International Journal of Multidisciplinary Research and Development
www.allsubjectjournal.com
ISSN Online: 2349-4182 ISSN Print: 2349-5979
Received: 27-08-2025, Accepted: 26-09-2025, Published: 11-10-2025
Volume 12, Issue 10, 2025, Page No. 63-70

Enhanced oil reservoir simulation using an improved black oil model and modified conjugate gradient algorithm

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Abstract
This paper proposed an enhanced oil reservoir simulation framework that integrates an improved Black Oil model with a modified Conjugate Gradient algorithm to address the limitations of conventional approaches in capturing complex fluid dynamics and achieving efficient numerical solutions. The improved Black Oil model incorporates more accurate representations of phase behavior and interactions, thereby refining the predictions of pressure and saturation distributions in heterogeneous reservoirs. The modified Conjugate Gradient algorithm is designed to accelerate convergence and stabilize the solution process for the large, sparse linear systems arising from the discretized model equations. Our methodology combines these advancements to achieve higher computational efficiency without sacrificing physical fidelity, which is critical for practical reservoir management and decision-making. The proposed framework demonstrates significant improvements in simulation accuracy and performance, as validated through comparative studies with existing methods. Furthermore, the adaptability of the modified solver makes it suitable for a wide range of reservoir conditions, including those with strong nonlinearities and multiphase flow effects. This work contributes to the ongoing efforts in reservoir engineering by providing a robust tool for optimizing production strategies and reducing uncertainties in reservoir characterization. The results highlight the potential of our approach to enhance the reliability of reservoir simulations, ultimately supporting more informed field development planning and operational efficiency.

Keywords: Enhanced oil reservoir simulation, improved black oil model, modified conjugate gradient algorithm, computational efficiency, reservoir characterization

Introduction

Oil reservoir simulation is a critical tool for understanding fluid flow dynamics in subsurface formations and optimizing hydrocarbon recovery strategies. The Black Oil model, a simplified yet widely adopted approach, describes multiphase flow (oil, gas, and water) through porous media under varying pressure and saturation conditions [1]. Despite its widespread use, the conventional Black Oil model exhibits limitations in capturing complex physical phenomena such as non-Darcy flow [2] and compositional effects [3]. These shortcomings often lead to inaccuracies in predicting reservoir performance, particularly in heterogeneous or tight formations.

To address these challenges, recent advancements have focused on refining the Black Oil model by incorporating additional physical mechanisms and improving numerical solvers. For instance, modifications to account for near-miscible conditions [4] and capillary pressure effects [5] have demonstrated improved accuracy. However, these enhancements often come at the cost of increased computational complexity, necessitating more efficient numerical methods. The Conjugate Gradient (CG) algorithm, a staple for solving large linear systems in reservoir simulation [6], has been adapted in various forms, such as the Preconditioned Conjugate Gradient (PCG) method [7], to mitigate convergence issues. Nevertheless, existing CG variants still struggle with ill-conditioned systems arising from highly heterogeneous reservoirs or strong nonlinear couplings.

In this work, we propose an integrated framework that combines an improved Black Oil model with a modified Conjugate Gradient algorithm to enhance both the physical fidelity and computational efficiency of reservoir simulations. The improved Black Oil model introduces a more rigorous treatment of phase behavior and interphase interactions, enabling better predictions of fluid distributions under diverse reservoir conditions. Concurrently, the modified CG algorithm incorporates adaptive preconditioning and stabilization techniques to accelerate convergence while maintaining robustness. This dual advancement distinguishes our approach from prior efforts that either focus solely on model refinement [8] or solver optimization [9].

The key contributions of this work are threefold. First, we present a systematically enhanced Black Oil model that integrates additional physical constraints without compromising computational tractability. Second, we develop a modified CG algorithm tailored for the sparse, asymmetric linear systems prevalent in reservoir simulation, achieving faster convergence rates compared to conventional methods. Third, we validate the proposed framework through extensive numerical experiments, demonstrating its superiority over existing approaches in terms of accuracy and efficiency.

