

## Teaching potentiation mediated by the digital game Tower of Hanoi

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### Abstract

According to the perspectives of the National Common Curricular Base (BNCC), digital technologies play an important role in terms of conceptions and relationships in the teaching-learning process. In this context, this article aims to present an experience report on the application of the digital game Tower of Hanoi, as a digital educational resource for the teaching of Mathematics, specifically the content of empowerment. The game Tower of Hanoi in its digital version was applied in a class of the 8th year, of Elementary School II, of a public school, located in the city of Denise-MT. In total, 25 students participated (identified as P1, P2, P3...P25). Data were qualitatively analyzed using semi-structured questionnaires. The results indicated that the students understood the rules of the game and were able to move the discs correctly, achieving success in the moves. In addition, it was possible to perceive that the Tower of Hanoi game enabled the development of skills and competences. It is expected that with the development of this methodological practice, through the use of Digital Technologies, there will be an effective contribution to the resignification and correlation of the mathematical contents with the objectives of the BNCC, in order to achieve the competencies and skills desired by the document.

**Keywords:** Teaching mathematics, digital games, tower of Hanoi, potentiation

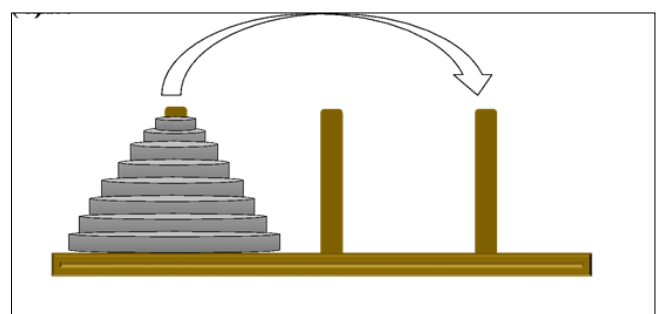
### Introduction

Humanity is marked by major technological transformations and is experiencing a historic moment. In which the widespread presence of electronic means of communication and information (ICTs) is providing students with multiple ways of relating to reality and their own learning. Thus, this work is justified by the growing demand for differentiated methodologies for teaching Science and Mathematics, which have the potential to transform relationships between the various protagonists in the educational scenario: student, teacher, institution, parents, among others (VALENTE, 1999, p. 67)<sup>[31]</sup>.

Currently in Brazil, competency-based teaching is an incessant search from the PCNs to the approval of the National Common Curricular Base (BNCC), which provides general competencies and specific skills for each curricular component (BRASIL, 2017<sup>[7]</sup>; VIEIRA, NICOLÓDI, DARROZ, 2021)<sup>[32]</sup>. From this perspective, the need for teaching based on competencies and a new teacher profile emerges (DIAS; FERREIRA; LUZ; MARINHO, 2021)<sup>[5]</sup>. For Prensky (2012)<sup>[22]</sup>, the great current challenge for educational institutions and teachers, specifically in Mathematics, is to ensure that the educational process follows the language of contemporary times. Gee (2007)<sup>[6]</sup> points out that educators need to implement the inclusion of games and new teaching technologies, thus enabling the use of contemporary language, with the aim of promoting dynamism in classes.

According to Laudares (2004)<sup>[14]</sup>, the use of interactive tools, such as digital games, brings enrichment to teaching, as they make Mathematics classes more dynamic and interesting, providing students with an approximation of everyday knowledge with scientific knowledge. Currently, students, defined as digital natives, end up demanding that teachers seek alternatives for teaching based on the development of skills and competencies, as recommended by the National Common Curricular Base (BNCC) (BRASIL, 2017)<sup>[7]</sup>.

In this context, the Tower of Hanoi game stands out, an interactive tool invented by the French mathematician Edouard Anatole Lucas, based on an Indian legend, and included in the third volume of his work *Récréations Mathématiques*, published in 1883. The manipulative game consists of a wooden base with three vertical rods attached, with eight wooden discs of decreasing diameters, drilled in the center (FIGURE 1), with significant costs and limitations on the number of discs. The jigsaw puzzle, in its digital format, offers users significant advantages over the manipulative version, since its digital version can be easily accessed and played on smartphones.



