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Intuitionistic fuzzy entropy measures and their applications: A Review

Priyanka Sahni, Rajeev Kumar
Department of Mathematics, Maharshi Dayanand University, Rohtak, India

Abstract
The concept of intuitionistic fuzzy set was introduced by Atanassov [1] and this concept is now being applied to every branch of knowledge. In information theory and telecommunications, the entropy is the basic subject and it measures the fuzziness in fuzzy sets. In the theory and applications of the fuzzy sets, many research investigations have been made. Intuitionistic fuzzy entropy is an important concept to describe the uncertainty of intuitionistic fuzzy sets. The aim of the present paper is to carry out literature survey of different types of intuitionistic fuzzy information measures and their applications in diversified fields.

Keywords: entropies, fuzzy sets, fuzzy entropy functions, intuitionistic fuzzy sets, intuitionistic fuzzy entropy functions

Introduction

Since Zadeh [19] put forward the concept of a fuzzy set in 1965, the fuzzy set theory has been widely applied to neural networks, medical diagnosis, and electronic communication, and has made great progress. However, due to the complexity of objective things and the limitation of subjective cognition, there is often a lack of information. Because of this situation, the American scholar Atanassov [1] established an intuitionistic fuzzy set, which contains the following three aspects of information: membership degree, non- membership degree, and hesitation degree. It generalizes the concept of fuzzy set as it includes the hesitation degree that is being applied in various areas of research. The basic difference between intuitionistic fuzzy set and fuzzy set is that the intuitionistic fuzzy set assigns the membership degree, non-membership degree and hesitation degree to each element and there is no degree of hesitation in the concept of fuzzy set. Some definitions are given below:

Entropy

Some uncertainty is always there for each and every probability distribution. The entropy provides a quantitative measure of this uncertainty. Entropy is particularly important in information theory and was introduced there by Shannon [14].

Intuitionistic Fuzzy Set

The concept of intuitionistic fuzzy set was introduced by Atanassov [1] as mentioned below:
"An Intuitionistic fuzzy set A on X, a universe is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}, \quad \text{where } \mu_A : X \to [0,1], \; \nu_A : X \to [0,1]$$

with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \quad \forall x \in X$$

The number $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and degree of non- membership of $x \in X$ to set A, respectively. For each intuitionistic fuzzy set A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),$$

then $\pi_A(x)$ is called intuitionistic index (or a hesitation degree) of the element $x \in X$ to A".

Review of Literature

Let $A_1, A_2, \ldots\ldots\ldots, A_n$ be the n possible outcomes of a random experiment with their respective probabilities $p_1, p_2, \ldots\ldots\ldots, p_n$, giving rise to the probability distribution

$$P = (p_1, p_2, \ldots\ldots, p_n); \quad \sum_{i=1}^{n} p_i = 1; \quad p_1 \geq 0, \ldots, p_n \geq 0$$

Some uncertainty always occurs when an experiment is performed. A brief review of literature is given as under:
Shannon [14] entropy measure is defined as

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{2.1}$$

Renyi [13] defined entropy of order α as

$$H_\alpha(P) = \frac{1}{1-\alpha} \log\left(\sum_{i=1}^{n} p_i^{\alpha}\right), \alpha \neq 1, \alpha > 0 \tag{2.2}$$

First non-negative entropy was introduced by Havrada and Charvat [8] as mentioned below:

$$H_\alpha(P) = \frac{\left[\sum_{i=1}^{n} p_i^{\alpha}\right] - 1}{2^{1-\alpha} - 1}, \alpha \neq 1, \alpha > 0. \tag{2.3}$$

Boekee and Lubbe [3] proposed the following R-norm information measure:

$$H_R(P) = \frac{R}{R-1}\left[1 - \left(\sum_{i=1}^{\infty} p_i^{R}\right)^{\frac{1}{R}}\right], R > 0, R \neq 1 \tag{2.4}$$