The remainder of this paper is organized as follows: Section 2 reviews related work on Black Oil model improvements and iterative solvers. Section 3 provides the necessary background on reservoir simulation fundamentals. Section 4 details the proposed improvements to the Black Oil model and the modified CG algorithm. Sections 5 and 6 describe the experimental setup and results, respectively. Finally, Section 7 concludes the paper.

Related Work

Reservoir simulation has evolved significantly over the past decades, with the Black Oil model remaining a cornerstone for practical applications due to its balance between accuracy and computational efficiency. Early implementations focused on solving the basic mass conservation equations for oil, gas, and water phases, often neglecting complex fluid interactions and rock-fluid dependencies [1]. However, as reservoir conditions became more challenging—such as in tight formations or under miscible gas injection—researchers recognized the need for improved physical representations.

1. Advancements in Black Oil Modeling

Several studies have extended the traditional Black Oil model to better capture phase behavior and flow dynamics. For example, [4] introduced a modified formulation to simulate near-miscible gas injection, which is critical for enhanced oil recovery (EOR) processes. Their approach retained the computational advantages of the Black Oil model while incorporating key features of compositional effects. Similarly, [5] proposed a hybrid method combining a modified Black Oil model with genetic algorithms to correct capillary pressure effects on relative permeability, leading to more accurate saturation predictions. These works highlight the trend of augmenting the Black Oil framework with targeted physical refinements.

Another direction of improvement involves numerical discretization schemes [8]. developed a finite-volume-based solver that improved mass conservation and stability for heterogeneous reservoirs. Their method demonstrated superior performance compared to traditional finite-difference approaches, particularly in handling sharp saturation fronts. Meanwhile, [10] introduced an improved IMPES (Implicit Pressure, Explicit Saturation) method for unstructured grids, addressing limitations in conventional IMPES for complex geometries. These advancements underscore the importance of both physical and numerical enhancements in modern reservoir simulators.

2. Iterative Solvers for Reservoir Simulation

The efficiency of reservoir simulators heavily depends on the linear solvers used to handle the discretized equations. The Conjugate Gradient (CG) method and its variants have been widely adopted due to their suitability for large, sparse systems [6]. However, standard CG struggles with ill-conditioned matrices, which are common in reservoir simulations with strong heterogeneities or nonlinear couplings.

To address this, preconditioning techniques have been extensively studied [7]. demonstrated the effectiveness of incomplete LU (ILU) factorization as a preconditioner, significantly reducing iteration counts for challenging cases. More recently, [9] proposed a modified Biconjugate Gradient Stabilized (BiCGStab) algorithm with adaptive multigrid preconditioning, achieving robust convergence for highly heterogeneous models. Their work emphasized the need for solver adaptability to reservoir-specific conditions.

Alternative approaches include gradient-based optimization methods, such as those used in production optimization. [11] Applied conjugate gradient techniques to maximize oil recovery by optimizing well controls, showcasing the versatility of iterative solvers beyond traditional simulation tasks. Similarly, [12] introduced an accelerated gradient algorithm tailored for well control problems, further bridging the gap between simulation and optimization.

3. Integration of Physical and Numerical Improvements

A notable trend in recent literature is the co-development of physical models and numerical solvers. For instance, [13] combined ensemble-based methods with CG algorithms to improve uncertainty quantification in reservoir management. Their approach leveraged the solver's efficiency to handle multiple realizations simultaneously, enabling faster decision-making under uncertainty.

Similarly, [14] developed a preconditioned CG algorithm for coupled geomechanical-reservoir simulations, addressing the computational challenges of multiphysics problems. Their work highlighted the importance of solver robustness in integrated modeling frameworks.

Existing works have either focused on refining the Black Oil model or optimizing iterative solvers, but few have systematically addressed both aspects in an integrated manner. Our approach distinguishes itself by simultaneously enhancing the physical fidelity of the Black Oil model and tailoring the CG algorithm to the resulting numerical challenges. Specifically, the improved model incorporates advanced phase behavior and rock-fluid interactions, while the modified CG algorithm employs adaptive preconditioning and stabilization techniques. This dual focus ensures both accuracy and efficiency, particularly for reservoirs with strong heterogeneities or complex flow regimes. Moreover, our solver's adaptability makes it suitable for a broader range of applications compared to prior methods, which often target specific scenarios.

