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The study on the characteristics of f distribution tending to normal distribution

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Abstract
In this paper, we studied the specific characteristics of F-distribution when its two degrees of freedom increase with the use of MATLAB software. The results showed that when the two degrees of freedom increase, the F distribution gradually changes from a positively skewed distribution to a normal distribution, its peak value gradually increases, and its points of maximum values gradually shift to the right.

Keywords: f distribution, normal distribution, degrees of freedom, skewness

1. Introduction
F distribution is an important one in probability statistics. When the two degrees of freedom of F distribution tend to infinity, it will gradually change to a normal distribution, which has been proved with various methods (Liang, 2014; Zeng and Zhang, 2015; Zhou *et al*., 2007; Li and Meng, 2004; Liang, 1995) [1, 2, 3, 4, 5]. However, no specific description of this approaching process was given by now. Therefore, we intends to use MATLAB software to study the specific changing process of F distribution when two degrees of freedom increase, and explore the characteristics of this change.

2. The situation when the first degree of freedom increases
The analytic formula of probability density function of the F distribution

Is $f_F(x;m,n)=\frac{\Gamma[\frac{(m+n)}{2}]}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}(\frac{m}{n})^{\frac{m}{2}}\frac{x^{\frac{m-2}{2}}}{[1+(\frac{m}{n})x]^{\frac{m+n}{2}}}$,

Which has two parameters, that is, the first degree of freedom *m* and the second degree of freedom *n* respectively. When *m* and *n* tend to infinity, the F distribution tends to be a normal distribution with a mean value of 1 and a variance of $\frac{2}{\sqrt{n}}$, that is $N(1,\frac{2}{\sqrt{n}})$ (Zeng and Zhang, 2015) [2].

In order to understand the specific details of the approximation process above, we set the second degree of freedom of the F distribution to a fixed value of 100 firstly, increased the first degree of freedom from 1 to 101, and set the step length to be 10, and drew the probability density function image of F distributions and a normal distribution. The MATLAB codes we used are as follows:

```
Clear all; clc;
x=-3:0.01:5;
n=linspace (1, 101, 10);
For i=1:10
B (i, :) =fp DF (x, n (i), 100) ;
End
y=normpdf (x, 1, 0. 2);
Plot (x, y, 'color', 'r', 'line width', 2. 1);
Hold on;
Plot (x, B, 'color', 'b', 'line width', 2);
X label ('x')
Legend (‘N (1, 0.2)','F (m, n), m from 1 to 101, n=100');
Title ('F distribution of m from 1 to 100');
Axis ([0 2 0 2.3]);
Grid on;
```

The obtained results were shown in figure 1.

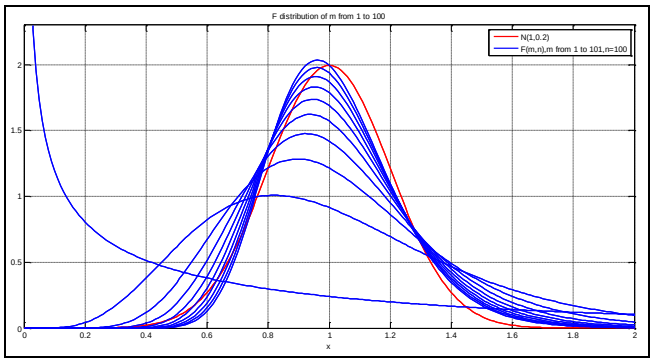

Fig 1: F distribution with the first degree of freedom increasing

It can be seen from Figure 1 that the probability density functions of F distribution are all positively skewed distribution; When the first degree of freedom of F distribution remains unchanged and the second degree of freedom increases, the function image shifts to the right and tends to be in a normal distribution overall, which means it gradually becomes symmetrical; Meantime, with the increase of the second degree of freedom, the peak value of F distribution gradually increases and the overall value is closer to its corresponding normal distribution; However, when the value of *m* reaches 101, The maximum value of the F function exceeds the maximum value of the normal distribution function $N(1,\frac{2}{\sqrt{n}})$.

2. The situation when the second degree of freedom increases

In order to understand the specific details of the above approximation process, we also set the first degree of freedom of the F distribution to a fixed value of 100 and increased the second degree of freedom from 1 to 101. The following codes were used to draw the image of the probability density function of F distribution and normal distribution $N(1,\frac{2}{\sqrt{n}})$.

```
Clear all; clc;
x=-3:0.01:5;
n=linspace (1, 101, 10);
For i=1:10
A (i, :) =normpdf(x, 1, 2/sort (n (i))) ;
B (i, :) =fpdf (x, 100, n (i)) ;
End
Plot (x, A, 'color', 'r', 'line width', 2.1);
Hold on;
Plot (x, B, 'color', 'b', 'line width', 2);
X label ('x')
Legend (‘N (1, 2/sqrt (n))’,'n from 1 to 101, m=100');
Title ('F distribution of second freedom from 1 to 100');
Axis ([0 2 0 2.3]);
Grid on;
```

