International Journal of Multidisciplinary Research and Development
www.allsubjectjournal.com
Online ISSN: 2349-4182, Print ISSN: 2349-5979
Received: 19-05-2022, Accepted: 03-06-2022, Published: 20-06-2022
Volume 9, Issue 6, 2022, Page No. 31-44

Mathematical study of blood flow through blood vessels under diseased condition

Jay Prakash Kumar, Mo Sadique, Sapna Ratan Shah

Mathematical Lab, School of Computational and Integrative Science, Jawaharlal Nehru University, New Delhi, India

Abstract

The notion of mathematical study for the flow of blood through blood vessel which is tapered and under diseased conditions is the topic of this dissertation. Heat, Arteries, and Blood vessels are the three major components of the cardiovascular system. We developed a mathematical model of pressure on artery walls caused by blood flow in this study. We employed Poiseuille's equation, the Navier-Stoke equation, and the continuity equation in this process. We developed a mathematical model for two-fluid blood flow using the Navier-Stoke equation, with non-Newtonian fluid in the core and Newtonian fluid in the periphery. For the scientific definition, a barrel-shaped coordinate framework was used. The framework's controlling conditions of movement are first sought in the Laplace change space, and its solution is reached by the use of a limited distinction conspire. The current model is designed to be applicable to both converging and diverging arteries. We investigated the variation of blood viscosity with regard to stenosis shape for various flow rates, as well as the variation of flow rate with respect to stenosis form for various pressures.

Keywords: cardiovascular system, navier-stoke equation, permeable wall, artery, blood

Introduction

Biomedical engineering blends computational sciences, physical, mathematical, chemical and engineering ideas to explore medicine, biology, behavior and healthcare. This produces revolutionary biomaterials, materials, implants devices, processes and techniques to preventing disease, diagnostics, and therapy, as well as patient rehabilitation and better health outcomes. This term is wide and can span several engineering disciplines; in reality, biomedical engineers can use the concepts of chemical, mechanical, medical engineering and electrical to do the analysis of biological tissue and their operational procedure and how they respond to environmental variables. Several biomedical engineers also concentrate on several other elementary engineering fields, like controls and systems difficulties via development of breakthrough rehabilitation, medical imaging and disease detection equipment, among others. Therefore, the essence of biomedical engineering has many applications due to the necessity of understanding various engineering concepts and physiology and applying insights from both areas to study. Main objective of biomedical engineering is to combine these subjects in order to understand these mechanisms or to design and manufacture devices for use in a biological or medical equipment. This work focuses on the relationship between mechanical bio-engineering ideas and fluid mechanics. It does not mean that the other engineering ideas could not be applicable to bio-fluid mechanics; they may. The majority of biofluid mechanics challenges involve defining the flow behavior in a specific tissue, and may be viewed as an extension or a very different case of fluid bio-mechanics. If we were to describe the flow of blood through the coronary artery, for instance, one could apply principles of fluid mechanics, but there is also a need of considering the mechanical properties of arteries and veins and how this can have an impact on the fluid flow. Similarly, if one has to create a novel implanted a device for cardiovascular system, then there is a need of understanding and evaluating not just mechanical flow principles, but also material qualities, electrical components, and the physiological implications the device can have on the circulatory system. Therefore, this type of difficulty gets close to the core of a biomedical engineer's duties, which are to create equipment to rectify physiological state which to quantify the effect of such equipment in physiologically relevant contexts. Many biomedical engineers are only concerned with engineering design, while others are more concerned with physiological applications.

Biofluid dynamics is a fascinating topic since it garners the interest of both scientists and the general population. We are concerned with circulatory systems of the human body. Studying the circulatory system and respiratory system is one of the primary issues of research and many engineering experts have carried out some fantastic work. The respiratory system is intricately intertwined with the circulatory system and extremely difficult to understand and grasp. In addition to curiosity, the study of biofluid dynamics is focused at uncovering answers to some of the human body associated disorders and issues. Unlike engineering applications, it is difficult to understand the biofluid dynamics of the human body. This is due to the difficulty of conducting in vivo experiments. Non-invasive diagnostics are beneficial however they may not always yield the desired results. Thus, both theoretical and computational biofluid dynamics are crucial to our understanding of the biofluid dynamics of the human body. By adhering to local government regulations, extracorporeal systems, such as medical equipment, can be experimentally evaluated. Real biofluids are often essential to assess these devices. Such fluids can only be accessed by approved entities. Thus, in the construction of medical devices both theoretical and computational biofluid dynamics play a key role in the early phases of design and development.

Blood has been exploited like applied fluid by humans for many years. Many years ago in the Stone Age painters employed various arrangements of resources to create their paintings. Blood was among many substances. Stone Age cave paintings (4,000-8,000 B.C.) throughout Africa, Europe, Asia, and Australia demonstrate the usage of blood like a painting material, and it is likely that they were aware of the value of human blood to the body during shooting, hunting and combat long before then. In old culture of Chinese, Huang Ti and Yellow Emperor, authored a text regarding circulation. And one of them Huang Ti is recognized for authoring of "internal classics," key doctrines of Chinese famous medicine. Though many Chinese scholars have their understanding about the book was composed in warring centuries by anonymous scribes. After that Egyptians has also began their blood rites some 3000 years ago. And they utilized bloodletting as a medicinal treatment, and afterwards other countries (e.g. Romans, Greeks, Arabs, Persians and Asians) adapted and maintained the Egyptian's practice. Early medical doctor in Persian dynasties have the understanding of the circulation of blood. As per Bundahishn circulation of blood is akin to water and rivers; following meal digestion, blood nourishes the physical body system and organs. In western civilization, various notions were noted. Hippocrates (B400BC) considered that there are basic four humors in humans body and they are phlegm, blood, yellow bile and black bile each humor was located in a distinct organ lung, brain, gall bladder and spleen.