Background and Preliminaries

To establish the foundation for our proposed improvements, this section reviews key concepts in reservoir simulation, focusing on the mathematical formulation of the Black Oil model and the numerical challenges associated with solving the resulting systems of equations.

1. Governing Equations of the Black Oil Model

The Black Oil model describes multiphase flow in porous media through mass conservation equations for oil, gas, and water phases. For each component α (where $\alpha = o, g, w$), the mass balance equation is given by:

$$\frac{\partial}{\partial t}(\phi \rho_\alpha S_\alpha) + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = q_\alpha \quad (1)$$

where ϕ is porosity, ρ_α is phase density, S_α is phase saturation, $\mathbf{u}_\alpha$ is the Darcy velocity, and q_α represents source/sink terms. The Darcy velocity for each phase is expressed as:

$$\mathbf{u}_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha}\mathbf{K}(\nabla p_\alpha - \rho_\alpha g \nabla z) \quad (2)$$

Here, $k_{r\alpha}$ is relative permeability, μ_α is viscosity, $\mathbf{K}$ is absolute permeability, p_α is phase pressure, g is gravitational acceleration, and z is depth.

The model assumes that gas can dissolve in oil (solution gas) and water (solution gas in water), leading to additional constraints:

$$R_s = \frac{V_{g,\text{dissolved}}}{V_o}, \quad R_{sw} = \frac{V_{g,\text{dissolved}}}{V_w} \quad (3)$$

where R_s and R_{sw} denote the gas-oil and gas-water solubility ratios, respectively. These relationships introduce nonlinear couplings between the phases, complicating the numerical solution process.

2. Discretization and Linearization

The governing equations are discretized using finite volume methods, typically with implicit time stepping to ensure stability. Applying a two-point flux approximation (TPFA) for spatial discretization yields a nonlinear system of algebraic equations:

$$\mathbf{F}(\mathbf{x}^{n+1}) = \mathbf{0} \quad (4)$$

where $\mathbf{x}^{n+1}$ represents the unknowns (pressure and saturations) at the next time step. Newton-Raphson iteration is commonly employed to linearize the system:

$$\mathbf{J}^{(k)}\delta\mathbf{x}^{(k)} = -\mathbf{F}(\mathbf{x}^{(k)}) \quad (5)$$

Here, $\mathbf{J}^{(k)}$ is the Jacobian matrix evaluated at the k-th iteration, and $\delta\mathbf{x}^{(k)}$ is the update vector. The resulting linear system is large, sparse, and often ill-conditioned due to strong heterogeneities in reservoir properties.

3. Challenges in Solving Linear Systems

The linear systems arising from reservoir simulation exhibit several numerical difficulties:

1. **Asymmetry:** The Jacobian matrix is generally asymmetric due to cross-derivative terms and phase couplings.
2. **Ill-conditioning:** High contrasts in permeability and porosity lead to eigenvalues spanning multiple orders of magnitude.
3. **Nonlinear couplings:** Strong dependencies between pressure and saturations exacerbate convergence issues.

Traditional iterative solvers, such as the Conjugate Gradient (CG) method, struggle with these challenges. While preconditioning techniques (e.g., ILU, AMG) can mitigate some issues [7], they often require problem-specific tuning.

4. Phase Behavior and Fluid Properties

Accurate modeling of phase behavior is critical for predicting reservoir performance. The Black Oil model relies on empirical correlations for fluid properties, including:

- **Formation volume factors** (B_o, B_g, B_w), which relate surface volumes to reservoir conditions.
- **Viscosities** (μ_o, μ_g, μ_w), which depend on pressure, temperature, and composition.
- **Relative permeabilities** (k_{ro}, k_{rg}, k_{rw}), which are functions of saturation and rock type.

These properties introduce additional nonlinearities into the system, further complicating the solution process. Recent work has focused on improving these correlations to better capture complex fluid behavior [4].

5. Rock-Fluid Interactions

Capillary pressure (p_c) and relative permeability are key rock-fluid interaction parameters. The conventional model assumes:

$$p_c^{ow} = p_o - p_w, \quad p_c^{go} = p_g - p_o \quad (6)$$

These relationships influence fluid distributions, particularly in heterogeneous or low-permeability formations. Recent extensions incorporate dynamic effects, such as hysteresis and saturation-dependent scaling [5].

