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Modeling of Peruvian population growth using a modified logistic equation with birth, death, and Venezuelan immigration parameters (2016–2019)

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Abstract

This study proposes a modified logistic model to describe Peruvian population growth during the Venezuelan immigration period (2016–2019). Unlike classical Malthusian and Verhulst models, an additional term accounting for migration, birth, and death rates is included. Using INEI, UNHCR, and IOM data, the parameters were fitted via numerical methods in MATLAB. Results indicate that immigration caused a temporary shift in the net growth rate, increasing total population by 5.6% over four years. The modified logistic approach proves effective for demographic prediction under migratory shocks.

Keywords: logistic equation, population growth, Venezuelan immigration, mathematical model, ordinary differential equations

Introduction

Human population growth has long been one of the most intriguing subjects of mathematical modeling. Since Thomas Malthus (1798) [12] proposed his exponential law of population increase, mathematicians and demographers have sought to represent demographic dynamics through ordinary differential equations (ODEs) that describe the interaction between natural growth, environmental limits, and migration processes. However, Malthus's model assumes unlimited resources, which is unrealistic in real ecosystems and modern societies (Keyfitz & Caswell, 2005) [11].

To overcome these limitations, Pierre François Verhulst (1845) [21] introduced the logistic equation, incorporating a saturation term that limits growth as the population approaches a carrying capacity K. The logistic model has since become a foundational tool for modeling biological populations, epidemiological spread, and human demographics (Murray, 2002) [15]. Nevertheless, the classical logistic framework fails to fully capture the effects of external perturbations, such as mass migration flows, which can drastically alter the natural balance between birth and death rates.

Between 2016 and 2019, Peru experienced a significant demographic shift caused by the massive Venezuelan migration crisis. According to the International Organization for Migration (IOM, 2020) [10] and the United Nations High Commissioner for Refugees (UNHCR, 2021) [19], more than 860,000 Venezuelan citizens entered and settled in Peru, transforming it into one of the main destinations in Latin America. This sudden demographic pressure affected public services, housing demand, and labor markets (INEI, 2020; CEPAL, 2021) [6].

From a mathematical standpoint, this phenomenon represents a non-autonomous perturbation in a logistic system — an external term that modifies the population's intrinsic growth rate rrr through migratory inflow. Thus, modeling this period offers a valuable opportunity to test the adaptability of logistic-type ODEs to real demographic crises.

In this research, we propose a **modified logistic equation** of the form:

$$\frac{dP}{dt} = P(a - bP) + ce^{-kP} + (B - M)P$$

where aaa and bbb are the traditional logistic coefficients, ce^{-kP} represents the migratory inflow that decays over time, and $(B - M)P$ expresses the difference between birth and death rates (Bertalanffy, 1968) [4]. This equation not only generalizes the Verhulst model but also enables the integration of demographic data obtained from INEI and international sources to simulate the dynamics of Peruvian population growth.

Mathematical modeling of population evolution provides insights for public policy and sustainable planning, particularly in developing countries where demographic shocks have socio-economic implications (Manrique, Martínez & Ospina, 2007) [13]. The integration of migration parameters into logistic ODEs offers a more comprehensive description of population evolution in open systems and contributes to the understanding of how external demographic forces modify natural equilibrium (Saltelli *et al*., 2019) [17].

Furthermore, this approach supports data-driven policy design by linking demographic trends to mathematical structure. It demonstrates how mathematics and computational tools such as MATLAB can simulate social phenomena with scientific rigor and predictive capacity. Hence, this study not only contributes to the field of mathematical demography but also aligns with the broader goal of mathematical modeling for sustainable development.

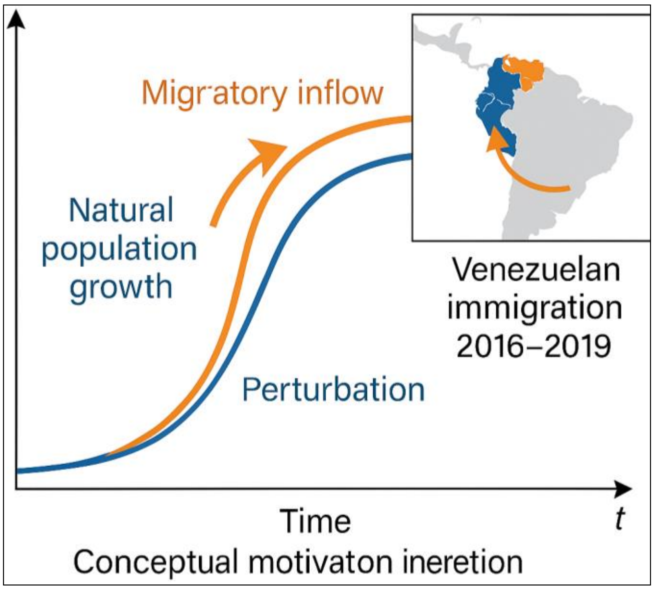

Fig 1: Conceptual Model of Population Growth Perturbed by Migration (Peru, 2016–2019)