After Hippocrates, Aristotle believed that main heart was having the responsibility of circulating blood and it is the center of the blood. A Greek physician, Praxagoras, established the contrast between veins (blood carriers) and arteries (air carriers, as he assumed) and one more founder of modern dissection Andreas Vasalius, has published a work "On the Fabric of the Human Body" in human structure. To illustrate the system of cardiovascular, he created many graphics depicting the architecture of the heart with veins. William Harvey, a forerunner, in the same area in the western domain and a physician of English who has done his medical degree in Italy, wrote an article on circulation of blood and described the heart's role of blood push in the circulatory structure. And it was demonstrated by him that blood is recycled rather than obsessive. It was calculated that the amount of blood that left the heart in only a couple of moments surpassed the total amount of blood. French physician and scientist Poiseuille learned mathematics and physics at the Ecole Polytechnique. He investigated the blood circulation in narrow arteries. It was discovered that

In order to determine the influence of stenosis on the blood flow through arteries, several investigations have been undertaken over the past few decades. Although the blood is a non- Newtonian fluid, but in many situations, it functions like Newtonian fluid which is solved by Navier-stokes equation combined with the continuity equation. Under certain assumptions, Navier-Stokes equations and continuity equation can be transformed into simple partial differential equations, known as the cardiovascular system equation. This model was developed for regular blood circulation by using the well-known Poiseuille's equation. Flow is an essential topic of physics of cardiovascular system. Because of to higher shear rate close to the wall of the artery, viscosity of the blood is lower and the cells concentration is higher in core and central zone. Recognizing the significance of the geometrical interplay between stenosis and the flow of blood, a number of research on the fluid dynamics of stenosed arteries have been conducted. This is done to analyze the pattern of the fluid flow, shear stresses of the wall, rate of fluid flow, and resistance by various mathematicians and scientists. The condition that annually claims millions of lives is, without a doubt, are disease related to the cardiovascular system.

A large number of cardiovascular disorders such as arterial occlusion, because of stenotic blockage, causes to considerable alterations in blood flow, wall shear stresses, rate of fluid flow and resistance. However, the fundamental reason of stenosis to arise is still not clearly apparent. Although it is well documented that fluid (blood) dynamics, plays a significant role in subsequent stenotic progression. Therefore, various investigations were undertaken throughout the past several decades to figure out the influence of stenosis on blood supply via arteries. Blood is red fluid that flows in human body vessels. The major purpose of blood is to transport nutrients and interchange the gases in our body. It also plays a crucial function in the body's resistance against infection. Red blood cells (RBC), plasma, white blood cells (WBC), and platelets are the significant blood component. Plasma is predominantly liquid part of the blood. Fresh oxygen is transported through the RBC. WBC also referred as Leukocyte safeguards against diseases and infection by consuming external materials and cellular wastages. Platelets are microscopic blood cells that support our body by creating clot to halt bleeding upon injury. This model has been given by Chandra, Shit and Roy. The subject of the investigation of this article is to represents the blood flow via a tapering and overlapped stenosed artery under the influence of an externally induced magnetic field.

In this analysis, variable viscosity of blood depending on the hematocrit is considered to improve resemblance to the real situation. Here, the analytical expression for velocity component, volumetric flow rate, wall shear stress, and pressure gradient are obtained. Experiments with hematocrit, magnetic field, and artery shape have a significant effect on the velocity profile, pressure gradient, and wall shear stress, since hematocrit is the most influential factor in determining the viscosity of whole blood. The current model is meant to be applicable to both converging and diverging arteries, depending on the situation. The essay addressed the mechanism under a variety of influences. Different flow properties of blood through a porous media under the influence of an external magnetic field have been expressed analytically. The flow characteristics are depicted visually together with relevant numerical data pertaining to blood flow. The data indicates that the central line velocity at the neck of secondary stenosis is approximately 30 percent lower than that of original stenosis. Put the central line velocity at z =2 (between the throats of two stenosis) lowers abruptly by approximately 55% compared to that of the secondary one. The observation leads to the flow circulation zone and may also result in plaque deposition. It has been noticed that the velocity at the stenosis's neck increases dramatically with length. Therefore, the influence of stenosis form plays a considerable role in the flow characteristics. The fundamental purpose of the study is to determine if the influence of primary stenosis on secondary stenosis is substantial in the case of diverging artery, and whether the flow velocity in the core area reduces gradually as magnetic field intensity increases. At the core area, the axial velocity decreases as the percentage of hematocrit increases.

It is a linear relationship between hematocrit and blood pressure. The starting value of hematocrit can lead a rise in blood cholesterol levels. Moreover, based on the current findings, it is possible to deduce that blood regulated by an external magnetic field. This study work focuses on the mathematical and theoretical study of blood flow via a tapering and overlapped stenosed artery under the influence of a magnetic field's pressure. In the presence of a magnetic field, blood flow in an axisymmetric two-dimensional tapered and overlapping stenosed artery is laminar, incompressible, and Newtonian. Because of maximum shear rate near the artery wall, blood viscosity is less and concentration of cell is high. Blood can therefore be considered as a Newtonian fluid with variable viscosity, especially in the case of big blood arteries. With an increase in magnetic strength, the axial velocity of blood flow drops dramatically along the artery's central axis. While the velocity close to the artery wall rises with rising Hartmann number M values (in order to maintain the constant volumetric rate). The axial velocity towards the center of the channel rises as the permeability parameter 'k' increases, but the opposite is true near the artery wall because 'k' depends on the matching condition and finally, we got the equation and expression of mathematical models.

Mathematical analysis and formulation of problem:

The Navier Stokes equation can be derived by considering an infinitesimal element of fluid and applying the priciple of conservation of momentum and other physical considerations.

Let us consider a differential fluid element as shown in the above figure. The mass of the infinitesimal element is dm and legth, width and height of the fluid element are dx, dy and dz respectively in x, y, z directions. Let the density of the fluid at centre O be ρ, assume the fluid element as our system and applying the Newtons second law of motion. Therefore the equation for the conservation of momentum is given by,