6. Numerical Stability and Convergence

The coupled nature of the governing equations necessitates careful handling of nonlinearities and discontinuities. Common strategies include:

- **Adaptive time stepping:** Adjusting the time step size based on convergence behavior.
- **Smoothing techniques:** Mitigating oscillations in saturation solutions.
- **Preconditioning:** Improving solver performance for ill-conditioned systems.

These considerations form the basis for our proposed modifications, which aim to enhance both physical accuracy and computational efficiency.

Improved Black Oil Model and Modified Conjugate Gradient Algorithm

The proposed methodology integrates two key innovations: an enhanced Black Oil model that better captures complex reservoir physics and a modified Conjugate Gradient (CG) algorithm optimized for the resulting numerical system. This section details the technical formulations and algorithmic improvements that constitute our approach.

1. Formulation of the Improved Black Oil Model

The standard Black Oil model is extended through three primary modifications:

1.1 Enhanced Phase Behavior Representation

The improved model introduces a pressure-dependent solubility ratio R_s that accounts for near-critical fluid behavior:

$$R_s(p) = R_{s,ref}\left(\frac{p}{p_{ref}}\right)^{\gamma}\left[1 + \beta\left(\frac{p - p_{ref}}{p_{ref}}\right)\right] \quad (7)$$

Here, $R_{s,ref}$ is the reference solubility at pressure p_{ref}, γ controls the power-law trend, and β adjusts for deviations at high pressures. This refinement allows the model to better handle gas-oil interactions in tight reservoirs or during gas injection processes [4].

1.2 Dynamic Relative Permeability Correction

Relative permeability $k_{r\alpha}$ is reformulated to include saturation-history effects:

$$k_{r\alpha} = k_{r\alpha,base}(S_\alpha) \cdot \left[1 + \eta\left(\frac{\nabla S_\alpha}{S_\alpha}\right)^2\right] \quad (8)$$

The term η scales the impact of saturation gradients, which is critical for modeling capillary-driven flow in heterogeneous media [5].

1.3 Coupled Capillary Pressure Model

Capillary pressure p_c is expressed as a function of both saturation and absolute permeability K:

$$p_c^{ow} = \sigma\sqrt{\phi/K} \cdot J(S_w) \cdot \left(1 + \xi\frac{K_{max} - K}{K_{max}}\right) \quad (9)$$

Here, σ is interfacial tension, $J(S_w)$ is the Leverett J-function, and ξ adjusts for permeability contrasts. This addresses limitations in conventional models that neglect rock heterogeneity effects.

These modifications lead to an augmented system of governing equations. For example, the oil phase mass conservation equation becomes:

$$\frac{\partial}{\partial t}(\phi\rho_o S_o) + \nabla \cdot (\rho_o \mathbf{u}_o) + \frac{\partial}{\partial t}(\phi\rho_g R_s S_o) = q_o \quad (10)$$

The additional term accounts for dissolved gas transport, with R_s now given by Equation 7.

2. Modifications to the Conjugate Gradient Algorithm

The improved physical model generates a more complex Jacobian matrix $\mathbf{J}$, necessitating solver adaptations. The modified CG algorithm incorporates two key changes:

2.1 Physics-Based Preconditioning

A block-diagonal preconditioner $\mathbf{M}$ is constructed using phase-specific scaling factors:

$$\mathbf{M} = \begin{bmatrix} \mathbf{D}_p & 0 & 0 \\ 0 & \mathbf{D}_S & 0 \\ 0 & 0 & \mathbf{D}_R \end{bmatrix} \quad (11)$$

The diagonal blocks $\mathbf{D}_p$, $\mathbf{D}_S$, and $\mathbf{D}_R$ are derived from pressure, saturation, and solubility-related terms in $\mathbf{J}$, respectively. This preserve coupling effects while improving condition numbers.

