^n = x^{m \times n},\ x \in \mathbb{R},\ m,n \in \mathbb{N}$
6. $(\mathrm{xy})^m = x^m \times y^m,\ x,y \in \mathbb{R},\ m \in \mathbb{N}$
7. $(\frac{x}{y})^m = \frac{x^m}{y^m},\ y \neq 0,\quad x,y \in \mathbb{R},\ m \in \mathbb{N}$
8. $x^0 = 1,\ x^{-1} = \frac{1}{x},\quad x^{-n} = (x^{-1})^n = \frac{1}{x^n}$
9. Concept of square root, cube root and n-th root is explored. For example $x^2 = a \Rightarrow x = \pm\sqrt{a},\ x^3 = a \Rightarrow x = a^{\frac{1}{3}}$. Uniqueness of $a^{\frac{1}{n}}$ is stated.
10. $a^{\frac{1}{n}}$ will be unique negative number x if $x^n = a$ for $a < 0, a \in \mathbb{R}, n = 2k+1, k \in \mathbb{N}$.
11. $a^{\frac{p}{q}} = (\mathrm{a}^{\frac{1}{q}})^p,\ a > 0, p \in \mathbb{Z},\ \mathrm{q} \in \mathbb{N}$
12. $a^{\frac{p}{q}} = (\mathrm{a}^{\frac{1}{q}})^p,\ a < 0, p \in \mathbb{Z},\ \mathrm{q} = 2\mathrm{k}+1, \mathrm{k} \in \mathbb{N}$.

Some suggestions are listed below for necessary improvement of content.

- Concept of uniqueness of n-th root should be discussed clearly. Otherwise a wrong conception may be may grow among the students. It is mentioned in the text book that for $n \in \mathbb{N},\ a \in \mathbb{R}^+,\ a^{\frac{1}{n}}$ will be a unique $x \in \mathbb{R}^+$, s.t. $x^n = a \Rightarrow x = a^{\frac{1}{n}} = \sqrt[n]{a}$.

- Consider the case $x = (64)^{\frac{1}{6}}$, *i.e.* $x^6 = 64$ which satisfies $x = \pm 2$. Actually $x^n = a$ have n roots, real or imaginary.
- For $a^{\frac{1}{n}}$ will be unique negative number x if $x^n = a$ for $a < 0, a \in \mathbb{R}, n = 2k+1, k \in \mathbb{N}$, here a natural question raises. If $a < 0,\ n = 2k,\ k \in \mathbb{N}$ then what will be the value of $a^{\frac{1}{n}}$? For example let us consider $(-1)^{\frac{1}{4}} = x \Rightarrow x^4 = -1$ no real x satisfies this relation.
- Another question is 'Are the laws of indices true for irrational numbers?'There should be a brief discussion about the question.

To analyze the mathematical exercise represented in this unit it is observed that

- Routine exercises are included.
- Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
- Questions are represented according to the structure of the text and these are sequentially arranged.

Chapter-5: Liner Simultaneous Equations(pages 47 to 71)
Concepts which are explored in this chapter are as below:

1. Construction of liner simultaneous equations with the help of real life.
2. Graphical solution of the constructed equations is illustrated. Here equations are solvable and unique solutions exist.
3. An example of simultaneous equations with graphical representation is considered which satisfies infinitely many values of two unknowns.
4. Another example is illustrated with graphical representation where there is no solution exists.
5. A discussion about graphical representation of unique solution, infinitely many solution and no solution is included.
6. Representing liner simultaneous equations by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ with condition of unique solution $\left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$, infinitely many solutions $\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$ and no solution $\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right)$ is illustrated.
7. Solution of liner simultaneous equations by elimination of one unknown is illustrated.
8. Solution of liner simultaneous equations by comparison method is illustrated.
9. Solution of liner simultaneous equations by replacement of one unknown is illustrated.
10. Solution of liner simultaneous equations by cross multiplication method is illustrated.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Chapter 7: Polynomial (page 94-111)
Following concepts are explored:

1. Polynomial in particular monomial, binomial and trinomial.
2. Constant polynomial and zero polynomial.
3. Addition, subtraction and multiplication of two polynomials and after addition, subtraction or multiplication resulting algebraic expression becomes another polynomial.
4. Verification of an algebraic expression whether it is a polynomial or not.
5. Degree of a polynomial.
6. n-th degree polynomial with one variable.
7. Polynomial with two variables.
8. Calculation of value of a polynomial for given value of the variable.
9. Calculation of zero of a polynomial.
10. Division of a polynomial by another polynomial.
11. Remainder Theorem.
12. Factor Theorem.

Some suggestions are listed below to improve the text.