$$dF = dm * a \quad (1)$$

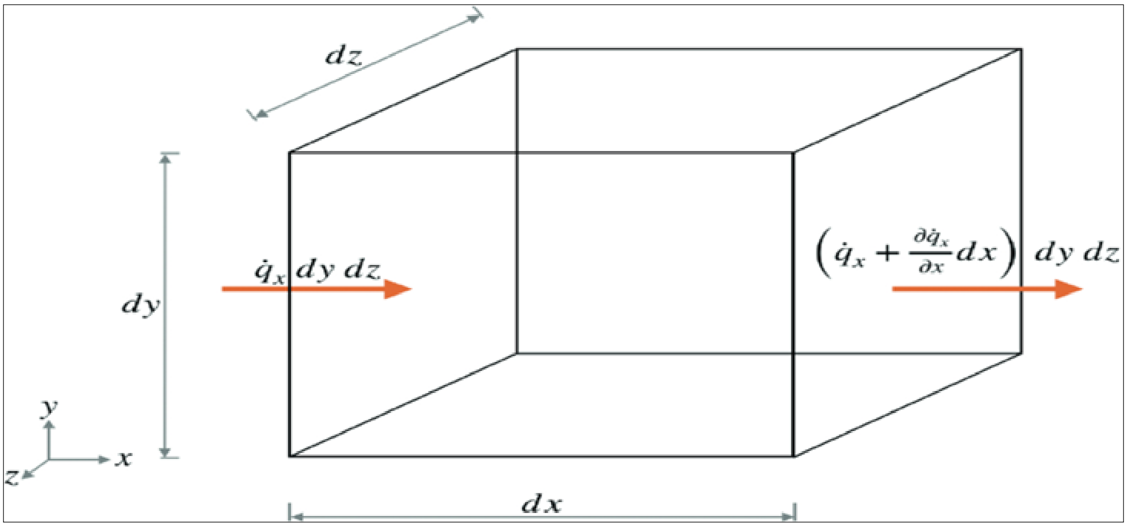

Fig 1: Normal stress at the fluid element and surface force.

The mass of the nelement of the fluid and volume of the element. Therefore we have,

$$dm = \rho . dv \quad (2)$$

$$dm = \rho \, dx . dy . dz \quad (3)$$

The acceleration of a fluid element is given by

$$a_p = \frac{DV}{Dt}$$

$$a_p = \frac{\partial V}{\partial t} + (V.\nabla)$$

$$a_p = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial t}$$

Where v, v and w arte the velocity of the fluid element in the axial directions.

The components of acceleration 'a_p' in the x, y, and z direction are as follows,

$$a_{x_p} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \quad (1)$$

$$a_{y_p} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \quad (2)$$

$$a_{z_p} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \quad (3)$$

Now we need to find out the forces acting on infinitesimal fluid element.

$$\vec{d}F = (dm) \quad (4)$$

Since the external force applied is body force, which is due to the gravity. Therefore these body force are equivalent to gravitaional force. Therefore we have,

$$dF_B = dm.g = \rho\ g\ dx\ dy\ dz \quad (5)$$

The surface forces are the forces exerted on the surfces of the fluid element from different normal directions. These surface forces are due to the normal and shear stresses. The surface forces acting on the six surfaces of the fluid element on the opposite directions are equal in magnitude and opposite in direction. Therefore the sum of these forces applying on fluid element is equal to zero. Let the stresses at the centre of the fluid element at point 'O' given by;

The surface force acting on the fluid element in x direction at the right face of the element is given by,

$$dF_{s_{(x,r)}} = (\ \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}.\frac{dx}{2})dy.dz \quad (6)$$

Now we right down the expression of surface exerting in x direction at all the remaining faces.
Now let us consider the left face, thus the focrce acting on the left face in x direction is given by,

$$dF_{s_{(x,l)}} = -(\ \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}.(-\frac{dx}{2}))dy.dz \quad (7)$$

For positive σ_{xx}, the force due to σ_{xx}, since the outward area normal for this face is;
Surface force acting on the front face is given by,

$$dF_{s_{(x,f)}} = (\ \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}.(\frac{dz}{2}))dx.dy \quad (8)$$

Surface force acting on the back face is given by,

$$dF_{s_{(x,bf)}} = -(\ \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}.(-\frac{dz}{2}))dx.dy \quad (9)$$

$$dF_{s_{(x,bf)}} = (-\ \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}.(\frac{dz}{2}))dx.dy \quad (10)$$

Surface force acting on the top face is given by,

$$dF_{s_{(x,t)}} = \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial z} . \left(\frac{dy}{2}\right)\right) dx.dz \quad (11)$$

Surface force acting on the bottom face is given by,

$$dF_{s_{(x,b)}} = -\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial z} . \left(-\frac{dy}{2}\right)\right) dx.dz \quad (12)$$

$$dF_{s_{(x,b)}} = \left(-\tau_{yx} + \frac{\partial \tau_{yx}}{\partial z} . \left(\frac{dy}{2}\right)\right) dx.dz \quad (13)$$

Total force acting in the x-direction = The sum of the body forces

$$dF_x = dF_{Bx} + dF_{sx}$$

$$dF_x = \rho g_x \, dx\, dy\, dz + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} . \frac{dx}{2}\right) dy.dz + \left(-\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} . \left(\frac{dx}{2}\right)\right) dy.dz + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} . \left(\frac{dz}{2}\right)\right) dx.dy + \left(-\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} . \left(\frac{dz}{2}\right)\right) dx.dy + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial z} . \left(\frac{dy}{2}\right)\right) dx.dz + \left(-\tau_{yx} + \frac{\partial \tau_{yx}}{\partial z} . \left(\frac{dy}{2}\right)\right) dx.dz$$

$$dF_x = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial z} + \frac{\partial \tau_{zx}}{\partial z}\right) dxdydz$$

Since dm is the mass of the fluid element. Therefore we have,

$$dm = \rho \, dx.dy.dz$$

The accelaration of the fluid element is given by,

$$a_{x_p} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

Therefore by using the principle of conservation of momentum, we get the following expression,

$$\left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial z} + \frac{\partial \tau_{zx}}{\partial z}\right) dxdydz = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

Thus the conservation equation for x direction is given by,

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

It can find out the momentum conservation equation for y and z directions. Therefore the momentum conservation equations for these directions are given as follows,

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \left(\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \left(\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)$$

Therefore the momentum conservation equations in x, y and z directiona are given as follows,

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \left(\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)$$
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \left(\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right) \quad (18)$$

These equations can be written as follows by using the definition of the material derivativce,

$$\rho\frac{Dv}{Dt}=\rho(\frac{\partial v}{dt}+(V.\nabla))=\rho g+\nabla.\bar{\bar{\sigma}} \tag{14}$$

where $\bar{\bar{\sigma}}$ is known as Cauchy stress tensor and it is defined

$$\bar{\bar{\sigma}}=\begin{bmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_{zz}\end{bmatrix}$$

Equation (19) is known as the Cauchy momentum equation.

