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Tree Related Relaxed Cordial Graphs

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Abstract
Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijection from V to {-1, 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$,<$K_{1,n}$: n>, $K_{1,2n}$XK_2,K_3*$K_{2,n}$, K_4*$K_{2,n}$ are Relaxed Cordial Graphs.

Keywords: Relaxed Cordial Labeling, Relaxed Cordial Graph. 2000 Mathematics Subject classification 05C78.

1. Introduction
A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e={uv} of vertices in E is called edges or a lineof G. In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, Subdivided star <$K_{1,n}$: n>, Book $K_{1,2n}$ X K_2,K_3*$K_{2,n}$, K_4*$K_{2,n}$are Relaxed Cordial Graphs. For graph theory terminology, we follow [2]

2. Preliminaries
Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijectionfrom V to {-1, 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.
The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph(RCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, Subdivided star<$K_{1,n}$: n>, Book $K_{1,2n}$ X K_2,K_3*$K_{2,n}$, K_4*$K_{2,n}$are Relaxed Cordial Graphs.

Definition: 2.1
A bipartite graph is a graph whose vertex set V(G) can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2 ; (V_1,V_2) is called a bipartition of G.If further, every vertex of V_1 is joined to all the vertices of V_2, then G is called a complete bipartite graph. The complete bipartite graph with bipartition (V_1, V_2) such that |V_1|=m and | V_2|=n is denoted by $K_{m, n.}$ A complete bipartite graph $K_{1, n}$ (or) $K_{n, 1}$ (or) S_n is called a star.

Definition: 2.2
The Bistar $B_{m, n}$ is a graph obtained from K_2 by identifying the centre of $K_{1, m}$ and $K_{1, n}$ at the end vertices of K_2 respectively.

Definition: 2.3
Subdivided star<$K_{1,n}$:n> is a graph obtained as one point union of n paths of path length 2.

Definition: 2.4
Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by D_2(G).

Definition: 2.5
Let Tr be any tree. Denote the tree obtained from T by considering two copies of T by adding an edge between them by T(2) and in general the graph obtained from T(n-1) and T by adding an edge between them is denoted by Tr(n).

3. Main Results

Theorem: 3.1

Star S_n, $(n \geq 2)$ is Relaxed Cordial Graph.

Proof:

Let the graph be a star S_n

Let $V(S_n) = \{u, [u_i : 1 \leq i \leq n]\}$

$E(S_n) = \{[(uu_i) : 1 \leq i \leq n]\}$

Define $f : V(G) \to \{-1, 0, 1\}$

The vertex labeling are

$f(u) = -1$

$f(u_i) = \begin{cases} 0 & i \equiv 1 (\mathrm{mod}\ 2) \\ 1 & i \equiv 0 (\mathrm{mod}\ 2) \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*[(u, u_i)] = \begin{cases} 1 & i \equiv 1 (\mathrm{mod}\ 2) \\ 0 & i \equiv 0 (\mathrm{mod}\ 2) \end{cases} \quad 1 \leq i \leq n$

And $e_f(0) = e_f(1)$

Therefore, the graph S_n satisfies this condition

$|e_f(0) - e_f(1)| \leq 1$

Hence, Star S_n is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of S_8 is shown in the figure 1

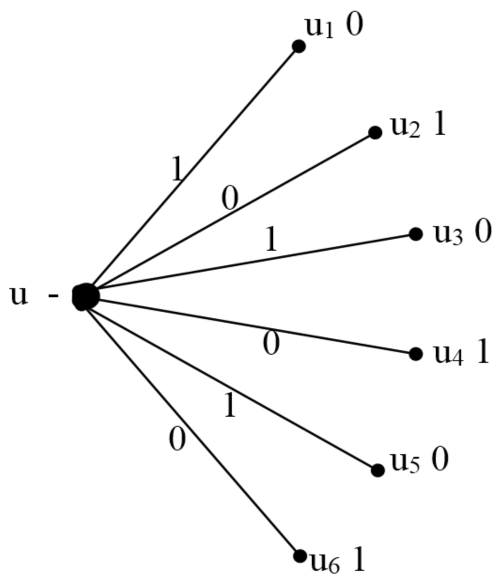

Fig 1: S_8

Theorem: 3.2

Bistar$B_{m, n}$, m, $n \geq 2$ is Relaxed Cordial Graph.

