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Magneto hydrodynamic convection in a stratified flow in an inclined porous channel

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Abstract

The convection in a stratified flow in an inclined porous channel bounded by two permeable layers under the influence of uniform magnetic field perpendicular to the channel is investigated. Here, we have assumed that the permeability of the fluid medium and the pressure gradient of the fluid flow are varying point to point. The flow between the permeable layers is governed by Navier—Stoke's equations, while the flow in the porous medium is governed by Darcy's law. The velocity field and temperature distribution are obtained and results are discussed with the help of various graphs.

Keywords: magneto hydrodynamic, porous channel, stratified flow, temperature distribution

1. Introduction

Convective flow problems of an inclined fluid layer involving porous media in the presence of a magnetic field have important applications in various branches of science and technology such as petroleum engineering, agricultural engineering, geophysics and bio-mathematics etc.

Flow in a porous medium is governed by either Darcy's law or its modified forms. Beavers and Joseph [1] investigated a slip boundary condition at the nominal surface of the porous bed and discussed the poiseuille flow over a permeable bed. Channabasappa and Ranganna [2] investigated the effect of stratification and slip velocity on the flow of viscous stratified fluid over a permeable bed Ramakrishna *et al*. Recently, MHD effects have been extensively investigated by Kumar *et al*. [3] Ramakrishna *et al*. [4] have studied the convection in a stratified fluid of variable viscosity in an inclined porous channel. Rudraiah and Wilfred [5] investigated the natural convection in an inclined channel bounded by porous media. Ball *et al*. [6] studied the MHD flow of a free convection heat transfer in an inclined square cavity field in a porous medium. The object of the present paper is to investigate the convection in a stratified flow in an inclined porous channel bounded by two permeable beds under the influence of a uniform transverse magnetic field perpendicular to the channel and an exact solution for the velocity profile has been obtained. The temperature distribution, the mass flow rate and its fractional increase are also obtained and discussed in detail with the help of various graphs.

2. Mathematical formulation of the problem

Let us Consider the flow of a stratified fluid of variable viscosity between two inclined porous permeable beds. A transverse magnetic field of constant strength is applied in the normal direction to the channel. The flow is Possible because of the imbalance between pressure and buoyancy force when the Grash off number is non-zero. The induced magnetic field has been neglected. Suppose that the channel is inclined at an angle ϕ with the horizontal. The upper and lower beds are kept at temperature $T_0 - \frac{1}{2} \Delta T$ and $T_0 + \frac{1}{2} \Delta T$ respectively, where T_0 is the ambient temperature of the fluid. The flow is in the x-direction and z-axis is normal to the beds. The physical model is shown in Fig. 2.1.

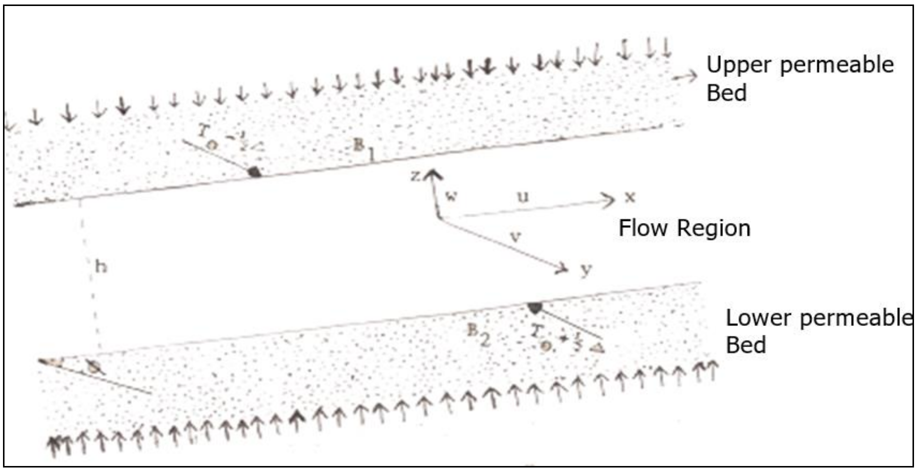

Fig 1: "Schematic Diagram"

In order to derive the basic equations of motion, we use the fó1lowing assumptions

1. The velocity component 'u' in the x-direction is a function of φ and z only.
2. The flow is steady.
3. The pressure 'p' is a function of x and z only.
4. The magnetic field is constant.
5. The dissipation function Φ is neglected and the transfer takes place by conduction only so that temperature distribution is linear in z.
6. The Grashoff number 'Gr' is small.
7. The permeability factor 'K', the pressure gradient $'\partial p/\partial x'$, the coefficient of viscosity $'\mu'$ and the density 'ρ' all vary from point to point and decay exponentially with z; that is, $\partial p/\partial x = \lambda e^{-\beta z}, K = K_0 e^{-\beta z} \mu = \mu_0 e^{-\beta z}$and $\rho = \rho_0 e^{-\beta z}$, where β > 0 is the stratification factor, $\lambda, K_0 \mu_0 and\, \rho_0$ are respectively the values of pressure gradient, permeability of the porous medium, coefficient of viscosity and density at z = 0.