Now since we have considered the fluid as a Newtonian fluid, therefore the shear stress at the opposite surfaces will be equal. That is the shear stresses are symmetric. The shear stresses for Newtonioan fluid is related with the angular deformation /shear rate in cartesian coordinates, therfeore we have

$$\begin{aligned}\tau_{xy}=\tau_{yx}&=\mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})\\ \tau_{yz}=\tau_{zy}&=\mu(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z})\\ \tau_{zx}=\tau_{xz}&=\mu(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})\end{aligned} \tag{20}$$

The expression for normal stress are given by,

$$\begin{aligned}\sigma_{xx}&=[P+(\frac{2}{3}\mu-k)\nabla.V]+2\mu\frac{\partial u}{dx}\\ \sigma_{yy}&=[P+(\frac{2}{3}\mu-k)\nabla.V]+2\mu\frac{\partial v}{dy}\\ \sigma_{zz}&=[P+(\frac{2}{3}\mu-k)\nabla.V]+2\mu\frac{\partial w}{dz}\end{aligned} \tag{15}$$

Now substituting the expression of normal and shear stresses given in equation (20) and (21) in the cauchy momentum equation (19), we get

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x+\frac{\partial}{\partial x}\left[-\left[P+\left(\frac{2}{3}\mu-k\right)\nabla.V\right]+2\mu\frac{\partial u}{\partial x}\right]$$

$$+\frac{\partial}{\partial y}[\mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\mu(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z})] \tag{16}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{dx}+\frac{\partial}{\partial x}\left[-\left(\frac{2}{3}\mu-k\right)\nabla.V\right]$$

$$+2\mu\frac{\partial u}{dx}+\frac{\partial}{\partial y}[\mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\mu(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z})]$$

Since the flow of the fluid is incompressible and the viscosity of the fluid is contant. Therefore we have,

$$\nabla.V=0$$

Now substituting the above value in equation (22) we obtain the following expression,

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\rho g_x-\frac{\partial p}{dx}+\frac{\partial}{\partial x}(2\mu\frac{\partial u}{dx})+$$

$$\frac{\partial}{\partial y}[\mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\mu(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z})]$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+2\mu\frac{\partial^2 u}{\partial x^2}+ \quad (17)$$

$$\mu\frac{\partial^2 v}{\partial x\,\partial y}+\mu\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 w}{\partial z\,\partial x}+\mu\frac{\partial^2 v}{\partial z^2}$$

Since the fluid is Newtonian, Incompressible, and the viscosity is constant. Therefore it can be obserevedc that,

$$\nabla.V=\mu\frac{\partial p}{\partial x}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$

$$\nabla.V=\mu\frac{\partial^2 v}{\partial x\,\partial y}+\mu\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 w}{\partial z\,\partial x}+\mu\frac{\partial^2 v}{\partial z^2}=0$$

Now substituting the above values in equation (23) we get the following expression,

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_x-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

Therefore the equation are given by,

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_x-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_y-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_z-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

In this section we are going to develop the equation of the cardiovascular system by using the Navier-Dtokes equations. The components for velosity in x, y and z direction are named u, v,w. P has taken as the pressure of blood, ρ as density of blood and μ as the kinematic viscosity. Also assuming g=0. The blood is an incompressible, Newtonian fluid. The Navier-Stokes equation in cartesion coordinates is given by,

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}) \quad (1)$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}) \quad (2)$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}) \quad (3)$$

Now we shall convert the above Navier-Stokes equation given in equations (1-3) into cylinderical co-ordinates by using the followwing varibles.

$$x=r\cos\theta,\quad y=r\sin\theta,\qquad z=z$$

Thus we get the following Navier-Stokes equation in cylinderical Co-ordinates,

$$-\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right)=\rho\left(v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}+\frac{\partial v_r}{\partial t}-\frac{v_\theta^2}{r^2}-\frac{v_r}{r^2}\right)$$

Now deviding the above equaion by ρ and rewritting the equation we get,

$$v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+v_z\frac{\partial v_r}{\partial z}+\frac{\partial v_r}{\partial t}-\frac{v_\theta^2}{r^2})=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}-\frac{v_r}{r^2}\right) \quad (4)$$

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{\partial v_\theta}{\partial t}+\frac{v_\theta v_r}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial r}+\delta\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta}{r^2}\right) \quad (5)$$

$$v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}+\frac{\partial v_z}{\partial t}+=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right) \quad (6)$$

where $\delta=\frac{\mu}{\rho}$ is the viscosity of the fluid.
Now due to the axisymmetric flow, we asume that there is no tangential velocity, that means ($v_\theta=0$), then above equation (4-6) can be reduced as follows,

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}\right) \quad (7)$$

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{\partial^2 v_z}{\partial z^2}\right) \quad (8)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}=0 \quad (9)$$

For the sake of simplicity, we can replace $v_r=f$ and $v_z=w$ in above equatio. Thus the above equations takes the following forms.

$$\frac{\partial w}{\partial t}+f\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}\right) \quad (10)$$

$$\frac{\partial f}{\partial t}+f\frac{\partial f}{\partial r}+w\frac{\partial f}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)+\frac{\partial^2 f}{\partial z^2}-\frac{f}{r^2}\right) \quad (11)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rf)+\frac{\partial w}{\partial z}=0 \quad (12)$$

$$\frac{\partial w}{\partial t}+f\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \quad (13)$$

$$\frac{\partial f}{\partial t}+f\frac{\partial f}{\partial r}+w\frac{\partial f}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\delta\left(\frac{\partial^2 f}{\partial r^2}+\frac{1}{r}\frac{\partial f}{\partial r}+\frac{\partial^2 f}{\partial z^2}-\frac{f}{r^2}\right) \quad (14)$$

The continuity equation is given by,

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho w)}{\partial z}=0 \quad (15)$$

Here new transformation

$$\gamma=\frac{\lambda r}{R(z,t)} \quad (16)$$

where R (z,t) is know as radius.

r = Radial direction.

γ = Viscosity.