**Fig 1:** The manipulable Tower of Hanoi game. The objective of the game is to transport the discs from tower A to tower C, using the intermediate tower B

Given the potential use of the Tower of Hanoi game, with emphasis on its digital version to guide the development of skills and abilities recommended by the BNCC, the following question arises: What is the potential for using the Tower of Hanoi game as a digital educational resource in teaching Mathematics, with the aim of enriching the learning process of exponentiation and making it more meaningful?

Thus, this paper aims to report the experience of using the digital Tower of Hanoi game as an educational resource in

teaching Mathematics, with emphasis on the content of exponentiation. The activity was developed with an 8th grade class of Elementary School II, in a public school located in the municipality of Denise, state of Mato Grosso. The choice of this digital resource is justified by the need to make teaching Mathematics more dynamic, meaningful and attractive to students, exploring strategies that stimulate logical reasoning, problem-solving and the construction of knowledge through playfulness. In this context, the Tower of Hanoi proved to be an effective teaching tool for linking mathematical concepts with the development of cognitive skills, favoring the understanding of exponentiation in a concrete and contextualized way.

## Theoretical Framework

### 1. The use of physical games and virtual games in teaching Mathematics

According to Grando (2000)<sup>[9]</sup> and Huizinga (2007)<sup>[10]</sup>, the use of games in the classroom is a methodological support suitable for all levels of education, as long as their purpose is clear, the activity is challenging and appropriate to the level of learning of each student. In this conception, the authors Smole, Diniz and Candido (2000)<sup>[28]</sup> mention that this methodological resource can be used in the classroom to develop the ability to deal with information and create cultural meanings for mathematical concepts. Furthermore, as pointed out by Martins and Amaral (2021)<sup>[18]</sup>, digital games help to create a more inclusive learning environment, where students can learn at their own pace, make mistakes, try again and, with this, build knowledge in an autonomous and meaningful way. We can see that the aforementioned authors argue that teaching through games is a way for the educator to develop more interesting, relaxed and dynamic classes, being able to compete on equal terms with numerous resources that the student has access to outside the school environment. "The evidence seems to justify the importance that games have been assuming in proposals for teaching mathematics" (KISHIMOTO, 2005, p. 73)<sup>[13]</sup>.

According to Pietruchinski (2015), the development of educational games should explore the pedagogical, cognitive, psychomotor and motivational dimensions, so that they can be used as auxiliary elements in the teaching-learning processes. Tarouco (2004)<sup>[30]</sup> reinforces that digital educational games show excellent pedagogical resources capable of enhancing teaching-learning. The adequate use of digital technologies depends on the way they are being worked on in the classroom. And also on the teacher's objectives, to diagnose the difficulties of students in the conceptual fields, in order to seek solutions to learning problems in these technologies.

Recently, the use of digital technologies in teaching Mathematics has shown itself to be an essential resource, since the transformations in the production of knowledge, resulting from the technological advances we are experiencing, have been increasingly rapid. However, the use of digital games can be a challenge, especially when other technologies such as educational software are associated with this context. According to Borba and Villarreal (2005), a new intelligence technology results in a new collective that produces knowledge, which, in turn, is qualitatively different from the knowledge produced by other collectives.

For Souto and Borba (2016)<sup>[29]</sup>, digital technologies in education involve students in learning processes, and the school, whether or not it allows it, finds itself immersed in a

new daily routine in which it is possible to explore situations that are part of each student's life. In mathematics teaching, games are especially effective in promoting logical reasoning, problem-solving and the construction of abstract concepts. As stated by Borba and Penteadó (2001)<sup>[3]</sup>, the use of digital technologies allows a more investigative approach, where the student assumes an active role in their learning, exploring mathematical concepts through simulations and interactions. A digital resource that has stood out in this scenario is the Tower of Hanoi game, known for its application in the development of logical reasoning, planning and resolution strategies. Gomes *et al* (2022)<sup>[8]</sup> in a recent study, point out that digital games in the teaching of Mathematics can provide more significant learning experiences, especially when associated with problem solving and logical reasoning."