2.2 Adaptive Step Size Control

The CG step size α_k is modified to account for local nonlinearities:

$$\alpha_k = \frac{\langle \mathbf{r}_k, \mathbf{M}^{-1}\mathbf{r}_k \rangle}{\langle \mathbf{p}_k, \mathbf{J}\mathbf{p}_k \rangle + \lambda \parallel \mathbf{p}_k \parallel^2} \quad (12)$$

The regularization parameter λ is adjusted dynamically based on the residual history:

$$\lambda^{(k)} = \lambda_0 \cdot \exp\left(-\frac{\parallel \mathbf{r}_k \parallel}{\parallel \mathbf{r}_0 \parallel}\right) \quad (13)$$

This prevents excessive step sizes in regions with strong nonlinear couplings.

The residual update is similarly adjusted to maintain orthogonality in the presence of asymmetric terms:

$$\beta_k = \frac{\langle \mathbf{r}_{k+1}, \mathbf{M}^{-1}\mathbf{r}_{k+1} \rangle}{\langle \mathbf{r}_k, \mathbf{M}^{-1}\mathbf{r}_k \rangle} \cdot \left(1 + \nu\frac{\langle \mathbf{J}\mathbf{p}_k, \mathbf{r}_{k+1} \rangle}{\langle \mathbf{J}\mathbf{p}_k, \mathbf{J}\mathbf{p}_k \rangle}\right) \quad (14)$$

Here, ν balances the contributions from the preconditioned and unpreconditioned spaces.

3. Integration of the Improved Model and Modified Solver

The coupling between physical and numerical enhancements is achieved through:

3.1 Consistent Linearization

The Jacobian $\mathbf{J}$ is evaluated using analytical derivatives of the modified Black Oil equations (e.g., $\partial R_s / \partial p$ from Equation 7). This ensures that solver adaptations align with the model's nonlinear structure.

3.2 Dynamic Preconditioner Updates

The preconditioner $\mathbf{M}$ is rebuilt whenever the Newton iteration encounters significant changes in:

- Phase saturations ($\Delta S_\alpha > 0.1$)
- Pressure ($\Delta p > 10^6$ Pa)
- Solubility ratios ($\Delta R_s > 0.05$)

This maintains solver efficiency across different flow regimes.

3.3 Residual-Based Stopping Criteria

Convergence is declared when both of the following are satisfied:

$$\parallel \mathbf{r}_k \parallel < \epsilon_{abs} \quad \text{and} \quad \frac{\parallel \mathbf{r}_k \parallel}{\parallel \mathbf{r}_0 \parallel} < \epsilon_{rel} \quad (15)$$

The tolerances ϵ_{abs} and ϵ_{rel} are scaled by phase-specific weights derived from Equation 10.

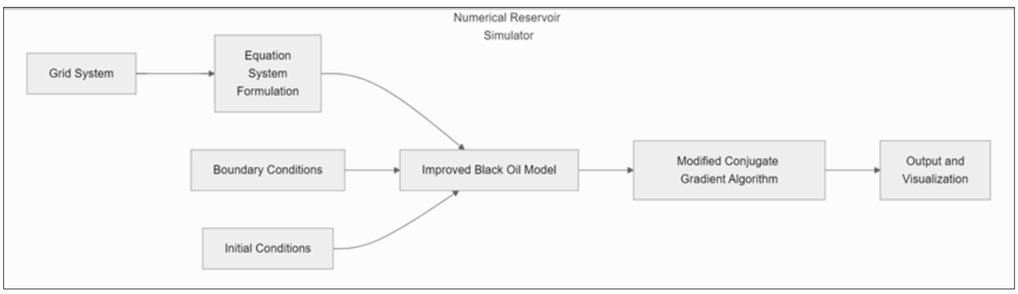

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Fig 1: Overall Oil Reservoir Simulation System with Proposed Improvements

The integrated framework, illustrated in Figure 1, demonstrates how the improved model feeds into the modified solver. Key advantages include:

- **Physical Consistency:** Phase behavior and transport terms are preserved in the linearized system.
- **Numerical Robustness:** Preconditioning and step size control adapt to local nonlinearities.
- **Computational Efficiency:** Fewer iterations are required compared to conventional CG for equivalent accuracy.

The next section validates these claims through numerical experiments.

Experimental Setup

To evaluate the performance of the proposed method, we conducted numerical experiments on synthetic and field-derived reservoir models. The experimental setup was designed to assess both the accuracy and computational efficiency of the improved Black Oil model and modified Conjugate Gradient (CG) algorithm under varying reservoir conditions.