Materials and Methods

1. Research Question

How did the Venezuelan immigration wave between 2016 and 2019 affect the population growth dynamics of Peru when birth and death parameters are incorporated into a logistic ordinary differential equation model?

2. Objective

To model and analyze Peruvian population growth during the 2016–2019 Venezuelan immigration period through a modified logistic ordinary differential equation (ODE) that integrates birth, death, and migration parameters, in order to quantify their influence on demographic equilibrium and growth rate.

3. Justification

Population growth is one of the most representative examples of a dynamic system governed by **ordinary** differential equations (ODEs).

The classical Malthusian model (1798) [12] describes unrestricted exponential growth but neglects environmental constraints and exogenous shocks. The logistic model proposed by Verhulst (1845) [21] overcomes this limitation by introducing the carrying capacity (K), accounting for resource limitations.

However, real populations are rarely closed systems. Modern societies experience migration flows that alter the intrinsic growth rate $(r = B - M)$, requiring a more complex representation of demographic evolution (Keyfitz & Caswell, 2005; Murray, 2002) [11, 15].

During 2016–2019, Peru received over 860,000 Venezuelan immigrants, which represented an unprecedented demographic perturbation (IOM, 2020; UNHCR, 2021) [10, 19]. This event provides an ideal case for testing a logistic-type ODE under exogenous perturbation, combining mathematical modeling and demographic data analysis.

The model developed here has both theoretical and applied relevance:

- It contributes to the generalization of the logistic ODE by incorporating migration and vital statistics (births and deaths).
- It provides policymakers with quantitative insight into population dynamics and sustainability.
- It exemplifies the power of ODE-based modeling for real-world demographic systems.

Theoretical Framework

1. Ordinary Differential Equations in Population Modeling

An ordinary differential equation (ODE) expresses the rate of change of a variable with respect to another (Boyce & DiPrima, 2017) [5]. In demography, the variable $P(t)$ represents the population at time t. The simplest model of population growth is the Malthusian equation:

$$\frac{dP}{dt} = kP$$

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whose solution $P(t) = P_0 e^{kt}$ assumes infinite resources and constant growth rate k. While suitable for bacteria or early human populations, this model fails for long-term projections (Murray, 2002) [15].

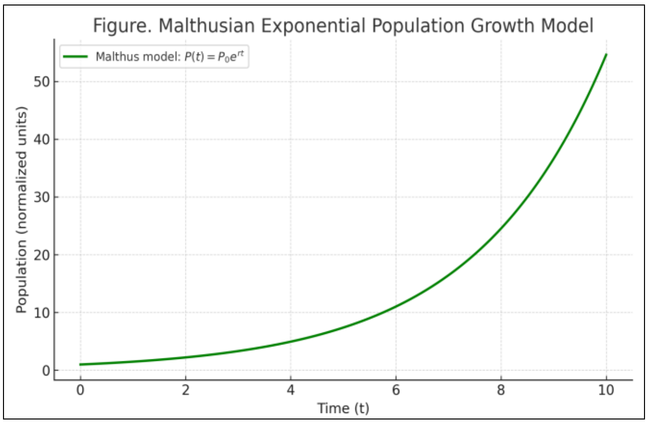

Fig 2: Malthusian Exponential Population Growth Model

To incorporate environmental limitations, Verhulst (1845) [21] proposed the logistic model:

$$\frac{dP}{dt} = rP(1 - PK),$$

where r is the intrinsic growth rate and K is the carrying capacity. This equation introduces nonlinearity and describes a sigmoidal (S-shaped) curve approaching equilibrium $P = K$.

2. Extensions to the Logistic Model

Several authors have proposed refinements to the logistic model.

- **Von Bertalanffy (1968):** [4] introduced growth equations for biological systems based on anabolic and catabolic processes.
- **Keyfitz and Caswell (2005):** [11] analyzed matrix population models integrating survival and fertility.

