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Cycle Related Homo-Cordial Graphs

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Abstract
Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that Cycle related graphs Cycle C_n(n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), Crown $C_n\Theta K_1$ are Homo-Cordial Graphs.

Keywords: Homo-Cordial Graph, Homo-Cordial Labeling. 2000 Mathematics Subject classification 05C78.

1. Introduction
A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that Cycle related graphs Cycle C_n(n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), Crown $C_n\Theta K_1$ are Homo-Cordial Graphs. For graph theory terminology, we follow [2]

2. Preliminaries
Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.
The graph that admits a Homo-Cordial Labeling (HoCL) is called a Homo-Cordial Graph (HoCG). We proved that Cycle related graphs Cycle C_n(n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), $C_n\Theta K_1$ are Homo-Cordial Graphs.

Definition: 2.1
A closed path is called a cycle and a cycle of length n is denoted by C_n.

Definition: 2.2
Graph obtained from a path P_n,by joining each end vertices of an edge with two isolated vertex. It is denoted by $C_2(P_n)$.

Definition: 2.3
Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by $D_2(G)$.

Definition: 2.4
Globe is defined as the two isolated vertex are joined by n paths of length 2. It is denoted by Gl(n).

Definition: 2.5
The corona $G_1\Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G. $C_n\Theta K_1$ is called a Crown.

3. Main Results

Theorem: 3.1

Cycle C_n (n-odd) is Homo-Cordial Graph.

Proof

Let $V(C_n) = \{[u_i : 1 \le i \le n]\}$ and

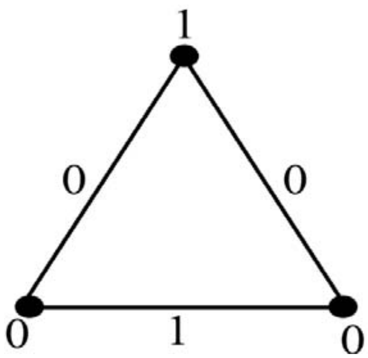
$E(C_n) = \{[(u_i u_{i+1}) : 1 \le i \le n-1] \cup [(u_1 u_n)]\}$.

Define $f: V(C_n) \to \{0,1\}$.

Case: 1

When n=3,

The labeling is,



Case: 2

When n>3,

The vertex labeling are,

$f(u_i) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases}$ $1 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$ $1 \le i \le n-1$

$f^*[(u_1 u_n)] = \begin{cases} 0 & n \equiv 3 \bmod 4 \\ 1 & n \equiv 1 \bmod 4 \end{cases}$

Here, $v_f(0) = v_f(1)+1$ for $n \equiv 3 \bmod 4$,

$v_f(1) = v_f(0)+1$ for $n \equiv 1 \bmod 4$,

$e_f(0) = e_f(1)+1$ for $n \equiv 3 \bmod 4$ and

$e_f(1) = e_f(0)+1$ for $n \equiv 1 \bmod 4$.

Therefore, Cycle C_n(n-odd) satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, Cycle C_n (n-odd) is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of C_5 is shown in figure 3.2

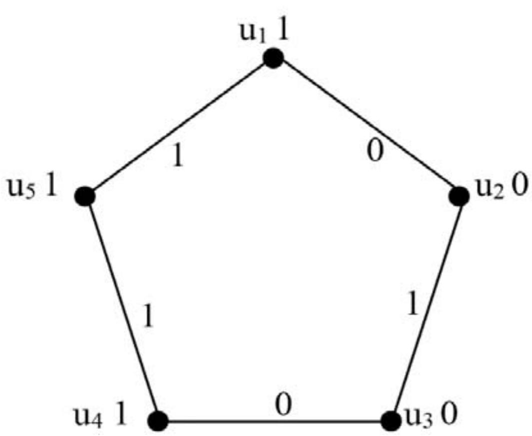


Fig 3.2: C_5

Theorem: 3.3

$D_2(C_n)$ is Homo-Cordial Graph.