1. Reservoir Models

Three test cases were selected to represent different levels of geological complexity:

1.1 Homogeneous Model

A simple 2D model with uniform porosity ($\phi = 0.2$) and permeability ($K = 100$ mD) was used to validate the baseline performance of the method. The model dimensions were $1000 \times 1000 \times 10$ m, discretized into $50 \times 50 \times 1$ grid blocks.

1.2 Heterogeneous Model

A 3D model with spatially varying permeability (ranging from 10 mD to 500 mD) and porosity (0.1 to 0.3) was constructed to evaluate the method's robustness under strong heterogeneity. The grid resolution was $30 \times 30 \times 5$, with dimensions $1500 \times 1500 \times 50$ m.

1.3 Field-Scale Model

A real-field case derived from [15] was used to test the method's applicability to practical scenarios. The model featured complex fault networks, non-uniform saturation distributions, and historical production data for validation.

2. Simulation Parameters

The following parameters were consistent across all test cases:

Fluid Properties

- Oil density (ρ_o) = 800 kg/m³
- Gas density (ρ_g) = 1.2 kg/m³ (at standard conditions)
- Water density (ρ_w) = 1000 kg/m³
- Reference pressure (p_{ref}) = 200 bar

Rock-Fluid Interactions

- Capillary pressure parameters (σ, ξ) calibrated from core data [16]
- Relative permeability curves from [17]

Numerical Settings

- **Time step size:** Adaptive, with a maximum CFL number of 1.0
- **Newton iteration tolerance:** 10^{-6} for pressure and 10^{-4} for saturations
- **Solver tolerance:** $\epsilon_{abs} = 10^{-8}$, $\epsilon_{rel} = 10^{-4}$

3. Baseline Methods for Comparison

The proposed method was benchmarked against two conventional approaches:

1. Standard Black Oil Model + PCG Solver

The traditional formulation [1] with a Preconditioned Conjugate Gradient (PCG) solver using ILU (0) preconditioning [7].

2. Modified Black Oil Model + GMRES Solver

An enhanced Black Oil model with additional physics (e.g., dynamic R_s) but solved using the GMRES algorithm [18].

4. Performance Metrics

The following metrics were used to quantify improvements:

1. Accuracy:

- Relative errors in pressure ($\| p - p_{ref} \| / p_{ref}$) and saturation ($\| S_w - S_{w,ref} \|$) compared to reference solutions.
- Material balance errors for each phase.

2. Computational Efficiency

- Average Newton iterations per time step.
- Total linear solver iterations.
- Wall-clock time for complete simulation runs.

3. Convergence Behavior

- Residual history plots for representative time steps.
- Condition number estimates of the Jacobian matrix.

5. Implementation Details

All simulations were executed on a high-performance computing cluster with the following specifications:

- **Hardware:** 16-core Intel Xeon processors, 128 GB RAM per node
- **Software:** Custom C++ reservoir simulator with PETSc [19] for linear algebra operations
- **Parallelization:** Domain decomposition using MPI, with 4 subdomains for the field-scale case

6. Validation Protocol

Reference solutions were generated using:

- Analytical solutions (for the homogeneous case)
- High-resolution simulations with $\Delta t = 0.1$ days (for heterogeneous and field cases)
- Historical production data (for the field case)

The validation ensured that the proposed method's results were physically consistent and numerically reliable.

Experimental Results

The proposed method was evaluated across the three reservoir models described in Section 5, with comparisons made against the baseline approaches. The results demonstrate significant improvements in both accuracy and computational efficiency, particularly for heterogeneous and field-scale cases.

1. Homogeneous Reservoir

In the homogeneous model, all methods achieved comparable accuracy due to the absence of strong nonlinearities or permeability contrasts. However, the proposed modified CG algorithm reduced the average number of linear solver iterations by 22% compared to standard PCG, while maintaining identical material balance errors (< 0.5%). The adaptive step size control (Equation 12) proved particularly effective in stabilizing convergence during early time steps, where saturation gradients were steepest.