In this context, the cell phone can now be seen as an extension of our body, or as Souto and Borba (2016)<sup>[29]</sup> discuss, as holders of technologies that permeate the human being, transforming him/her. The smartphone, specifically, is a technology that has become part of several collectives of human-beings-with-media. The central concept of this construct is the notion of reciprocal molding, that is, the feedback given by a given media, which influences the reasoning of those who interact with it. In other words, according to Borba and Villarreal (2005)<sup>[4]</sup>, the media shapes the human being, but human beings also shape the media as they use it. It is from this perspective that digital technologies are included, contemplated in one of the general competencies of education.

Several researchers, including Borba and Penteadó (2002)<sup>[2]</sup>, Pais (2002)<sup>[20]</sup>, Scheffer (2002)<sup>[26]</sup>, Scheffer and Dallazen (2005/2006)<sup>[27]</sup> and Bairral (2005)<sup>[1]</sup>, indicate the use of technologies as allies in the teaching of Mathematics, as an integral part of the process of "discovery", encouraging understanding and meaning. Therefore, the role of the educator is to use digital technologies to promote teaching-learning situations. Since these pedagogical actions favor and challenge students, stimulating their creativity and providing them with paths to satisfactory learning.

### 2. The digital game Tower of Hanoi as an option for teaching Mathematics

Within the context of using a digital game in teaching Mathematics, the game Tower of Hanoi will be presented as a digital educational learning resource. Initially, it is necessary to know some aspects of the puzzle, both in its technical and playful nature. One digital resource that has stood out in this scenario is the Tower of Hanoi game, known for its application in the development of logical reasoning, planning and problem-solving strategies. According to Rodrigues (2017)<sup>[23]</sup>, the game can be an excellent teaching tool to introduce and explore concepts such as exponentiation and geometric progression, since the minimum number of moves required to solve the game follows the formula  $2^n - 1$ , where  $n$  represents the number of disks.

According to Machado (1995)<sup>[17]</sup>, the Tower of Hanoi is a very simple game that involves challenges with increasing degrees of difficulty and that can even be explored with the help of computers. Even for students who are not used to playing on smartphones and have questions about the gameplay, it is possible to check the step-by-step instructions, and even start a tutorial teaching the user how

to play. Regarding the technical data of the Tower of Hanoi game, it is compatible with Android (free) and iOS (paid), reasoning and strategy category, size 8.52 MB, age rating 5+ (developer Felipe Reigosa). Observe Figure 2, after performing 20 movements.

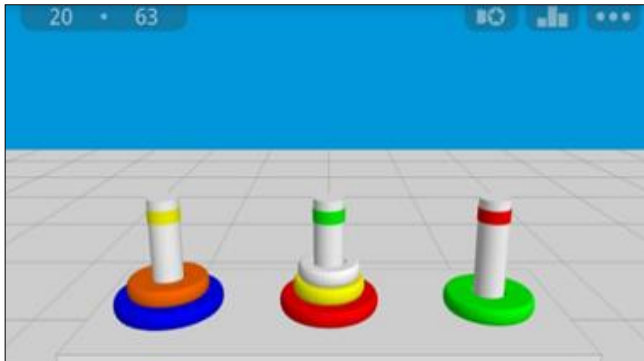


Fig 2: Screenshot of the Tower of Hanoi Game, after 20 moves

The challenge of this game is to move all the disks from one rod to another using an auxiliary rod and respecting the following rules: only one disk can be moved at a time, and a larger disk can never be positioned on a smaller disk. Many authors such as Kamii and Devries (1991) [12] and Grando (2000) [9] highlighted in their works the importance of using games in school as a means to favor the development and learning of mathematical concepts by students.

In this context, it is clear that the production of knowledge through digital technologies can mediate the development of skills and abilities by elementary school students, based on the proposals of the BNCC (VIEIRA, NICOLINI, DARROZ, 2021) [32]. It can be seen that the Tower of Hanoi game in its digital format meets what is proposed by Savi and Ulbricht (2008) [25], when it is observed that it is possible to teach disciplinary content through the game, in addition to promoting the development of skills and abilities related to the cognitive capacity of students, as proposed by the BNCC.