Proof:

Let the graph be Bistar$B_{m, n}$

Let $v(B_{m, n}) = \{[u_1, u_2, v_i, v_j\ 1 \leq i \leq m ; 1 \leq j \leq n]\}$

$E(B_{m, n}) = \{[(u_1u_2)] \cup [u_1v_i : 1 \leq i \leq m] \cup [u_2w_i : 1 \leq i \leq n]\}$

Define $f : V(G) \to \{-1, 0, 1\}$

The vertex labling are $f(u_i) = 0$ $i = 1, 2$

$f(v_i) = \begin{cases} 0 & i = 0 (\mathrm{mod}\ 2) \\ 1 & i = 0 (\mathrm{mod} 2) \quad 1 \leq i \leq n \\ -1 & i = 1 (\mathrm{mod}\ 2) \end{cases}$

$f(w_i) = \begin{cases} 0 & i = 0 (\mathrm{mod}\ 2) \\ 1 & i = 0 (\mathrm{mod} 2) \quad 1 \leq i \leq n \\ -1 & i = 1 (\mathrm{mod}\ 2) \end{cases}$

The induced edge labeling are

$f^*(u_1u_2) = 0$

$f^*(u_1, v_i) = \begin{cases} 0 & i \equiv 0 (\mathrm{mod}\ 2) \\ 1 & i \equiv 1 (\mathrm{mod}\ 2) \end{cases} \quad 1 \leq i \leq n$

$f^*(u_2, w_i) = \begin{cases} 0 & i \equiv 0 (\mathrm{mod}\ 2) \\ 1 & i \equiv 1 (\mathrm{mod}\ 2) \end{cases} \quad 1 \leq i \leq n$

Case 1:

When n=m

$| e_f(0) - e_f(1) | = 1$ if n, m are both even and odd

Case 2:

When n>m

$| e_f(0) - e_f(1) | = \begin{cases} 1 & if\ n - m\ is\ even \\ 0 & if\ n - m\ is\ odd \end{cases}$

Case 3:

When n<m

$| e_f(0) - e_f(1) | = \begin{cases} 1 & if m - n\ is\ even \\ 0 & if\ m - n\ is\ odd \end{cases}$

Therefore, the graph Bistar$B_{m, n}$ satisfies the condition

$| e_f(0) - e_f(1) | \leq 1$.

Hence, Bistar $B_{m, n}$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of bistar $B_{7, 7}$ is shown in the figure 18

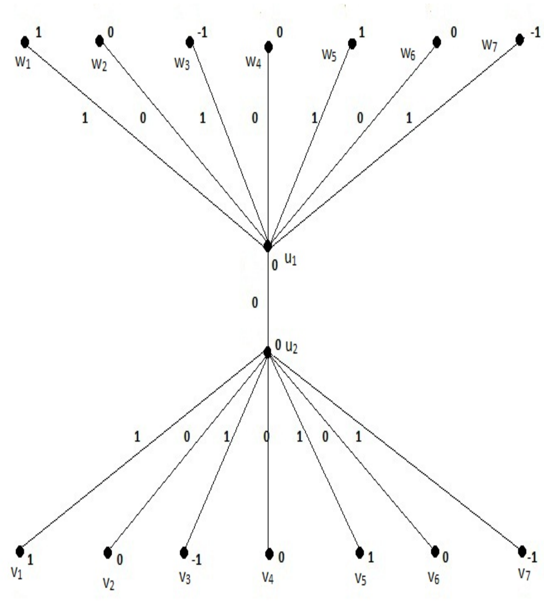

Fig 2: Bistar $B_{7, 7}$

Theorem: 3.3

$\langle K_{1, n} : n \rangle$ $n \geq 2$ is Relaxed Cordial Graph.

Proof:

Let G be $\langle K_{1, n} : n \rangle$

Let $V(G) = \{u, [v_i, w_i : 1 \leq i \leq n]\}$

Let $E(G) = \{[(uv_i) : 1 \leq i \leq n] \cup [(v_iw_i) : 1 \leq i \leq n]$

Define $f : V(G) \to \{-1, 0, 1\}$

The vertex labeling are

f(u) = 0

$f(v_i) = 1 \quad 1 \le i \le n$

$f(w_i) = -1 \quad 1 \le i \le n$

The vertex edge labeling are

$f^*(uv_i) = 1 \quad 1 \le i \le n$

$f^*(v_iw_i) = 0 \quad 1 \le i \le n$

And $e_f(0) = e_f(1)$

Therefore, the graph G satisfies the condition

$|e_f(0) - e_f(1)| = 1$

Hence, $<K_{1,n} : n>$ $(n \ge 2)$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $<K_{1,7} : 7>$ is shown in the figure 3