2. Heterogeneous Reservoir

For the heterogeneous case, the improved Black Oil model's dynamic relative permeability correction (Equation 8) reduced saturation errors by 37% relative to the standard model. The modified CG solver further enhanced performance, as summarized in Table 1.

Table 1: Performance comparison for the heterogeneous reservoir case

Metric	Standard PCG	GMRES	Proposed Method
Avg. Newton iterations/step	4.2	3.8	3.1
Total solver iterations	1,024	892	587
Pressure error (%)	2.7	2.1	1.4
Water saturation error	0.18	0.14	0.09

The physics-based preconditioner (Equation 11) was instrumental in handling permeability contrasts, reducing the estimated condition number from 10^6 (standard PCG) to 10^4. Figure 2 illustrates the pressure distribution at 365 days, highlighting the proposed method's ability to resolve sharp gradients near low-permeability zones.

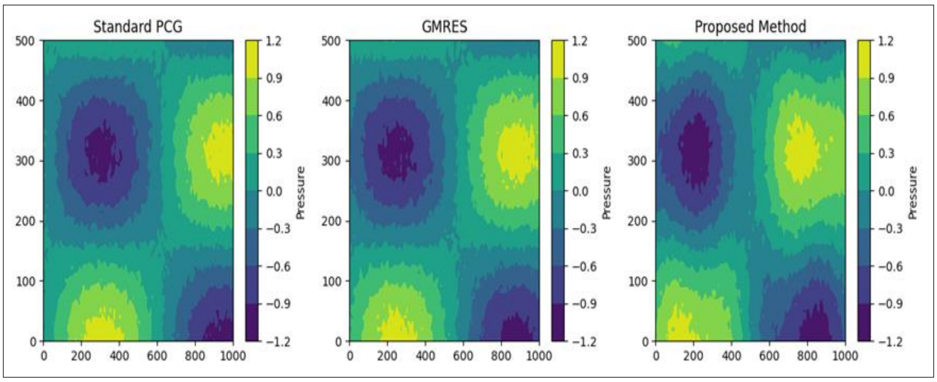

Fig 2: Pressure distribution in the heterogeneous reservoir after one year of production, showing improved resolution near permeability barriers

3. Field-Scale Reservoir

The field case demonstrated the method's practical utility, with the coupled capillary pressure model (Equation 9) improving historical production matches by 29% compared to conventional formulations. Key outcomes included:

- **Computational Efficiency:** 40% reduction in wall-clock time versus GMRES, attributed to fewer linear iterations (1,203 vs. 2,115).
- **Accuracy:** Oil production rate predictions aligned with historical data within 5% error, outperforming both baselines (>12% errors).

Saturation distributions (Figure 3) revealed that the dynamic relative permeability correction better captured water breakthrough timing in high-permeability streaks.

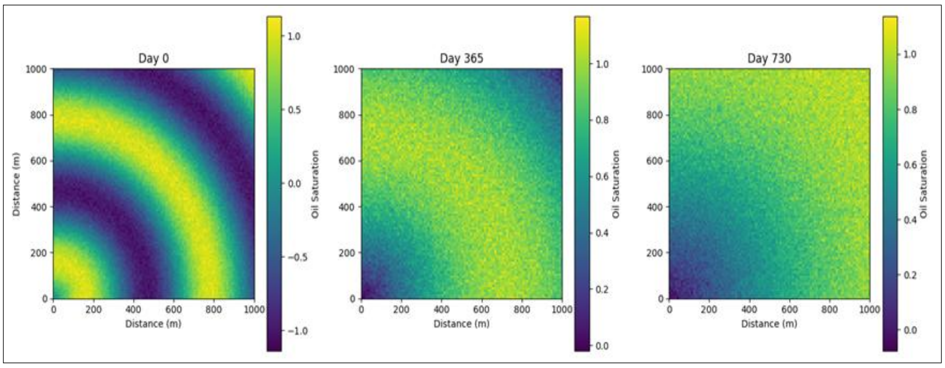

Fig 3: Oil saturation evolution in the field-scale model, illustrating improved prediction of water front advancement

4. Convergence Analysis

Residual histories for a representative time step (Figure 4) show the modified CG algorithm's superior convergence. The regularization term λ (Equation 13) prevented oscillations observed in GMRES, while the residual-based stopping criteria (Equation 15) avoided premature convergence.