**Methodological Approach**

Methodologically, this research adopts a qualitative approach, the choice of which is justified by the

interpretative nature of the study, focused on understanding the experiences of participants in a real teaching context. According to Lincoln and Guba (1985) [15], qualitative research seeks direct involvement with participants, promoting a relationship of trust and credibility, which allows the researcher to capture meanings, perceptions and interpretations constructed in the environment investigated. This approach favors a more in-depth analysis of educational phenomena, by considering the sociocultural context and interactions that occur during the application of digital games, such as the Tower of Hanoi, in the classroom. Given this context, the work is an experience report.

The Tower of Hanoi game in its digital and manipulable version was applied in a public school, located in the city of Denise-MT. And the research subjects were 25 students, from the 8th grade of Elementary School II. The students were identified as P1, P2, P3... P25. It is worth mentioning that in this proposal we emphasized the digital version, encouraging the use of smartphones in the classroom. Most of the students had cell phones, and it was possible to see how each student was mobilizing the concept of exponentiation with the digital version of the Tower of Hanoi game. Above all, students who did not have smartphones were invited to sit with students who did have cell phones.

**1. Methodological dynamics of the proposed activities**

Initially, the students downloaded the application and some instructions were explained with the help of the multimedia projector and the manipulable Tower of Hanoi: how to move the pieces – the emergence of the game, and also mentioning the legend that surrounds it.

In order to promote reflections around the research question defined in the introduction of the work, an activity was carried out with the objective of relating the minimum number of moves with the concept of exponentiation. Considering that the students had already studied exponentiation in the previous grade, it can be introduced/deduced that the minimum number of moves in the game is  $2^n - 1$  and, for this, the student must know the exponentiation operation. After the activities, a semi-structured questionnaire was prepared to verify the students' acceptance of this teaching proposal. Table 1 summarizes all the moments of the methodological dynamics of the activity.

Table 1: Description of each moment of the implementation of the methodological dynamics adopted in the classes

Moment – Classes	Description of the Methodological Dynamics
1st moment – 2 classes	Introduction to the concept of exponentiation in the form of an expository class, with exercise resolution.
2nd moment – 2 classes	Student interaction with the Tower of Hanoi game, a manipulative version.
3rd moment – 2 classes	Initial discussion on the concept of exponentiation: to provide elementary school students with the opportunity to reflect and discuss the use of smartphones in the classroom and to show the possible relationships between mathematics and digital technologies.
3rd moment – 2 classes	After the presentation of the class and discussions on the use of digital technologies in the classroom, the entire mathematical concept of exponentiation was explored immediately afterwards, explaining the relationship between the Tower of Hanoi game and the concept of exponentiation.
4th moment – 4 classes	Systematization of mathematical knowledge – Exponentiation using the Tower of Hanoi game.
5th moment – 2 classes	Application of the semi-structured questionnaire.
6th moment – 2 classes	Discussion and application of the satisfaction survey of the Tower of Hanoi game

This research was conducted during regular class hours. Questions were asked to analyze students' views on the use of digital technologies in the production of mathematical knowledge, in addition to the semi-structured questionnaire. A satisfaction survey was conducted with the digital version

of the Tower of Hanoi game, with the aim of observing and evaluating whether the technology used allows students to acquire certain skills and abilities. The questions asked to the students are presented in Table 2, totaling seven questions.

**Table 2:** Questions asked to students during the workshops

Semi-structured questionnaire
1. What did you think of learning the concept of exponentiation through the Tower of Hanoi game? Justify your answer.
2. Which version of the game did you like playing, digital or manipulative? Why?
3. Mention at least one difference, if any, between learning math by playing on your smartphone and learning math using expository classes? Give your opinion.
4. Do you believe that math classes become more dynamic and interesting with the use of digital games? Justify your answer.
5. What is your biggest difficulty when relating the Tower of Hanoi game to the content of exponentiation?
6. Would you like to have other experiences using your smartphone to learn math? If so, justify your answer.
7. Survey on satisfaction with the digital version of the Tower of Hanoi game.

The semi-structured questionnaire used to obtain data had the consent of the students' guardians, since the students who participated in the workshops were minors. These differentiated activities, together with the questionnaire, lasted 12 hours and served as an instrument for data production: they were carried out during regular Math class hours with the authorization of the School Administration.