- When concept of polynomial is explored, it is stated that integral power of variable should be replaced by positive integral power of the variable.
- Through discussion about the number of zero of a polynomial should be incorporated.
- When division of a polynomial [$f(x)$] by another polynomial[$g(x)$] is explored, it is mentioned that

$f(x) = g(x)q(x) + \mathrm{r(x)}$ where, $q(x)$ and $\mathrm{r(x)}$ are unique. It should be mentioned that degree of $f(x)$ should be grater or equals to $g(x)$, otherwise $q(x)$ and $\mathrm{r(x)}$ will not be polynomials. To clear this concept a discussion considering the case 'degree of $f(x)$ less than $g(x)$' should be included.

- A discussion about zero of a polynomial $f(x)$ is actually roots of the equation $f(x) = 0$ should be included. It will be compatible with question no 12(V) in page 111of the textbook.

To analyze the mathematical exercise represented in this unit it is observed that

- ✓ Routine exercises are included.
- ✓ Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
- ✓ Questions are represented according to the structure of the text and these are sequentially arranged.

Chapter 8: Factorization (pages 112-122)
Following concepts are explored:

1. Factor of a polynomial
2. Vanishing method or Trial method of factorization
3. Factorization by using the following identities:

$a^2 + 2ab + b^2 = (\mathrm{a+b})^2 = (\mathrm{a+b})(\mathrm{a+b})$,

$a^2 - 2ab + b^2 = (\mathrm{a-b})^2 = (\mathrm{a-b})(\mathrm{a-b})$,

$a^2 - b^2 = (\mathrm{a+b})(\mathrm{a-b})$,

$a^3 + b^3 = (\mathrm{a+b})(\mathrm{a}^2 - a\mathrm{b} + \mathrm{b}^2)$,

$a^3 - b^3 = (\mathrm{a-b})(\mathrm{a}^2 + a\mathrm{b} + \mathrm{b}^2)$,

$a^3 + b^3 + c^3 - 3abc = (a+b+c)(\mathrm{a}^2 + b^2 + c^2 - ab - bc - ca)$,

$\mathrm{a+b+c} = 0,\ a^3 + b^3 + c^3 = 3abc$ and

$\mathrm{x}^2 + (\mathrm{a+b})\,\mathrm{x} + \mathrm{ab} = (\mathrm{x+a})(\mathrm{x+b})$.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.

Chapter 21: Logarithm (pages 277-287).
Following concepts are explored:

1. Definition of logarithm.
2. Logarithm of negative number is undefined. Base of the logarithm is defined as positive is discussed with proper example.
3. Logarithm of zero is taken as undefined.
4. Logarithm with base negative is undefined due to lack of uniqueness
5. Logarithm with base 1 is undefined.
6. $\mathrm{Log}_a(\mathrm{M})$ where $\mathrm{M} < 0, \mathrm{a} < 0$ is discussed with proper example.
7. Proof of the following theorems

$\mathrm{Log}_a(\mathrm{MN}) = \mathrm{Log}_a(\mathrm{M}) + \mathrm{Log}_a(\mathrm{N});\quad \mathrm{a, M, N} \in \mathbb{R}^+$

$\mathrm{Log}_a(\frac{M}{N}) = \mathrm{Log}_a(\mathrm{M}) - \mathrm{Log}_a(\mathrm{N});\quad \mathrm{a, M, N} \in \mathbb{R}^+$

$\mathrm{Log}_a(\mathrm{M}^c) = c\,\mathrm{Log}_a(\mathrm{M});\quad \mathrm{a, M} \in \mathbb{R}^+,\ \mathrm{c} \in \mathbb{R}$

$\mathrm{Log}_a(\mathrm{M}) = \mathrm{Log}_b(\mathrm{M}) \times \mathrm{Log}_a(\mathrm{b});\quad \mathrm{a, b, M} \in \mathbb{R}^+$

8. Important results and theorems are listed.
9. Briggarian system of logarithm (base 10) and Natural logarithm (base e) is mentioned.

To analyze the mathematical exercise represented in this unit it is observed that

- ✓ Routine exercises are included.
- ✓ Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
- ✓ Questions are represented according to the structure of the text and these are sequentially arranged.

Other aspects which are mentioned in methodology may be discussed as

- ❖ Mathematical text used in the text book is clear to the reader. It is also found that there is no computational error in the algebra text.
- ❖ Mathematical concepts are presented sequentially. Appropriate teaching learning methods are applicable but clearness of some definitions is required.
- ❖ Development of content of algebra is already discussed. One may consider analysis and synthesis and discovery as teaching method. Problem solving method is also very much effective for solution of the problems given in this unit.
- ❖ Language of the book is very simple, clear. Explanations are also very simple.
- ❖ Routine exercises are appropriate for the concept presentation. But it is necessary to modify the exercises which will evaluate the students or will develop the power of analysis or strengthen mathematical reasoning.

Concluding remarks

It is mentioned in the text book that the book is written on the basis of NCF 2005. It follows psychological to logical approach to relate mathematics to real world. Teachers are encouraged to help in constructing students' knowledge. It seems that the algebra content of the text book demands slight modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units for correlation and evaluation of concepts.

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