Proof

Let $V(D_2(C_n)) = \{[u_i, v_i : 1 \le i \le n]\}$ and

$E(D_2(C_n)) = \{[(u_i u_{i+1}) \cup (u_i v_{i+1}) \cup (v_i u_{i+1}) \cup (v_i v_{i+1}) : 1 \le i \le n-1] \cup [(u_1 u_n) \cup (v_1 v_n) \cup (u_1 v_n) \cup (v_1 u_n)]\}$.

Define $f : V(D_2(C_n)) \to \{0,1\}$.

The vertex labeling are,

$f(u_i) = 0$ $1 \le i \le n$

$f(v_i) = 1$ $1 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 1$ $1 \le i \le n-1$

$f^*[(v_i v_{i+1})] = 1$ $1 \le i \le n-1$

$f^*[(u_i v_{i+1})] = 0$ $1 \le i \le n-1$

$f^*[(v_i u_{i+1})] = 0$ $1 \le i \le n-1$

$f^*[(u_1 u_n)] = 1$

$f^*[(v_1 v_n)] = 1$

$f^*[(v_1 u_n)] = 0$

$f^*[(u_1 v_n)] = 0$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, the graph $D_2(C_n)$ satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, $D_2(C_n)$ is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of $D_2(C_5)$ and $D_2(C_6)$ are shown in the following figure 3.4 and figure 3.5 respectively.

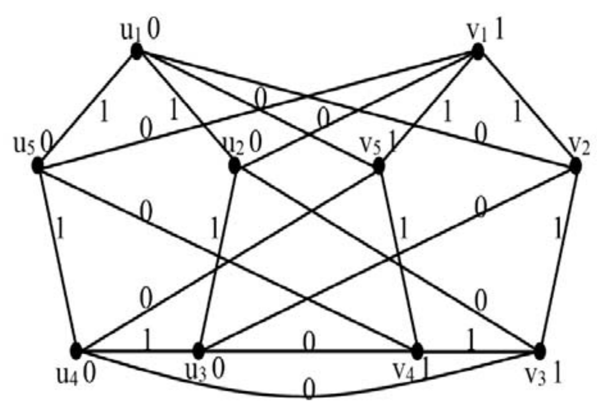


Fig 3.4: $D_2(C_5)$

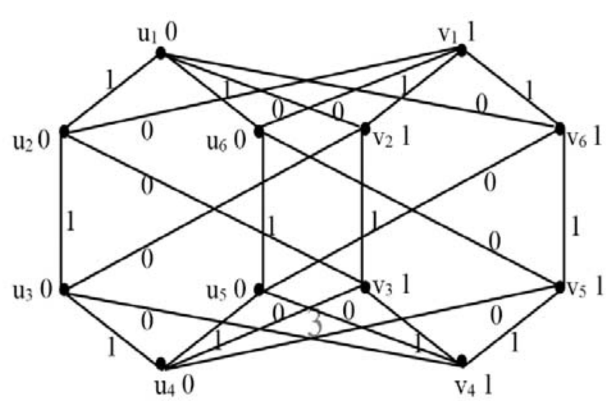


Fig 3.5: $D_2(C_6)$

Theorem: 3.6
Globe Gl(n) : (n-even) is Homo-Cordial Graph.

Proof
Let V(Gl(n)) ={[u,v,u_i : 1≤i≤n]} and
E(Gl(n)) = {[(uu_i) ∪ (vu_i) : 1≤i≤n]}.
Define f : V(Gl(n))→{0,1}.