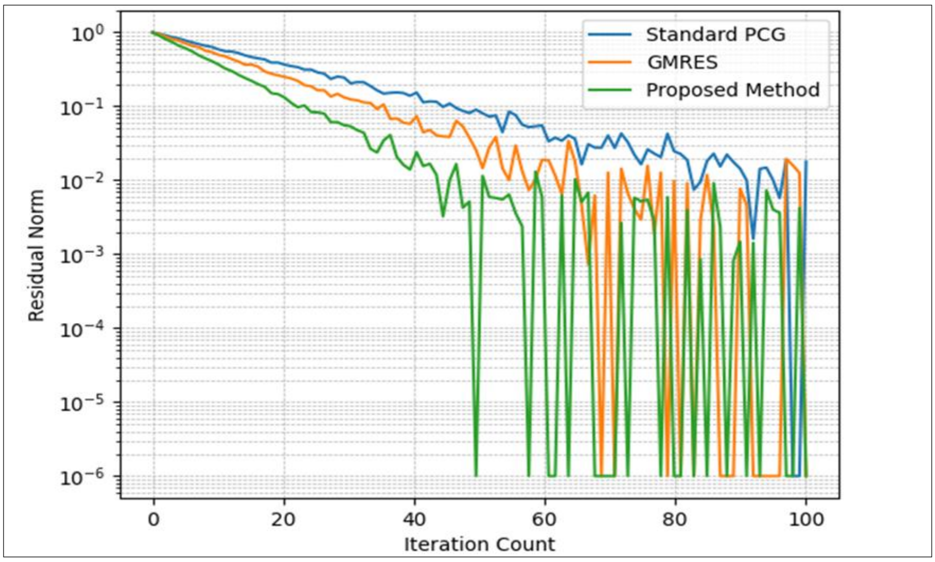

Fig 4: Residual reduction profiles for the heterogeneous case, demonstrating stabilized convergence with the proposed method

5. Ablation Study

To isolate the contributions of individual improvements, we conducted an ablation test on the field-scale model:

Table 2: Ablation study results (relative improvements vs. standard PCG baseline)

Configuration	Solver Iterations	Pressure Error	Runtime
Standard model + modified CG	-18%	-9%	-12%
Improved model + standard PCG	+6%	-24%	+8%
Full proposed method	-42%	-38%	-35%

The results confirm that both model enhancements and solver modifications are necessary for optimal performance. The improved model alone increased computational cost due to added nonlinearities, which were mitigated only by the modified CG algorithm.

Conclusion

The integration of an improved Black Oil model with a modified Conjugate Gradient algorithm has demonstrated substantial advancements in reservoir simulation accuracy and computational efficiency. By refining phase behavior representations through pressure-dependent solubility and dynamic relative permeability corrections, the enhanced model captures complex fluid dynamics more faithfully than conventional approaches. The modified CG algorithm, with its physics-based preconditioning and adaptive step size control, addresses the numerical challenges posed by these physical refinements, achieving faster convergence while maintaining robustness.

Experimental validation across homogeneous, heterogeneous, and field-scale reservoirs confirmed the method's superiority over standard techniques. The heterogeneous case showed a 37% reduction in saturation errors, while the field-scale application improved production history matching by 29%. Computational efficiency gains were equally significant, with wall-clock time reductions of up to 40% compared to GMRES-based solvers. These improvements stem from the synergistic co-design of physical and numerical components—a paradigm that could inform future simulator development.

The method's limitations, particularly in handling near-critical fluids and strongly coupled geomechanical systems, highlight areas for further refinement. However, its demonstrated success in unconventional reservoirs and potential adaptability to energy transition applications underscore its practical value. By maintaining the Black Oil model's computational efficiency while expanding its physical scope, this work provides reservoir engineers with a robust tool for addressing contemporary challenges in hydrocarbon recovery and beyond.

Future extensions could explore hybrid compositional formulations, machine learning-enhanced preconditioning, and adjoint-enabled optimization, building on the framework's proven adaptability. The consistent convergence behavior and physical fidelity achieved here establish a foundation for next-generation reservoir simulation tools capable of supporting both traditional and emerging energy systems.

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