The above defined measure is called R-norm information measure and this measure is an extension of Shannon's entropy, i.e. when R →1, it tends to Shannon's entropy.
Classical Shannon information entropy was analyzed by Pal and Pal [11] and they proposed an exponential entropy as mentioned below:

$$H(P) = \sum_{i=1}^{n} p(x_i)\left(e^{(1-p(x_i))} - 1\right) \tag{2.5}$$

These authors conclude that the exponential entropy has an advantage over Shannon's entropy. For the uniform probability distribution $P = \left(\frac{1}{n}, \frac{1}{n}, \ldots\ldots, \frac{1}{n}\right)$, exponential entropy has a fixed upper bound

$$\lim_{n\to\infty} H\left(\frac{1}{n}, \frac{1}{n}, \ldots\ldots, \frac{1}{n}\right) = e - 1$$

which is not the case for Shannon's entropy.

Fuzzy Entropy
Hooda and Raich [9] discussed the fuzzy entropy as:
"The measure of uncertainty is adopted as a measure of information. Hence, the measure of fuzziness is known as fuzzy information measures. The measure of the amount of fuzzy information obtained from a fuzzy set or fuzzy system is known as fuzzy entropy". On the basis of this, De Luca and Termini [10] proposed the following measure of fuzzy entropy corresponding to Shannon [14] entropy:

$$H(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\right] \tag{3.1}$$

Corresponding to Renyi's [13] entropy of order α (given in (2.2)), Bhandari and Pal [2] has taken the following fuzzy entropy measure of order α:

$$H_\alpha(A) = \frac{1}{1-\alpha}\sum_{i=1}^{n}\log\left[\mu_A{}^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right], \alpha \neq 1, \alpha > 0 \tag{3.2}$$

Pal and Pal [12] gave the exponential fuzzy entropy corresponding to the entropy given by Pal and Pal [11] in (2.7) as

$$H(A) = \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\left[\mu_A(x_i)e^{(1-\mu_A(x_i))} + (1-\mu_A(x_i))e^{\mu_A(x_i)} - 1\right] \tag{3.3}$$

Gupta *et al.* [9] defined α -exponential fuzzy entropy as

$$H_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}\left[\mu_A^\alpha(x_i)e^{1-\mu_A^\alpha(x_i)} + (1-\mu_A(x_i))^\alpha e^{1-(1-\mu_A(x_i))^\alpha} - 1\right], \quad 0 < \alpha \leq 1 \tag{3.4}$$

when $\alpha = 1$, the above measure reduces to Pal and Pal [12] fuzzy entropy measure.

Intuitionistic Fuzzy Entropy

Szmidt and Kacprzyk [15] proposed a fuzzy information measure for IFSs on extending De Luca and Termini [10] for fuzzy sets. Here Hooda and Raich [9] give a new definition on the lines of Szmidt and Kacprzyk [15] as follows:

"A real function E: IFSs(X) $\rightarrow [0,1]$ is called an intuitionistic fuzzy information measure on IFSs(X) if it satisfies the following properties:

1. $E(A) = 0$, iff A is a crisp set.
2. $E(A) = 1$, iff $\mu_A(x_i) = \nu_A(x_i)$ for all i.
3. $E(A) \leq E(B)$, iff A is crisper or sharper than B.
   i.e. if $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for* $\mu_B(x) \leq \nu_B(x)$
   or $\mu_A(x) \geq \mu_B(x)$ *and* $\nu_A(x) \leq \nu_B(x)$ *for* $\mu_B(x) \geq \nu_B(x)$.
4. $E(A) = E(A^c)$ where $A^c = \{(x, \nu_A(x), \mu_A(x)) / x \in X\}$".

Vlachos and Sergiadis [18] introduced an intuitionistic fuzzy entropy measure corresponding to De Luca and Termini [10] entropy given in (3.1) as

$$E(A) = -\frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix}\mu_A(x_i)\log\mu_A(x_i) + \nu_A(x_i)\log\nu_A(x_i)\\ -(1-\pi_A(x_i))\log(1-\pi_A(x_i)) - \pi_A(x_i)\end{bmatrix} \tag{4.1}$$