The vertex labeling are,
$f(u) = 0$
$f(v) = 1$
$f(u_i) = 0 \quad 1 \le i \le \frac{n}{2}$
$f(u_i) = 1 \quad \frac{n}{2}+1 \le i \le n$

The induced edge labeling are,
$f^*[(uu_i)] = 1 \quad 1 \le i \le \frac{n}{2}$
$f^*[(uu_i)] = 0 \quad \frac{n}{2}+1 \le i \le n$
$f^*[(vu_i)] = 0 \quad 1 \le i \le \frac{n}{2}$
$f^*[(vu_i)] = 1 \quad \frac{n}{2}+1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and
$e_f(0) = e_f(1)$ for all n.

Therefore, Globe Gl(n) : (n-even) satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.
Hence, Globe Gl(n) : (n-even) is Homo-Cordial Graph.
For example, the Homo-Cordial Labeling of Gl(4) is shown in figure 3.7

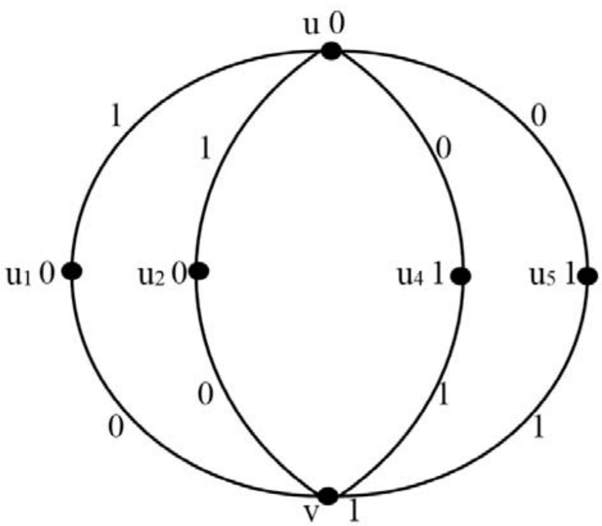


Fig 3.7: Gl(4)

Theorem: 3.8
Globe Gl(n) : (n-odd) is Homo-Cordial Graph.

Proof
Let V(Gl(n)) ={[u,v,u_i : 1≤i≤n]} and
E(Gl(n)) = {[(uu_i) ∪ (vu_i) : 1≤i≤n]}.
Define f : V(Gl(n))→{0,1}.

The vertex labeling are,
$f(u) = 0$
$f(v) = 1$
$f(u_i) = 0 \quad 1 \le i \le \frac{n-1}{2}$
$f(u_i) = 1 \quad \frac{n+1}{2} \le i \le n$

The induced edge labeling are,
$f^*[(uu_i)] = 1 \quad 1 \le i \le \frac{n-1}{2}$
$f^*[(uu_i)] = 0 \quad \frac{n+1}{2} \le i \le n$
$f^*[(vu_i)] = 0 \quad 1 \le i \le \frac{n-1}{2}$
$f^*[(vu_i)] = 1 \quad \frac{n+1}{2} \le i \le n$

Here, $v_f(1) = v_f(0)+1$ for all n and
$e_f(0) = e_f(1)$ for all n.

Therefore, Globe Gl(n) : (n-odd) satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.
Hence, Globe Gl(n) : (n-odd) is Homo-Cordial Graph.
For example, the Homo-Cordial Labeling of Gl(5) is shown in figure 3.9

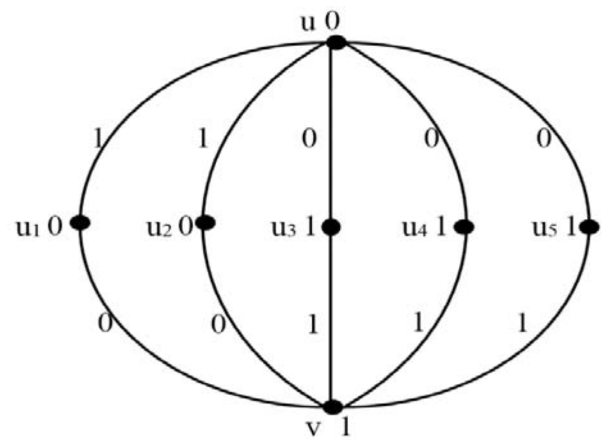


Fig 3.9: Gl(5)

Theorem: 3.10
Triangular snake $C_2(P_n)$ is Homo-Cordial Graph.