By using radial coordinates transformation (0.0.14) for the z component of momentum in (13), we get

$$\frac{\partial w}{\partial t} + (\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial R}\frac{\partial R}{\partial t}) + f(\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial r}) + w(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial R}\frac{\partial R}{\partial z})$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial r} + \delta[(\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial r})(\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial r}) + \frac{1}{r}(\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial r}) + ((\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial z}))((\frac{\partial w}{\partial \gamma}\frac{\partial \gamma}{\partial z}))] \quad (17)$$

Now substituting these values in above equation.

$$\frac{\partial \gamma}{\partial R} = \frac{\partial}{\partial R}(\frac{\lambda r}{R}) = -\frac{\lambda r}{R^2}$$

$$\frac{\partial \gamma}{\partial r} = \frac{\partial}{\partial R}(\frac{\lambda r}{R}) = \frac{\lambda}{R}$$

And

$$\frac{\partial \gamma}{\partial z} = \frac{\partial}{\partial R}(\frac{\lambda r}{R}) = 0$$

$$\frac{\partial w}{\partial t} + (\frac{\partial w}{\partial \gamma}(\frac{\lambda r}{R^2})\frac{\partial R}{\partial t}) + f(\frac{\partial w}{\partial \gamma}(\frac{\lambda}{R})) + w(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial \gamma}(-\frac{\lambda r}{R^2})\frac{\partial R}{\partial z})$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial r} + \delta[(\frac{\partial w}{\partial \gamma}(\frac{\lambda}{R}))(\frac{\partial w}{\partial \gamma}(\frac{\lambda}{R})) + \frac{1}{r}(\frac{\partial w}{\partial \gamma}(\frac{\lambda}{R}))] \quad (18)$$

from simplify the equation (0.0.18),we get

$$\frac{\partial w}{\partial t} - \frac{\lambda r}{R}[(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial z}) - f]\frac{\partial w}{\partial \gamma}(\frac{\partial w}{\partial \gamma}(\frac{\lambda}{R})) + w(\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\lambda^2 \delta}{R^2}[(\frac{\partial^2 w}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial w}{\gamma}] \quad (19)$$

now take continuity equation and take trasformation (0.0.15),we get

$$\frac{1}{r}\frac{\partial}{\partial r}(rf) + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial f}{\partial r} + \frac{f}{r} + \frac{\partial w}{\partial z} = 0 \quad (20)$$

now applying the radial coordinate transformation in eqation (0.0.20)

$$\gamma = \frac{\lambda r}{R}$$

$$\frac{\lambda}{R}\frac{\partial f}{\partial \gamma} + \frac{f}{r} - \frac{\partial w}{\partial \gamma}\frac{\gamma}{R}.\frac{\partial R}{\partial z} + \frac{\partial w}{\partial z} = 0 \quad (21)$$

for the velocity components, simplyfied the above equation,

$$\frac{\gamma}{r}\frac{\partial f}{\partial r} + f - \frac{\gamma^2}{\lambda}\frac{\partial w}{\partial r}.\frac{\partial R}{\partial z} + (\frac{\gamma R}{\lambda})\frac{\partial w}{\partial z} = 0 \quad (22)$$

Integrating the above equation with respect to γ from the limits 0 to γ, we get

$$\int_0^{\gamma} \frac{\gamma}{r}\frac{\partial f}{\partial r}\, ds + \int_0^{\gamma} f\, ds + \int_0^{\gamma} (\frac{\gamma R}{\lambda})\frac{\partial w}{\partial z}\, ds - \int_0^{\gamma} \frac{\gamma^2}{\lambda}\frac{\partial w}{\partial r}.\frac{\partial R}{\partial z}\, ds = 0 \quad (23)$$

using boundary condition

$$w(o,z,t) = f(0,z,t) = 0$$
$$f(\gamma,z,t) = \frac{-R}{\gamma}\int_0^{\gamma}(\gamma)\frac{\partial w}{\partial z}\,ds + \gamma\frac{\partial R}{\partial z}w - \frac{2\lambda}{\gamma}\frac{\partial R}{\partial z}\int_0^{\gamma} w\gamma\,ds \tag{24}$$

$$\text{let} \quad f(\gamma) = \frac{-2}{N}\sum_{k=1}^{N}\frac{k+1}{k}(\gamma^{2k}-1)$$

we solving above equation, we get

$$-\int_0^1 \frac{\partial w}{\partial z}\,d\gamma = \int_0^1 \gamma\left[\lambda\frac{f(\gamma)}{R}\frac{\partial R}{\partial t} + \frac{2w\lambda}{R}\frac{\partial R}{\partial z}\right]d\gamma \tag{25}$$

we get,

$$-\frac{\partial w}{\partial z} = \lambda\frac{f(\gamma)}{R}\frac{\partial R}{\partial t} + \frac{2w\lambda}{R}\frac{\partial R}{\partial z} \tag{26}$$

after solving the equation (0.0.26) and putting the f(γ)

$$f(\gamma,z,t) = \gamma\frac{\partial R}{\partial z}w + \gamma\frac{\partial R}{\partial t} - \frac{\gamma}{N}\frac{\partial R}{\partial t}\sum_{k=1}^{N}\frac{1}{k}(\gamma^{2k}-1) \tag{27}$$

we assume the velocity profile in the axial direction can be expressed as the following polynomial form :

$$w(\gamma,z,t) = \sum_{k=1}^{N} q_k(\gamma^{2k}-1) \tag{28}$$

say, n =1, above equation can be expressed as

$$f(\gamma,z,t) = \frac{\partial R}{\partial z}\gamma w + \frac{\partial R}{\partial t}\gamma - \frac{\partial R}{\partial t}\gamma(\gamma^{2k}-1) \tag{29}$$

$$w(\gamma,z,t) = q(z,t)(\gamma^2-1) \tag{30}$$

using the equation and putting the in equation (29)and (0.0.30)and (19), we the get Navier Stokes equation as below

$$\frac{\partial q}{\partial t} - \frac{4q}{R}\frac{\partial R}{\partial t} - \frac{2q^2}{R}\frac{\partial R}{\partial z} + \frac{4\mu}{R}q + \frac{1}{\rho}\frac{\partial P}{\partial z} = 0 \tag{31}$$

$$2\frac{\partial R}{\partial t} + \frac{R}{2}\frac{\partial q}{\partial z} + q\frac{\partial R}{\partial z} = 0 \tag{32}$$

we multiplying equation(0.0.32) with π and using chain rule $\frac{\partial Q}{\partial z}$ and $\frac{\partial s}{\partial t}$, we gets

$$\frac{\partial Q}{\partial t} + \frac{3Q}{s}\frac{\partial s}{\partial z} - \frac{Q^2}{s^2}\frac{\partial s}{\partial z} + \frac{4\pi\mu}{s}Q + \frac{s}{2\rho}\frac{\partial P}{\partial z} = 0 \tag{33}$$

where,

$$\frac{\partial Q}{\partial z} = -\frac{\partial s}{\partial t}$$

We are developing the model of flow of the blood as a two-fluid with axisymmetric stenosis in an artery within the core as a non-Newtonian fluid and the peripheral as a Newtonian (plasma).