Verma and Sharma [16] defined exponential intuitionistic fuzzy entropy corresponding to the exponential fuzzy entropy given by Pal and Pal [12] as

$$e^{E(A)} = \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\begin{bmatrix}\dfrac{(\mu_A(x_i)+1-\nu_A(x_i))}{2}e^{1-\frac{(\mu_A(x_i)+1-\nu_A(x_i))}{2}}\\ +\left(1-\dfrac{(\mu_A(x_i)+1-\nu_A(x_i))}{2}\right)e^{\frac{(\mu_A(x_i)+1-\nu_A(x_i))}{2}} - 1\end{bmatrix} \tag{4.2}$$

Verma and Sharma [17] defined intuitionistic fuzzy entropy of order- α as

$$E_\alpha(A) = \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log\begin{bmatrix}\mu^\alpha{}_A(x_i) + \nu^\alpha{}_A(x_i) \times (\mu_A(x_i) + \nu_A(x_i))^{1-\alpha}\\ +2^{1-\alpha}\pi_A(x_i)\end{bmatrix}, \alpha > 0, \alpha \neq 1 \tag{4.3}$$

The measure defined above reduces to Bhandari and Pal [3] measure (given in eq. (3.2)), when $\alpha \rightarrow 1$.

Gupta *et al.* [7] extended the measure for intuitionistic fuzzy set corresponding to the measure given by Gupta *et al.* [6], given in (3.4) as

$$E_\alpha(A)=\frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}\left[\begin{array}{l}\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)^{\alpha}e^{1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)^{\alpha}}\\+\left(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)^{\alpha}e^{1-\left(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)^{\alpha}}-1\end{array}\right] \quad (4.4)$$

Garg *et al.* [5] defined the entropy measure of order α and degree β as

$$E_\alpha^\beta(A)=\frac{2-\beta}{n(2-\beta-\alpha)}\sum_{i=1}^{n}\log\left[\begin{array}{l}\left(\mu^{\frac{\alpha}{2-\beta}}{}_A(x_i)+\nu^{\frac{\alpha}{2-\beta}}{}_A(x_i)\right)(\mu_A(x_i)+\nu_A(x_i))^{1-\frac{\alpha}{2-\beta}}\\+2^{1-\frac{\alpha}{2-\beta}}(1-\mu_A(x_i)-\nu_A(x_i))\end{array}\right], \quad (4.5)$$

where the log is to the base two, $\alpha>0,\ \beta\in[0,1],\ \alpha+\beta\neq 2.$

The above entropy measure is the generalization of Verma and Sharma [17] entropy when $\beta=1$ and reduces to Bhandari and Pal [2] entropy when $\alpha\to 1,\ \beta\to 1.$

Garg and Kapur [4] defined a new (R, S) - norm information measure as:

$$E_R^S(A)=\begin{cases}\frac{R\times S}{n(R-S)}\sum_{i=1}^{n}\left[\begin{array}{l}(\mu_A^S(x_i)+\nu_A^S(x_i)+\pi_A^S(x_i))^{\frac{1}{S}}\\-(\mu_A^R(x_i)+\nu_A^R(x_i)+\pi_A^R(x_i))^{\frac{1}{R}}\end{array}\right], & \text{either } R>1,\ 0<S<1 \text{ or } 0<R<1,\ S>1\\ \frac{R}{n(R-1)}\sum_{i=1}^{n}\left[1-(\mu_A^R(x_i)+\nu_A^R(x_i)+\pi_A^R(x_i))^{\frac{1}{R}}\right], & S=1;\ 0<R<1\\ \frac{S}{n(1-S)}\sum_{i=1}^{n}\left[(\mu_A^S(x_i)+\nu_A^S(x_i)+\pi_A^S(x_i))^{\frac{1}{S}}-1\right], & 0<S<1;\ R=1\\ \frac{-1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i)+\nu_A(x_i)\log\nu_A(x_i)+\pi_A(x_i)\log\pi_A(x_i)\right], & R=1=S\end{cases} \quad (4.6)$$

The above measure is equivalent to Boekee and Lubbe [3] measure when $R=1$ and $R\neq S.$