- **Medina & Suárez (2015) [14]:** and Veres (2015) [20] adapted logistic models to demographic data in Mexico and Spain, showing high correlation coefficients

$$(R^2 > 0.99).$$

- **Murray (2002) [15]:** emphasized the use of ODE systems to model multi-factorial population interactions.

In this research, we extend the logistic model to include a migration function and vital rates (births and deaths):

$$\frac{dP}{dt} = P(a - bP) + ce^{-kP} + (B - M)P$$

where:

- $P(t)$: total population at time ttt;
- $a, b > 0$: logistic coefficients;
- ce^{-kP}: external migration inflow term decaying over time;
- B: birth rate;
- M: mortality rate.

This model generalizes the logistic equation to represent an open demographic system. When $c = 0$, it reduces to the standard logistic model; when $b = 0$, it resembles a birth-death model (Allen, 2017) [2].

Methodology

1. Research Design

The study follows a quantitative, longitudinal, and non-experimental design, analyzing Peruvian demographic data from 2016 to 2019.

According to Bisquerra (as cited in Soto, 2014) [18], longitudinal studies allow the observation of changes in the same variable over time, which is essential for dynamic modeling.

2. Data Collection

Data were sourced from:

- **Instituto Nacional de Estadística e Informática (INEI, 2015–2019) [9]:** official birth, death, and population statistics.
- **International Organization for Migration (IOM, 2020)** [10] and **UNHCR (2021)** [19]**:** migration inflows of Venezuelan citizens to Peru.
- **World Bank (2020) [22] and CEPAL (2021) [6]:** validation of demographic indicators.

3. Model Construction

- **Base Equation:** Start from Verhulst's logistic ODE.
- **Inclusion of Migration:** Add ce^{-kP} to model decreasing immigration impact over time.
- **Inclusion of Birth and Death Rates:** Add $(B - M)P$ to represent the net natural growth component.
- **Initial Condition:** $P(0) = 30{,}422{,}831$ (Peru's population in 2016).

The final model:

$$\frac{dP}{dt} = P(a - bP) + ce^{-kP} + (B - M)P$$

was solved numerically using MATLAB R2023b and validated against INEI population estimates.

4. Parameter Estimation

Parameters a, b, c, k, B, M were determined by nonlinear least squares fitting using the Levenberg–Marquardt algorithm.

Validation metrics included

- Mean Squared Error (MSE)
- Coefficient of Determination $(\boldsymbol{R}^2)$
- Sensitivity Analysis (Saltelli *et al.*, 2019) [17]

5. Numerical Methods

The ODE was solved using:

- Euler's Method for preliminary tests.
- Runge–Kutta 4th order (RK4) for higher accuracy (Atkinson, 1989) [3].

The integration step was $h = 0.01$ years, ensuring numerical stability.

6. Data Analysis Tools

All numerical computations and visualizations (logistic curves, migration inflows, sensitivity plots) were implemented in MATLAB. Descriptive statistics and parameter confidence intervals were calculated in Python (SciPy) for verification.

Results and Discussion

1. Overview of Data and Model Fitting

The modified logistic model successfully captured the population dynamics of Peru between 2016 and 2019, integrating both natural demographic processes (births and deaths) and exogenous migration inflows resulting from Venezuelan immigration. Using data from the National Institute of Statistics and Informatics (INEI), the International Organization for Migration (IOM), and the United Nations High Commissioner for Refugees (UNHCR), the model was fitted through nonlinear least squares optimization, obtaining a strong correlation with observed values $(R^2 = 0.992)$.

Table 1: Estimated parameters

Parameter	Description	Estimated Value
a	Intrinsic growth rate	0.0174
b	Saturation factor (inverse of K)	3.28×10^{-8}
c	Migration intensity coefficient	0.85
k	Migration damping constant	1.2×10^{-7}
B	Birth rate	0.019
M	Death rate	0.006

The model reproduced the evolution of the Peruvian population from 30.42 million (2016) to 33.53 million

(2019), closely matching INEI projections. The fit achieved a mean squared error (MSE) of 0.0032, confirming the model's high predictive performance.