The fluid in a core region of an arteryis assumed as a Herschel-Bulkley and casson fluid has non- Newtonian fluid. To finding the mathematical modelling uses perturpation technique, system of non-linear partial differential equations.

Here taking the two -layered axisymmetric flow of blood through an axisymmetric stenosis. the geometry of stenosis is given by

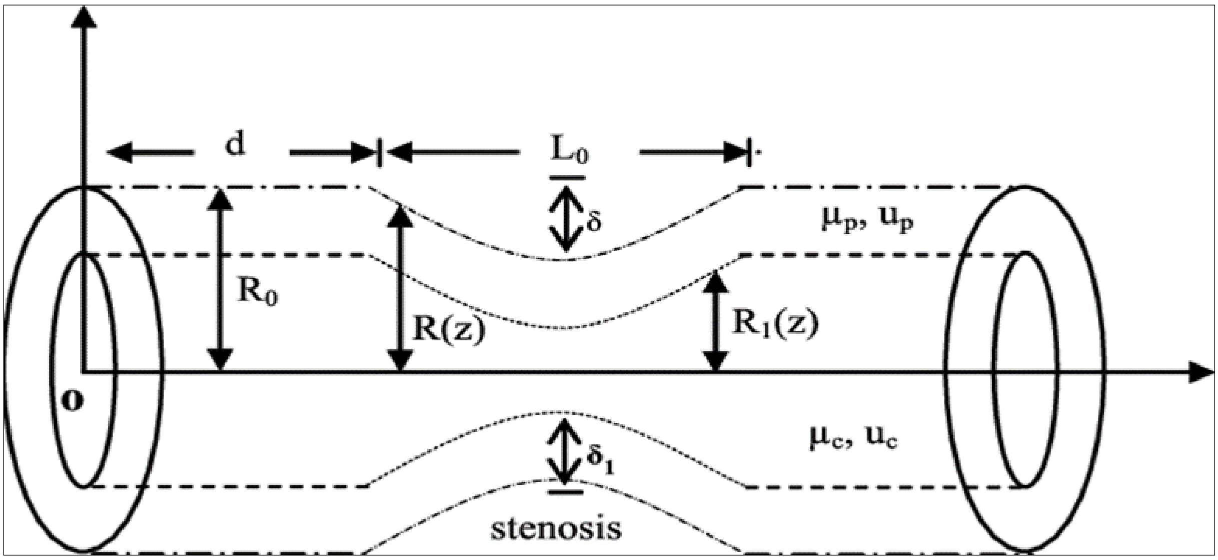

Fig 2: The shape of the central layer

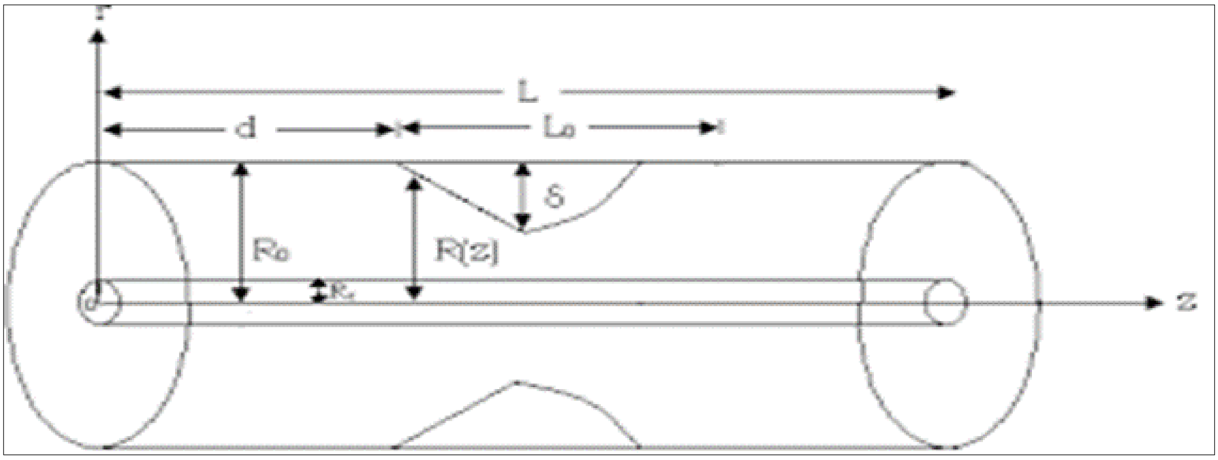

Fig 3: 1 Geometry of a composite stenosis in a catheterized artery

$$\frac{R(z)R_1(z)}{R_0} = \begin{cases} (1,\beta) - \frac{\lambda\lambda_1}{2R_0}\left[1+\cos\left(\frac{2\pi}{L_0}\right)\left(z-d-\frac{L_0}{2}\right)\right], & d \le z \le d+L_0 \\ (1,\beta), & \text{Otherwise} \end{cases} \qquad (1)$$

Where the different parameters which are mentioned in the above equation are defined as follows,

R_1 = Radius.

μ = shows velocity of fluid.

v =show the velocity of fluid.

r = radial coordinates.

z = axial coordinates.

R_0 = the radius of the normal.

L_0 = the length of stenosis.

d = location of stenosis.

β = Ratio

(λ,λ_1) = maximum stenosis and the bulging.

The suffix p and c denote the periphered and central layers.