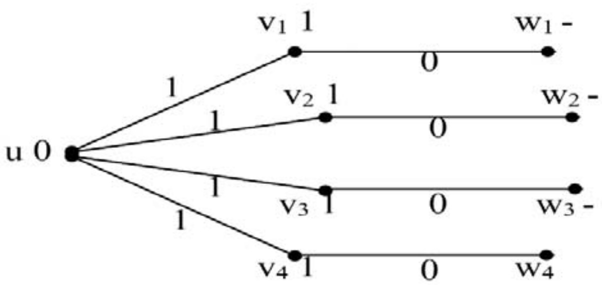

Fig 3: $<K_{1,7} : 7>$

Theorem: 3.4

$D_2(S_n)$ $(n \ge 2)$ is Relaxed Cordial Graph.

Proof:

Let the graph G be $D_2(S_n)$

Let $V(G) = \{u, v, v_i : 1 \le i \le n\}$

Let $E(G) = \{[(uv_i) : 1 \le i \le n]$ U $[(wv_i) : 1 \le i \le n]$

Define f : V(G) -> {-1, 0, 1}

The vertex labeling are

f(u) = 0

f(w) = -1

$f(v_i) = 1 \quad 1 \le i \le n$

The induced edge labeling are

$f^*(uv_i) = 1 \quad 1 \le i \le n$

$f^*(wv_i) = 0 \quad 1 \le i \le n$

and $|e_f(0) - e_f(1)| = 0$

Therefore, It satisfies the condition

$|e_f(0) - e_f(1)| \le 1$

Hence, the graph $D_2(S_n)$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $D_2(S_6)$ is shown in the figure 4

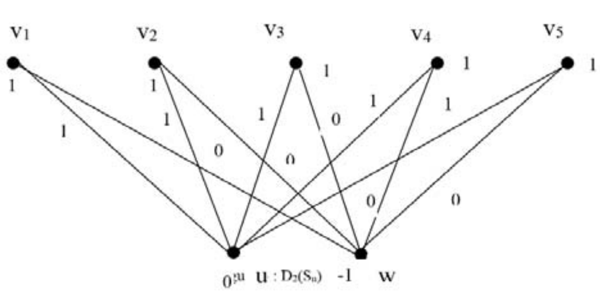

Fig 4: $D_2(S_n)$

Theorm: 3.5

Tr_n is relaxed cordial Graph Graph.

Proof:

Let the graph be Tr_n

Let $V(Tr_n) = \{[u_i: 1 \le i \le n], [u_{i1}, u_{i2}, v_i, w_i : 1 \le i \le n]\}$

Let $E(G) = \{[(u_iu_{i+1}) : 1 \le i \le n-1]$ U $[(u_iu_{i1}) : 1 \le i \le n]$ U$[(u_{i1}u_{i2}) : 1 \le i \le n]U[(u_{i1}v_i)U(u_{i2}w_i : 1 \le i \le n]$

Define f : $V(Tr_n)$ -> { -1, 0, 1}

The vertex labeling are

$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \le i \le n$

$f(u_{i1}) = -1 \quad 1 \le i \le n$

$f(u_{i2}) = 1 \quad 1 \le i \le n$

$f(v_i) = 1 \quad 1 \le i \le n$

$f(w_i) = 0 \quad 1 \le i \le n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 1 \quad 1 \le i \le n-1$

$f^*(u_iu_{i1}) = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \le i \le n$

$f^*(u_{i1}u_{i2}) = 0 \quad 1 \le i \le n$

$f^*(u_{i1}v_i) = 0 \quad 1 \le i \le n$

$f^*(u_{i2}w_i) = 1 \quad 1 \le i \le n$

$|e_f(0) - e_f(1)| = \begin{cases} 0 & if\ n\ is\ odd \\ 1 & if\ n\ is\ even \end{cases}$

Therefore, the graph Tr_n satisfies the condition

$|e_f(0) - e_f(1)| \le 1$.

Hence, Tr_n is Relaxed Cordial.

For example, the Relaxed Cordial labeling of Tr_7 is shown in the figure 5

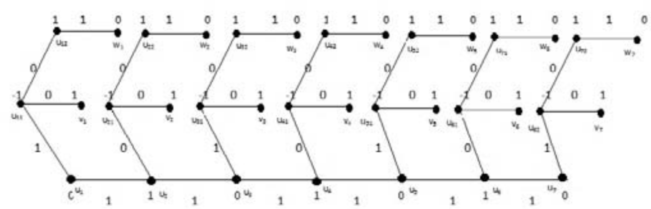

Figure 5: Tr_7

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