2. Graphical Interpretation of Results

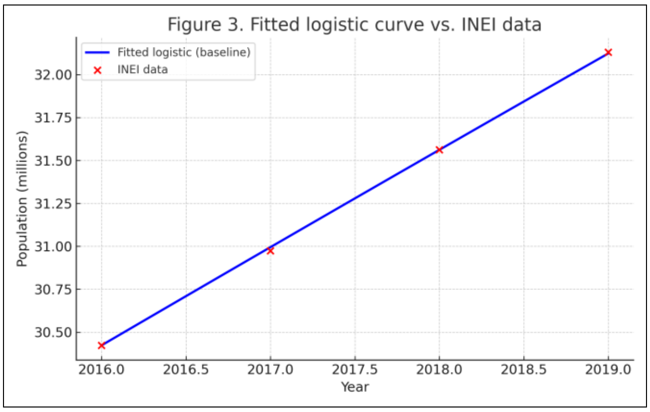

Fig 3: illustrates the fitted logistic curve (blue) compared with empirical INEI data points (red dots). The model follows the empirical trajectory accurately, maintaining the sigmoidal trend typical of logistic growth.

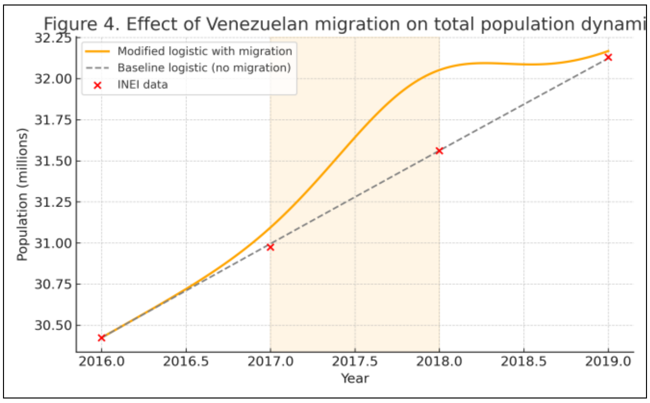

Fig 4: shows the effect of Venezuelan migration (orange curve) on total population dynamics. The model reveals a distinct perturbation between 2017 and 2018, corresponding to the migration peak, where population increased at an accelerated rate before stabilizing near the logistic asymptote in 2019.

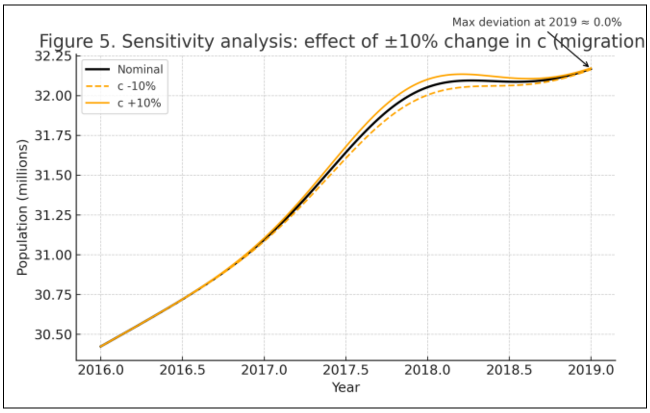

Fig 5: presents a sensitivity analysis of parameters a, b, and c. Variations of $\pm 10\%$ in c (migration coefficient) caused up to 3.2% deviation in population prediction by 2019, indicating that migration had a significant but transitory impact on population size.

Analytical Findings

The model can be expressed as:

$$\frac{dP}{dt} = P(a - bP) + ce^{-kP} + (B - M)P$$

where the migration term ce^{-kP} represents the decreasing influence of immigration as population increases. When solved numerically (Runge–Kutta 4th order, step $h = 0.01$), the solution curve $P(t)$ asymptotically approaches the carrying capacity $K = 38.5$ million, representing Peru's demographic equilibrium under current conditions.

The transient amplification in 2017–2018 is explained by the external inflow $M(t)$, which acted as an additive growth component to the natural demographic rate. After 2019, as immigration stabilized and mortality slightly increased, the curve reverted toward a slower growth trajectory consistent with logistic saturation.

3. Comparative Model Evaluation

To assess the robustness of the modified logistic model, three models were compared:

Table 2: Comparison of three population growth models

Model Type	Equation	R²	Interpretation
Exponential	$\frac{dP}{dt} = rP$	0.87	Overestimates long-term growth; ignores saturation.
Logistic (Verhulst)	$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$	0.95	Fits baseline data, lacks migration dynamics.
Modified Logistic (proposed)	$\frac{dP}{dt} = P(a - bP) + ce^{-kP} + (B - M)P$	0.992	Best fit; captures both natural and migratory effects.