**Results And Discussion**

Considering the use of the game in the educational context, we will show below how it can be used to learn the concept of exponentiation with  $2^n-1$ , which can be proven by the principle of finite induction, when n disks can always be

removed with  $2^n-1$ . We will not demonstrate this here, as it is beyond the focus of the investigation in this article. The challenge of the Tower of Hanoi game is to move all the disks from one rod to another using an auxiliary rod and respecting the following rules: only one disk can be moved at a time, and a larger disk can never be placed on a smaller disk. The movements represented by the figure represent the movements that were used by the students in the activities developed in the classroom.

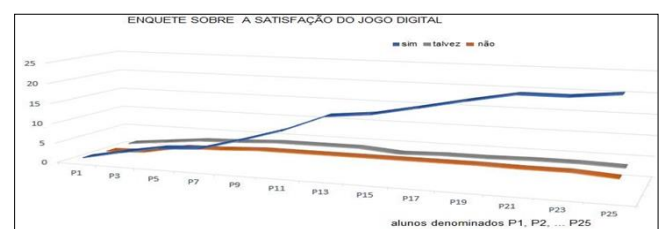
During the classes, the students were asked to solve a list of movements as shown in Table 3, using the Tower of Hanoi game. At this point, emphasis was given to the digital version, as previously reported.

**Table 3:** Activities from the students on exponentiation x Tower of Hanoi game

Number of pieces (n)	Number of moves for $n 2^n - 1$	Base	Exponent	Power
1	$2^1 - 1 = 1$	2	1	2
2	$2^2 - 1 = 2 \cdot 2 = 4 - 1 = 3$	2	2	4
3	$2^3 - 1 = 2 \cdot 2 \cdot 2 = 8 - 1 = 7$	2	3	8
4	$2^4 - 1 = 2 \cdot 2 \cdot 2 \cdot 2 = 16 - 1 = 15$	2	4	16
5	$2^5 - 1 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 - 1 = 31$	2	5	32
6	$2^6 - 1 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64 - 1 = 63$	2	6	64
7	$2^7 - 1 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128 - 1 = 127$	2	7	128
8	$2^8 - 1 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256 - 1 = 255$	2	8	256

It is worth noting that these activities took place in a procedural manner, making it difficult to assess within the context of the students' skills and abilities. According to Rodrigues (2017) [23], the game can be an excellent teaching tool to introduce and explore concepts such as exponentiation and geometric progression, since the minimum number of moves required to solve the game follows the formula  $2^n-1$ , where n represents the number of disks. It was observed that the assessment of knowledge related to the subject became easier in this way. This fact opens space for discussion about which is the best teaching method to be chosen, when there is no consensus on the correct way to assess, according to Perrenoud (1999) [21]. Given the above, the need to apply assessment procedures is evident, based on the context and learning conditions, using tools and resources that can guide teaching based on the skills and abilities sanctioned by the BNCC, with the use of digital technologies to improve the performance of the school, educators and students (BRASIL, 2017) [7]. In this context, Nakamura and Csikszentmihalyi (2002) [19] report that there is a need for a balance between the challenges generated in the activities and the skill of those who are executing them, so as not to generate anxiety due to

difficulty and not to cause boredom due to ease. As mentioned by (P2) and (P24) in different situations in the game, when they were asked about the use of digital games in Mathematics classes: whether these facts make them more interesting and dynamic. The majority indicated that they approve of this pedagogical resource, as shown in Figure 3.



**Fig 3:** Satisfaction survey with the digital Tower of Hanoi game by students

Some studies indicate that technology promotes engagement, active learning and playfulness, and that new assessments are needed to verify whether the student has in fact acquired the proposed skills (SANTOS; FRANÇA;

SILVA, 2020)<sup>[24]</sup>. It was observed that the application of the Tower of Hanoi game as a digital teaching resource not only sparked the interest of students, but also enabled concrete advances in their logical and mathematical thinking structure.

In particular, the game proved to be effective in understanding the content of exponentiation, since its operating logic is directly related to the mathematical formula  $2^n - 1$ , where  $n$  represents the number of disks. In this way, students were led to apply and understand, in a practical way, the concept of powers of base 2 when solving the proposed challenge, which facilitated the abstraction and internalization of the content.