The results obtained were shown in Figure 2:

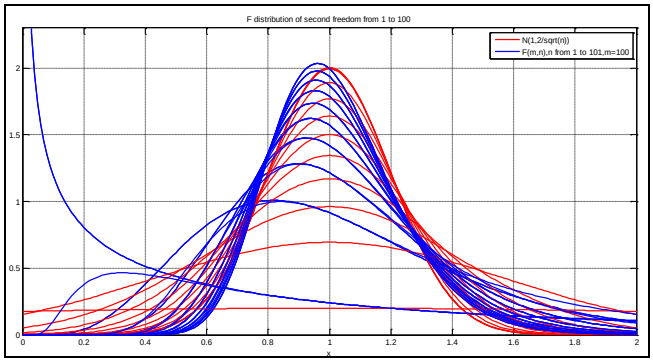

Fig 2: F distribution and normal distribution with the second degree of freedom increasing

It can be seen from Figure 2 that when *n* increases, the symmetry axis of the normal distribution $N(1,\frac{2}{\sqrt{n}})$ remains at 1, but its peak value increases gradually, and the function image gradually becomes taller and thinner; For the F distribution, when the second degree of freedom increases, the function changes from positively skewed distribution to normal distribution; At the same time, its peak value also increases. Therefore, the maximum value points of the two distributions are gradually approaching, so that the two function images of distribution are gradually approaching overall.

4. The situation when two degrees of freedom increase simultaneously

In order to understand the details of the approximation process above, we increased the two degrees of freedom of F distribution from 1 to 101 at the same time. The following codes were used to draw the probability of the image of F distribution and normal distribution $N(1,\frac{2}{\sqrt{n}})$.

```
Clear all; clc;
x=-3:0.01:5;
n=linspace (1, 101, 10);
For i=1:10
A (i, :) =normpdf (x, 1, 2/sqrt (n (i))) ;
B (i, :) =fpdf (x, n (i), n (i)) ;
End
Plot (x, A, 'color', 'r', 'line width', 2.1);
Hold on;
Plot (x, B, 'color', 'b', 'line width', 2);
X label ('x')
Title ('F distribution of two freedoms from 1 to 100');
Axis ([0 2 0 2.3]);
Grid on;
```

The obtained results are shown in figure 3

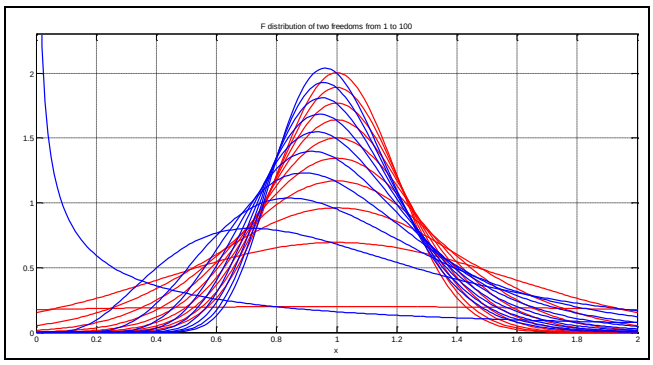

Fig 3: F distribution of two degrees of freedom changing simultaneously

It can be seen from Figure 3 that the image of F distribution with two parameters changing and its image with only one parameter changing are of great similarity. They all change from positively skewed distribution to normal distribution, and its peak value gradually approaches the peak value of normal distribution. The function image of F distribution approaches the image of corresponding normal distribution overall, and the approaching degree is better and faster than that with only one parameter increasing.

5. Conclusion

Based on the conclusions of previous study, this article analyzed the characteristics of F distribution with the increase of degrees of freedom further by using MATLAB software. According to the analysis above, the conclusions obtained are as follows:

1. When the two degrees of freedom of the F distribution increase, the F distribution approaches the corresponding normal distribution, no matter only one of the parameters is increased or both parameters increase simultaneously, and the approaching trends are same;
2. When the parameters increases, the F distribution gradually changes from a positively skewed distribution to the normal distribution with the symmetry axis of 1, and its peak value increases gradually, approaching the corresponding normal distribution overall;
3. When two parameters are increased at the same time, the approaching degree of F distribution to normal

distribution is better than that with only one parameter increased.

In order to make the function images easier to observe and analyze, this study changed the degrees of freedom of the F distribution from 1 only to 101, did not discuss the situation that the degrees of freedom is bigger than 101. If the degree of freedom is further increased, some more elaborate characteristics can probably be discovered. This study only analyzed the image of the F distribution function and the image of the normal distribution function and did not calculate the kurtosis value and skewness value of the F distribution function during the change of its degrees of freedom. If these two values were analyzed further, more properties would be supposed to be found.

6. References

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