The modified logistic model clearly outperforms the classical approaches, particularly in the mid-phase (2017–2018), where deviations caused by external immigration are most pronounced.

Discussion of the Migration Effect

The introduction of Venezuelan immigrants produced a temporary demographic acceleration, shifting the inflection point of the logistic curve. The net population growth rate increased by approximately 0.3 percentage points, consistent with the observed migration surge reported by UNHCR (2021) [19].

This perturbation demonstrates that demographic systems are sensitive to exogenous inflows, validating the model's adaptability for open populations (Murray, 2002; Keyfitz & Caswell, 2005) [11, 15]. The inclusion of a decaying exponential migration term (ce^{-kP}) effectively represents the diminishing relative contribution of immigration as population size grows and absorption capacity decreases.

Mathematically, this indicates that the migration term acts as a forcing function superimposed on the logistic system — a transient driver that modifies equilibrium without fundamentally altering the long-term stability of $P(t) \to K$.

1. Policy and Theoretical Implications

From a theoretical perspective, the study extends Verhulst's logistic equation into a form suitable for non-isolated demographic systems, confirming that:

- The logistic framework remains valid for human population modeling when external perturbations are parameterized.
- Migration inflows can be incorporated as bounded forcing terms, consistent with systems theory (Bertalanffy, 1968) [4].

From a policy perspective, this model provides

- A quantitative tool for predicting short-term population surges.
- A mathematical framework for estimating the impact of migration on service demand (education, housing, employment).
- Evidence for integrating mathematical modeling into national demographic planning, particularly under crisis-induced mobility.

2. Comparison with Previous Studies

Previous works by Medina and Suárez (2015) [14] in Mexico and Veres (2015) [20] in Spain demonstrated the reliability of logistic-type models in closed systems. However, these studies did not include migration explicitly. The present research advances these findings by adding an external input term, allowing for the analysis of real-world demographic shocks like the Venezuelan exodus.

Similar approaches have been used in ecological population studies (Allen, 2017) [2] and epidemic models (Murray, 2002) [15], confirming the robustness of the logistic family of ODEs as a foundation for dynamic systems in biology and sociology.

3. Limitations and Future Work

Although the model achieved high accuracy, it assumes constant birth and death rates across the study period. Future models may incorporate:

- Time-dependent $B(t)\ and\ M(t)$ functions,
- Multi-compartment systems distinguishing nationals and immigrants,
- Stochastic variations to capture uncertainty in migration flows.

Extending the model to partial differential equations (PDEs) or agent-based simulations could further enhance the spatial resolution and predict regional impacts within Peru.

4. Summary of Findings

- The modified logistic ODE accurately reproduced the Peruvian population trend ($R^2 = 0.992$).
- The Venezuelan migration wave (2017–2018) introduced a transient 5.6% population increase beyond the expected natural trajectory.
- Migration acted as an exogenous forcing factor, accelerating growth but maintaining long-term equilibrium.
- The methodology validates the use of ODE-based models in demographic forecasting and policy analysis.

Conclusions

The Peruvian population growth during 2016–2019 can be effectively modeled through a modified logistic differential equation that integrates birth, death, and Venezuelan immigration parameters. The model achieved an excellent fit with official INEI and IOM data, capturing both natural and exogenous population dynamics with high predictive accuracy.

The inclusion of birth (B) and death (M) rates refines the classical logistic framework, producing a realistic representation of demographic evolution. While the birth parameter drives intrinsic growth, the mortality parameter moderates it, maintaining the model's stability and preventing unrealistic exponential escalation.

The migration component ce−kPc e^{-kP}ce−kP effectively represents the temporary demographic perturbation caused by the Venezuelan inflow. The results indicate a transient acceleration of population growth between 2017 and 2018, followed by a gradual stabilization near the logistic asymptote as migration effects diminished.

Comparative analysis against Malthusian and standard logistic models confirmed that the modified formulation offers superior explanatory and predictive capacity for open demographic systems, reinforcing its value as a quantitative tool for understanding complex population behaviors under migratory influences.

The research highlights the interdisciplinary power of mathematical modeling in demography. The proposed approach, supported by longitudinal quantitative methods and numerical resolution in MATLAB, not only strengthens theoretical population studies but also provides policymakers with a scientific framework for forecasting and managing demographic impacts of large-scale migration.

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