References

1. Atanassov KT. Intuitionistic fuzzy sets, Fuzzy Sets and Systems,1986:20(1):87-96, URL: https://doi.org/10.1016/S0165-0114(86)80034-3.
2. Bhandari D, Pal NR. Some new information measures for fuzzy sets, Information Science,1993:67:204-228. URL: https://doi.org/10.1016/0020-0255(93)90073-U.
3. Boekee DE, Lubbe JCA. The R- norm information measure, Information and Control,1980:45:136-155. URL: https://doi.org/10.1016/S0019-9958(80)90292-2.
4. Garg H, Kapur J. A novel (R, S)-norm entropy measure of intuitionistic fuzzy sets and its applications in multi-attribute decision making, Mathematics,2018:6(92):1-19. URL: https://doi.org/10.3390/math6060092.
5. Garg H, Agarwal N, Tripathi A. Generalized intuitionistic fuzzy entropy measure of order α and degree β and its applications to multi-criteria decision making problem International Journal of Fuzzy System Applications,2017:6(1):86-107, DOI: 10.4018/IJFSA.2017010105.
6. Gupta P, Arora HD, Tiwari P. On some generalized exponential entropy for fuzzy sets, Proceeding of 3rd International Conference on Reliability, Infocom Technologies and Optimization (ICRITO) (Trends and Future Directions), 2014, 309-311. DOI: 10.1109/ICRITO.2014.7014702.
7. Gupta P, Arora HD, Tiwari P. Generalized entropy for intuitionistic fuzzy sets, Malaysian Journal of Mathematical Sciences,2016:10(2):209-220. DOI: https://einspem.upm.edu.my/journal/fullpaper/vol10no2may/6.%20arora.pdf
8. Havrada JH, Charvat F. Quantification methods of classification process: Concept of structural $\alpha-$ entropy, Kybernetika,1967:3(1):30-35, URL: http://dml.cz/dmlcz/125526.

9. Hooda DS, Raich V. Fuzzy information measures with applications, Alpha Science Interntional Limited, 2015. URL: http://www.narosa.com/books_display.asp?catgcode=978-81-8487-411-2
10. Luca D, Termini S. A definition of a non-probabilistic entropy in the settings of fuzzy sets theory, Information and Control,1972:20:301-312, DOI: 10.1016/S0019-9958(72)90199-4.
11. Pal NR, Pal SK. Object-background segmentation using new definitions of entropy, IEE Proceedings E: Computers and Digital Techniquesm,1989:136(4):284-295, URL: https://www.isical.ac.in/~sankar/paper/47.pdf.
12. Pal NR, Pal SK. Entropy: A new definition and its applications, IEEE transactions on systems, Man and Cybernetics,1991:21(5):1260-1270. URL: https://www.isical.ac.in/~sankar/paper/TSMC_91_NRP_SKP.pdf.
13. Renyi. On measures of entropy and information, Proceedings 4th Berkeley Symposium on Mathematical Statistics and Probability,1961:1:547-561. URL: https://www.scirp.org/(S(czeh2tfqyw2orz553k1w0r45))/reference/ReferencesPapers.aspx?ReferenceID=2031029
14. Shannon CE. The mathematical theory of communication, Bell System Technical Journal, 1948:27:423-467. URL: https://doi.org/10.1002/j.1538-7305.1948.tb01338.x.
15. Szmidt E, Kacprzyk J. Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems,2001:118(3):467-477. URL: https://doi.org/10.1016/S0165-0114(98)00402-3.
16. Verma RK, Sharma BD. Exponential entropy on intuitionistic fuzzy sets, Kybernetika,2013:49(1):114-127. URL: http://dml.cz/dmlcz/143243.
17. Verma RK, Sharma BD. On intuitionistic fuzzy entropy of order-α, Advances in Fuzzy Systems (2014), DOI: 10.1155/2014/789890.
18. Vlachos IK, Sergiadis GD. Intuitionistic fuzzy information- applications to pattern recognition, Pattern Recognition Letters,2007:28(2):197-206. DOI: 10.1016/j.patrec.2006.07.004.
19. Zadeh LA. Fuzzy sets, Information and Control,1965:8:338-353. DOI: 10.1016/S0019-9958(65)90241-X.