Proof
Let V($C_2(P_n)$) ={[u_i: 1≤i≤n+1], [v_i,w_i : 1≤i≤n]} and
E($C_2(P_n)$) = {[(u_iv_i) ∪ (u_iw_i) ∪ ($u_{i+1}v_i$) ∪ ($u_{i+1}w_i$) ∪(u_iu_{i+1}):1≤i≤n]}
Define f : V($C_2(P_n)$)→{0,1}.

The vertex labeling are,
$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1\le i\le n+1$
$f(v_i) = 0 \quad 1\le i\le n$
$f(w_i) = 1 \quad 1\le i\le n$

The induced edge labeling are,
$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1\le i\le n$
$f^*[(u_iv_i)] = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1\le i\le n$
$f^*[(u_iw_i)] = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1\le i\le n$
$f^*[(u_{i+1}v_i)] = \begin{cases} 0 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1\le i\le n$
$f^*[(u_{i+1}w_i)] = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1\le i\le n$

Therefore, Triangular snake $C_2(P_n)$ satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.
Hence, Triangular snake $C_2(P_n)$ is Homo-Cordial Graph.
For example, the Homo-Cordial Labeling of $C_2(P_5)$ and $C_2(P_4)$ are shown in figure 3.11 and figure 3.12 respectively.

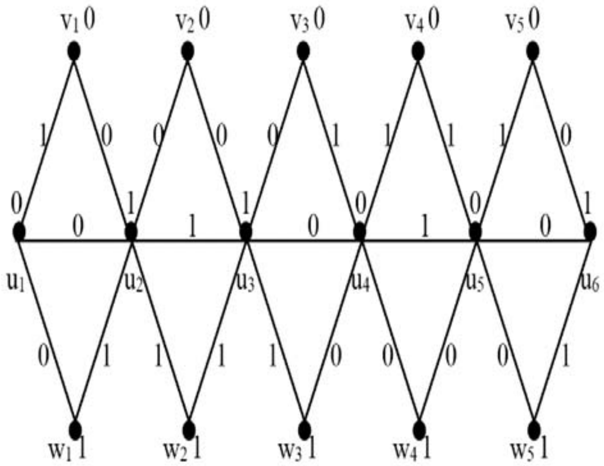


Fig 3.11: $C_2(P_5)$

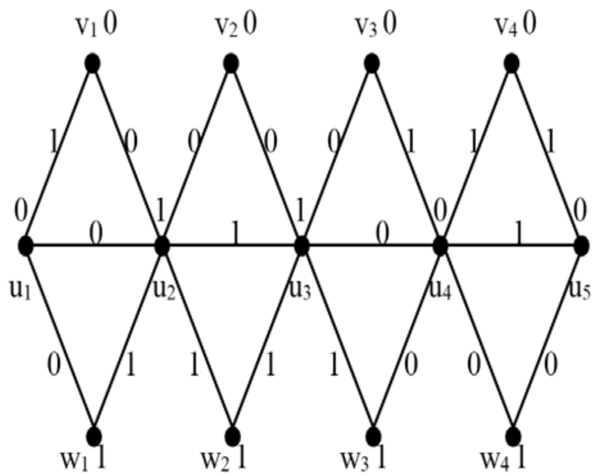


Fig 3.12: $C_2(P_4)$

Theorem: 3.13
Crown $C_n \Theta K_1$ (n-even) is Homo-Cordial Graph.