The two layered Newtonian fluid has been assumed. The equation for laminar steady, one dimensional flow whithin side the case of a throughts stenosis:

$$\frac{\partial P}{\partial z} = \frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)v_p, \qquad R_1 \le r \le R(z) \qquad (2)$$

$$\frac{\partial P}{\partial z} = \frac{\mu_c}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})v_c, \qquad 0 \le r \le R_1(z) \tag{3}$$

Here,

$$\mu_p = \text{viscosity } R_1(z) \le r \le R(z)$$
$$v_p = \text{viscosity } R_1(z) \le r \le R(z)$$
$$\mu_c = \text{viscosity } 0 \le r \le R_1(z)$$
$$v_c = \text{viscosity } 0 \le r \le R_1(z)$$

using boundary conditions,

$$\frac{\partial v_c}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{4}$$

$$v_p = v_c \text{ and } \mu_p\frac{\partial v_p}{\partial r} = \mu_c\frac{\partial v_c}{\partial r} \quad \text{at} \quad r = R_1(z) \tag{5}$$

$$v_p = v_c \text{ and } \mu_p\frac{\partial v_p}{\partial r} = \frac{\alpha}{\sqrt{k}}(v_B - v_{porous}) \quad \text{at} \quad r = R(z) \tag{6}$$

where

$$v_{porous} = \frac{-k}{\mu_p}\left(\frac{dP}{dz}\right) = \text{velocity in permeable boundary.}$$

v_B = slip velocity
k = darcy's number.
α = slip parameter.

The solutions of equation (2),(3) under with (4), (5), and (6), we get the expression of velocity v_p and v_c as

$$v_p = \frac{R_0^2}{4\mu_p}(\frac{dP}{dz})[(\frac{R}{R_0})^2 + (\frac{r}{R_0})^2 - 2\frac{R\sqrt{k}}{R_0^2\alpha} + 4\frac{k}{R_0}] \tag{7}$$

$$v_c = \frac{R_0^2}{4\mu_p}(\frac{dP}{dz})[(\frac{R}{R_0})^2 + \mu(\frac{r}{R_0})^2 - (1-\mu)(\frac{R_1}{R_0})^2 - 2\frac{R\sqrt{k}}{R_0^2\alpha} + 4\frac{k}{R_0}] \tag{8}$$

With $\mu = \frac{\mu_p}{\mu_c}$ The volumetric flow rate Q is given by

$$Q = 2\pi[\int_0^L rv_c\, dr + \int_0^L rv_p\, dr]$$

Putting the values of v_c and v_p and simplifying, we get

$$Q = -\frac{\pi R_0^4}{8\mu_p}(\frac{dP}{dz})[(\frac{R}{R_0})^4(1-\mu)(\frac{R_1}{R_0})^2 + \frac{8k}{R_0^2}(\frac{R}{R_0})^2 - 4\frac{R\sqrt{k}}{\alpha R_0}(\frac{R}{R_0})^3)] \tag{9}$$

Results and Discussion:
Flow rate with blood viscosity
The mathematical model's equation can be solved by MATLAB software. Figure 4: shows The variation of flow rate with viscosity for different value of stenosis shape. And x-axis represent the stenosis shape(cm), y-axis denotes the viscosity of blood (Pascal-second). It can reprsents the solution of different value of stenosis shape. In this plot, we can observe that the viscosity decreases with radius of stenosis artery. i.e $\mu \propto r$ and the parmeter decreases as the flow rate increases.

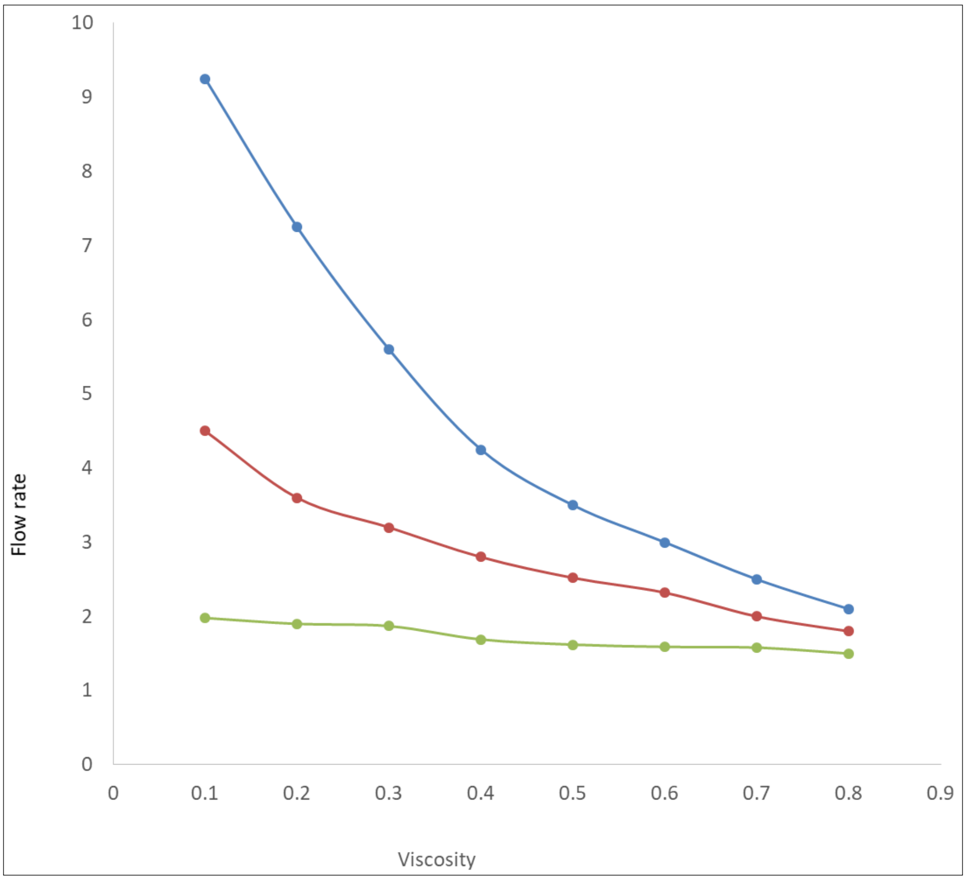

Fig 4: Variation of flow rate with viscosity for different values of stenosis shape

Flow Rate with Pressure

The equation of model can solve by MATLAB. Figure.5 shows flow rate with pressure for different stenosis shape where x-axis denotes shape of stenosis(cm) andy-axis denotes flow rate of blod (cm-sec). it can represent the flow rate with pressure for different stenosis shapes. As we can observe from the plot when pressure increases the flow rate of blood also increases.