It is clear that most students demonstrated great appreciation for the use of the digital game in the classroom, which favored the learning environment and the motivation to explore mathematical content. According to Macedo, Petty and Passos (2000)<sup>[16]</sup>, this playful approach allows the transfer of cognitive strategies developed in the game to other learning situations, strengthening planning, anticipation and problem-solving skills. The observed data indicate that the students understood the rules of the game and applied them correctly, moving the discs successfully, which also reflected in the assimilation of the concept of exponentiation in a meaningful and contextualized way.

Given the conjecture of using digital technologies to guide the development of skills and abilities provided for in the National Common Curricular Base (BRASIL, 2017)<sup>[7]</sup>, the games arouse greater interest in the content studied in students, as they provide them with greater interaction.

When asked whether the construction of Table 3 had helped them better understand the rules of the game and discover the secret to playing well, it was reported that making the calculations in advance helped students not waste time and moves, and in this perspective, a significant number of students mentioned the need to think calmly and mentally make possible moves. In line with what Prensky (2001) highlights about the potential of digital games, educational digital games can combine entertainment and learning, promoting engagement and the development of cognitive skills.

At this point, it is clear that logical mathematical reasoning and strategy are relevant factors in this game, as well as the establishment of a pattern or sequence of algorithms. We can better understand this perception of the digital game through some of the students' answers.

"With the game, you don't have to do math in your notebook; it's more fun to learn math on your cell phone" (P7).

"When you do all the movements correctly, the math is correct, but when you put the piece in the wrong place, the math is wrong" (P1).

"It's cool because we train our minds more to try not to put the big piece on top of the small one, but always correctly. If we make any mistake, the number of moves increases and so does the chance of losing." (P17).

At the end of the activity, it was observed that students with significant difficulties regarding mathematical concepts and algorithms performed well in the game, overcoming their initial difficulties. From the moment the student is in contact with the virtual game Tower of Hanoi within a school space, it is expected that the concept that was previously considered spontaneous will acquire a greater degree of abstraction and logical-mathematical reasoning.

Furthermore, as Martins and Amaral (2021)<sup>[18]</sup> point out, digital games help to create a more inclusive learning environment, where students can learn at their own pace, make mistakes, try again and, with this, build knowledge in an autonomous and meaningful way.

Educational games used appropriately support the activities of the teacher who wants to awaken the investigative spirit of their students. With the exponential advancement of digital technologies, as previously mentioned, education systems have rejected the concept of education based on content and memorization, taking into account that this is not the only function of the school, that is, it is necessary to form citizens with a critical view of the world.

### Final Considerations

The inclusion of the Tower of Hanoi game in the teaching-learning process allowed us to work on skills and abilities achieved through the use of the game, in accordance with the National Common Curricular Base. With the use of mathematical processes and tools, including available digital technologies, to model and solve everyday, social and other knowledge area problems, validating strategies and results. Returning to the research question discussed here: "What is the potential for using the Tower of Hanoi game as a digital educational resource in teaching Mathematics?" It is clear that the digital technology used during the research process contributed to the development of skills provided for in the BNCC. It is worth noting, above all, that the BNCC itself requires assessment methods that assist educators in this research process, and thus, support and guarantee the success of teaching development, pointing out more efficient solutions. The game stands out for its pedagogical potential and playfulness, especially in its digital version, which can be accessed on smartphones by users of the *Android (free)* and *iOS (paid)* operating systems.

The game has the potential to develop logical reasoning and this proposal fits into the Mathematics curricular component, and can be used to address various mathematical contents, from early grades to higher education. In this work, the object of contextualized knowledge was exponentiation: from a series of attempts, produced in the construction of a movement relationship during the game, it was possible to deduce that the minimum number of movements in the game is  $2^n - 1$  and, therefore, the paths taken so far make us reflect on the importance of games, specifically digital games, in teaching Mathematics for the development of skills and abilities proposed by the BNCC. Concluding the game here, the use of the Tower of Hanoi as a learning object represents an innovative and effective strategy for teaching exponentiation in Elementary School, promoting not only the learning of mathematical content, but also the development of essential cognitive skills in the educational process.

### Acknowledgments

MCDN would like to thank the "Colégio de Aplicação Pedagógica" (UEM) for the activities developed for students with advanced abilities and for the activities developed at the Planetariums.

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