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An overview of differential equation with physics

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Abstract
In the world all the system undergoing change either decay directly or growth/decay exponentially can be described with the help of differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of discipline such as physics, chemistry, biology and engineering. A differential equation is one which is written in the form of dy/dx=................ Some of these can be solved to get y =............. Simply by integrating or other require much more regrous mathematical method and the result comes by solving the diff. equations which will explain the very interesting phenomenon relating to physics and other disciplines also.

Keywords: ordinary differential eqn, partial diff. eqn, carbon dating, radioactive decay, integral, potential difference

Introduction

The mathematical theory of differential equation first developed together with the sciences where the eqns. had originated and where the results found application. However diverse problems sometimes originating in quite distinct scientific fields, may give rise to identical differential equations, whenever this happens, mathematical theory behind the eqns. can be viewed as a unifying principle behind diverse phenomena, e.g. consider propagation of light and sound in the atmosphere and of waves on the surface of a pond. All of them may be described by the same second order partial diff. eqn., the wave equation which allows us to think of light and sound as forms of waves much like familiar waves in the water. An example of modeling a real world problems using differential eqns. is the determination of the velocity of a ball falling through the air, considering only gravity and air resistance. The ball's acceleration towards the ground is the acceleration due to gravity minus the accelerating due to air resistance. The means that the ball's acceleration which is derivatives of its velocity, depends on time (i.e. $\frac{dv}{dt}$). Similarly, Newton's second law of motion expressed in its most common form. Acceleration is the time derivative of velocity and velocity is the time derivative of position i.e.

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} \qquad (1)$$

Hence, Newton's second law of motion is expressed as second order differential equation to be solved for position. Also, the differential equation can be used for instance to determine the motion of a simple pendulum by deriving the differential equation of the form :

$$\frac{d^2\theta}{dt^2} + \frac{g}{h}\theta = 0 \qquad (2)$$

Here are some more details and examples related to the importance and the uses of differential equations mentioned below:

Newton's law of cooling

It states that the rate at which a body loses heat (or cool) is directly proportional to the difference of temperatures between the body and its surroundings. Mathematically, we can write

$$-\frac{dQ}{dt} \propto (T - T_0) \text{ or } \frac{dQ}{dt} = -k(T - T_0) \qquad (3)$$

Where k is the proportionality constant.
Now, by using the required formula of physical quantity and integrating both sides, we get

$$\ln(T - T_0) = \frac{-k}{ms}t + c \qquad (4)$$

where c is integrating constant and S is sp. heat capacity of body. This eqn (4) is the equation of straight line, it means the quantity *ln* $(T-T_0)$ is directly proportional to time, which verifies Newton's law of cooling.
Also, the time of death of murdered person can be determined with the help of modeling through differential equation and Newton's law of cooling.

R–L–C Circuit

Consider discharge of a capacitor C may be oscillatory in character an inductance L is placed in series with it, the circuit also possesses some resistance R, shown in fig.–1

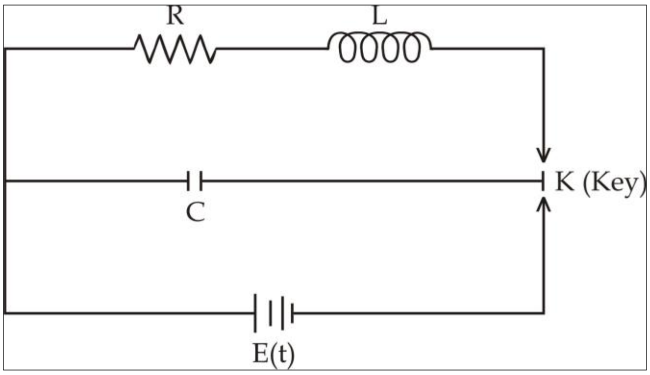

Fig 1: R–L–C Circuit

When the Key 'K' is pressed downwards the capacitor C is charged. Let q_0 be the charge on it and when 'K' is released the capacitor circuit is completed through the inductance and the resistance. The capacitor slowly loses its charge due to which there is a current I at any instant. The current varies as the rate $\frac{dI}{dt}$. If q is the charge on the capacitor some time later then the sum of pot. diff. across the components of the circuit when on source of e.m.f, i.e.

$$\frac{q}{C}+RI=-L\frac{dI}{dt} \text{ or, } \frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{1}{LC}q=0 \qquad (5)$$

which is the required differential equation form of the RLC circuit and solving this differential equation by using various mathematical formulae and after that it is found fruitful results. this result shows that the charge decaying exponentially with time.

Radioactive decay and carbon dating

Consider a sample of radioactive source containing N_0 number of radioactive atoms. After time t, N is the final number of atoms remained. Let $\frac{dN}{dt}$ be the rate of number of radioactive atoms disintegrating with respect to time. According to Rutherford and Soddy, the rate of disintegration is directly proportional to the number of un–decayed atom ie.

$$\frac{dN}{dt}\infty N$$
$$\frac{dN}{dt}=-\lambda N \qquad (6)$$

Where λ is proportionality constant known as decay constant and negative sign shows that the number of atoms decreases. The solution of the above eqn. (6) is

$$N=Noe^{-\lambda t} \qquad (7)$$

hence, from eqn (7) it is clear that the no. of radioactive atoms in terms of initial number after time t is exponentially decayed with time as shown in fig.–2.

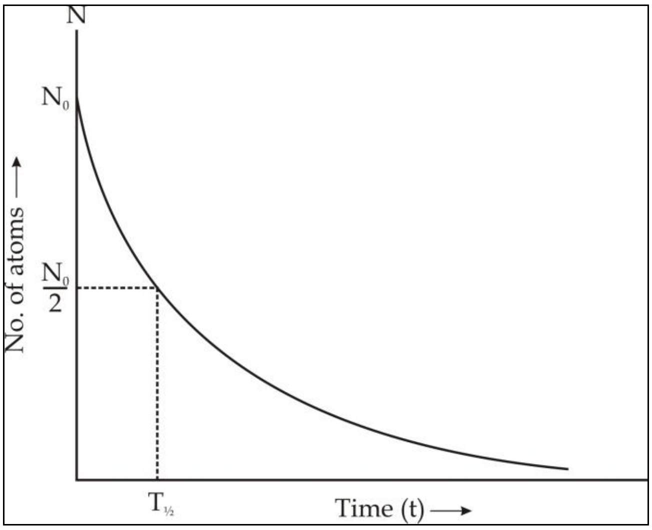

Fig 2

This result is used to estimate the age of archeological and geological objects like rocks, minerals etc is known as carbon dating or simply dating. The dating of the specimen is done by the measurement of the concentration of radioactive isotopes. For dating radioactive isotope of carbon–14 ($_6C^{14}$) is used.

This is to say a few, there are still more wonderful applications with using differential equation.

Conclusion

The laws of the natural and physical world are usually written and modeled in the form of differential equations. Differential equations involves the derivates of a function or set of functions. These equations arise in the study of rates of change and of quantities or things that change. They play significant role in physics. They can be used to describe a wide variety of phenomena such as Newton's law of cooling in thermodynamics, Radioactivity decay in nuclear physics, Euler–langrange eqn. & Hamilton's eqn. in classical mechanics, Maxwell's eqns. in electrodynamics and Schrödinger's eqn. in quantum mechanics etc.

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