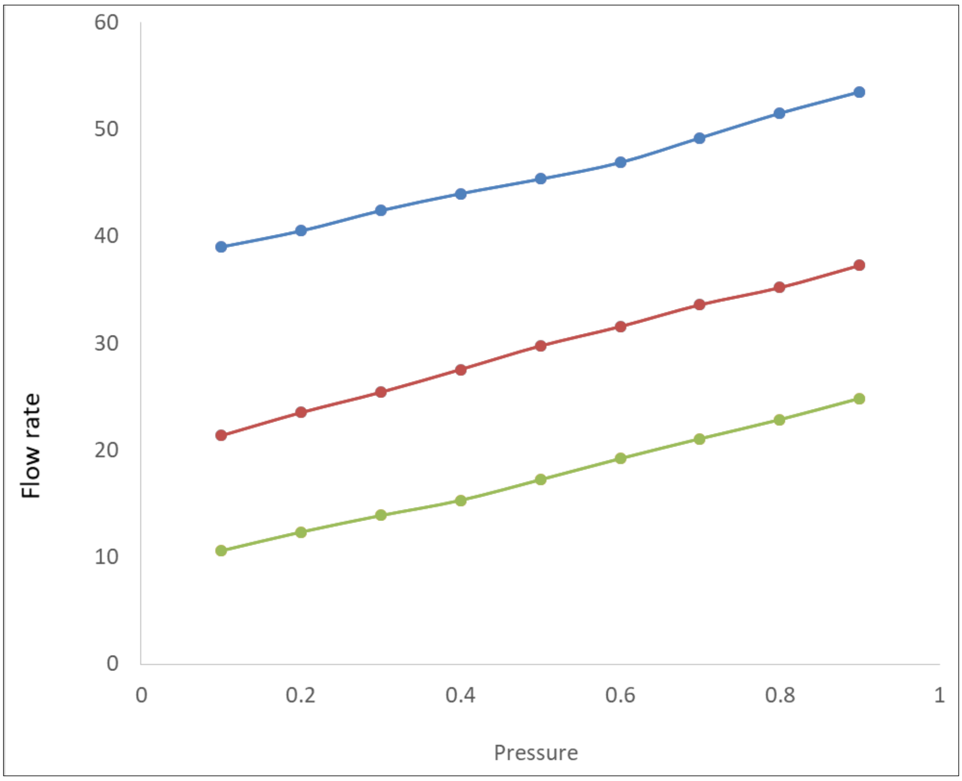

Fig 5: Variation of flow rate with pressure for different values of stenosis shape parameter

Conclusion

In this project a blood flow problem has been considered to obtained the effects of shape parameters on blood flow parameters. The main aim of this project is to develop a mathematical model of blood flow and extended the model for two -fluid blood flow through artery. We concluded the important result for variation between flow rate (Q) and viscosity (μ) with different values of shape parmeter. The second result which shows that the flow rate increases with the viscosity increases. Similarly when the flow rate will increases then pressure also increases.

References

1. Petrila T, Tarif D. Basic of fluid mechanics and introduction to computational fluid dynamics,ISBN0-387-23837-3,Springer U.S.A., 2005.
2. Latex, Making Presentations with LATEX, Guidelines, Xavier Perseguers.
3. Ellahi R, Rahman SU, Mudassar M, Nadeem, Vafai K. Mathematical study of non-newtonian micropolar fluid in arterial blood flow through composite stenosis, aplied mathematics and information Science,2014:8(4):1567-1573.
4. Ku DN. Blood floe in arteries, Annual Review of fluid Mechanics,1997:29:399434.
5. chakrvarty S, Mandal PK. Mathematical Modelling of Blood Flow Through an Overlapping Arterial Stenosis.
6. Dr. Sapan Ratan shah, Rohit kumar. Mathematical modelling of blood flow with the suspension of nanoparticles through a tapered artery with a blood clot.
7. JC Misra, Gc Shit. Blood flow through arteries in a pathological state: A theoretical study, international journal of engineering science 44(2006),662-671.
8. Gopal Chandra, Malay roy. Mathematical modelling of blood flow through a tapered overlapping stenosed artery with variable viscosity.
9. Pramod Kumar Pant, Dr. Ajay Kumar Gupta, Dr. Mohammad Miyan. Analytical Survey on the Two-Fluid Blood Flow through Stenosed Artery with Permeable Wall.
10. Misra JC, Shit GC. ROle of slip velocity in blood flow through stenosed arteries:A non- Newtonian model, journal of mechanics in medicine and biology,2007:7:337-353.
11. Young DF. Effect of a time -dependent stenosis on flow through a tube, J.Engn. Ind, trans ASME,1968:90:284-254.
12. forrester JH, Young Df. flow through a converging diverging tube and its implications in occlusive vascular disease, J. Biomech,1970:3:297-316.
13. Lee JS, Fung YC. Flow in locally constricted tubes at low Reynolds numbers,J. Appl.Mech,1970:37:9-16.
14. Young DF, Tsai FY. flow characteristics in models of arterial stenosis- I, steady flow, j. biomech,1973:6:395-411.
15. ECharm S, Kurland G. Viscometry of human blood for shear rates 0-100,00sec^{-1} Nature,1965:206:617-618.
16. Whitemore RL. Rheology of the Circulation, pergamon press, New york, NY, 1968.
17. Chakravarty S, Datta A. Effects of stenosis on arterial rheology through a mathematical model, meth. Comp. Modelling,1961:12:1601-1612.
18. Chakravarty S. Effect of stenosis on the flow behaviour of blood in an artery, It. J. Engng. Sci,1987:25:1003-1018.
19. McDonald DA. blood flow in arteries, Edward, 1974.
20. Milnor WR. Hemodynamics, Williams and Williams, Baltimore, MD, 1982.
21. thurston GB. effects of viscoelasticity of blood on wave propagation in the circulation, J. Biomech,1976:9:13-20.
22. Fry DL. Acute vascular endothelial changes associated with increased blood velocity gradient, circulation Res,1968:22:165-197.
23. Chakravarty S, Datta A. Dynamic response of stenotic blood flow in vivo, Math Comp. modelling,199216:3-20.
24. Pramod kumar Pant, Ajay Kumar Gupta, Mohammad Miyan. Analytical Survey on the two-fluid blood flow through stenosed artery with permeable wall.
25. S Oka. Cardiovascular hemorheology, Cambridge University Press, london, 1981, 28.
26. Rupesh K Srivastav, Qazi Shoed Ahmad, Abdul Wadood Khan. Two-Phase model of blood flow though a composite stenosis in the presence of a peripheral layer.