Proof
Let $V(C_n \Theta K_1) = \{[u_i, v_i : 1 \le i \le n]\}$ and
$E(C_n \Theta K_1) = \{[(u_i u_{i+1}): 1 \le i \le n-1] \cup [(u_1 u_n)] \cup [(u_i v_i): 1 \le i \le n]\}$.
Define $f: V(C_n \Theta K_1) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})]$	=	0	$1 \le i \le n-1$
$f^*[(u_1 u_n)]$	=	0	
$f^*[(u_i v_i)]$	=	1	$1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and
$e_f(0) = e_f(1)$ for all n.

Therefore, Crown $C_n \Theta K_1$ satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, Crown $C_n \Theta K_1$ (n-even) is Homo-Cordial Graph.
For example, the Homo-Cordial Labeling of $C_6 \Theta K_1$ is shown in figure 3.14

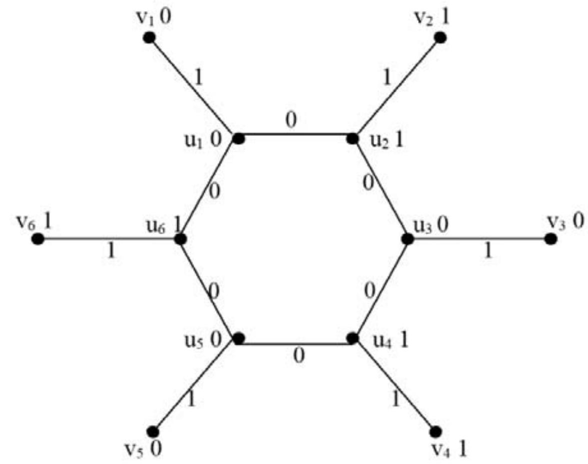


Fig 3.14: $C_6 \Theta K_1$

Theorem: 3.15
Crown $C_n \Theta K_1$(n-odd) is Homo-Cordial Graph.

Proof
Let $V(C_n \Theta K_1) = \{[u_i, v_i : 1 \le i \le n]\}$ and
$E(C_n \Theta K_1) = \{[(u_i u_{i+1}): 1 \le i \le n-1] \cup [(u_1 u_n)] \cup [(u_i v_i): 1 \le i \le n]\}$.
Define $f: V(C_n \Theta K_1) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u_i) \quad = \quad \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$

$f(v_1) \quad = \quad 1$

$f(v_i) \quad = \quad \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 2 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})]$	=	0	$1 \le i \le n-1$
$f^*[(u_1 u_n)]$	=	1	
$f^*[(u_1 v_1)]$	=	0	
$f^*[(u_i v_i)]$	=	1	$2 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and
$e_f(0) = e_f(1)$ for all n.

Therefore, Crown $C_n \Theta K_1$(n-odd) satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.
Hence, Crown $C_n \Theta K_1$ (n-odd) is Homo-Cordial Graph.
For example, the Homo-Cordial Labeling of $C_5 \Theta K_1$ is shown in figure 3.16

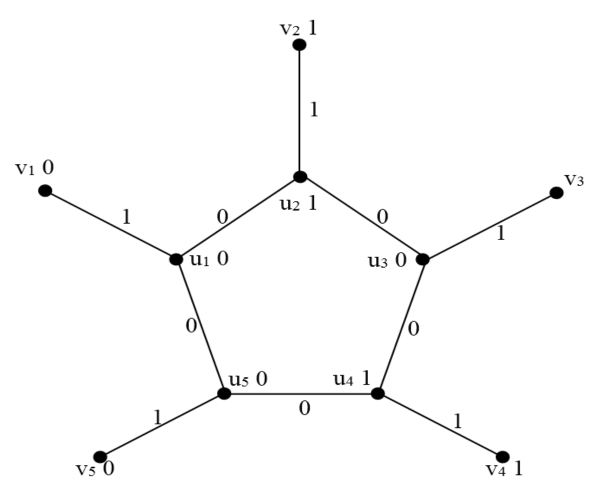


Fig 3.16: $C_5 \Theta K_1$

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