Thus, if h be the distance between the upper and lower permeable beds and axis of the channel is chosen from the origin, then under these assumptions the flow is governed by the following system of equations:

$$\frac{d^2u}{dz^2} - \beta\frac{du}{dz} - \frac{\sigma B_0^2}{\rho v}u = \frac{1}{\mu_0}\left(\lambda e^{-\beta z} + \rho g \sin \quad \varphi\right)e^{\beta z}, \qquad \ldots (1)$$

$$\frac{\partial p}{\partial z} + \rho g \cos\varphi = 0 \qquad \ldots (2)$$

$$\frac{\partial u}{\partial x} = 0 \qquad \ldots (3)$$

$$\frac{d^2T}{dz^2} = 0 \qquad \ldots (4)$$

and
$$\rho = \rho_0\{1 - \beta(T - T_0)\} \qquad \ldots (5)$$

where $B_0 = |\vec{B}|$ is the magnitude of the magnetic induction $\vec{B}$; u is the velocity component of the fluid in the x-direction; T is the temperature of the fluid; σ is the electrical conductivity of the fluid; g is the acceleration due to gravity; υ is the kinematic viscosity of the fluid and φ is the angle inclination of the channel to the horizontal.

The Darcy velocity of the fluid is given as

$$q = q_0 e^{\beta z} \qquad \ldots (6)$$

where
$$q_0 = -\frac{K}{\mu_0}e^{-\beta z}\left(e^{-\chi z} + \rho g \sin \quad \varphi\right). \qquad \ldots (7)$$

The boundary conditions are given as

$$\left.\begin{array}{lll} z & = & \frac{h}{z} : u = u_{B_1};\ \frac{du}{dz} = -\frac{\alpha}{\sqrt{K}}\left(u_{B_1} - q_{B_1}\right); T = T_0 - ½\,\Delta T \\ z & = & -\frac{\pi}{2} : u = u_{B_2};\ \frac{du}{dz} = \left(u_{B_2} - q_{B_2}\right); T = T_0 + ½\,\Delta T \\ x & = & 0,\ z = 0 : p = 0. \end{array}\right\} \qquad \ldots (8)$$

Here, α is the slip parameter and K is the permeability coefficient of porous medium.

3. Solution of the problem

Now in order to non-dimensionalize equations (1) to (8), we make use of following transformation:

$\bar{x}$ = x/h, $\bar{z}$ = z/h, $\bar{T}$ = T/Δ T, $\bar{u}$ = uh/υ,

$\bar{q}$ = qh/υ, $\bar{p}$ = p/$\rho_0 gh$, $\bar{\lambda}$ = λ/$\rho_0 g$,

γ = β h, η = h/$\sqrt{K}$, δ = gh^3/υ^2, $\bar{M}^*$ = M*/μ_0, M = B$_0$h $\sqrt{\sigma}/\mu$, Gr = β δΔ T.... ... (9)

Using the above non-dimensional quantities and after removing bars over the variables for convenience, equations (1) to (7) in the non-dimensional form become:

$$\frac{d^2u}{dz^2} - \gamma\frac{du}{dz} - M^2u + Pe^{\gamma z} + (T - T_0)Gr\sin\varphi e^{\gamma z} = 0, \quad \text{... (10)}$$

$$\frac{\partial p}{\partial z} + \left\{1 - (T - T_0)\frac{Gr}{\delta}\right\}\cos\varphi = 0, \quad \text{... (11)}$$

$$\frac{\partial u}{\partial x} = 0, \quad \text{... (12)}$$

$$\frac{d^2T}{dz^2} = 0, \quad \text{... (13)}$$

$$\rho = \rho_0\{1 - \beta(T - T_0)\Delta T\} \quad \text{... (14)}$$

and $q = -\frac{1}{\eta^2}[-P + zGr\sin\varphi]$, ... (15)

where $P = \delta(\lambda e^{-\gamma z} + \sin\varphi)$, ... (16)

The boundary conditions (8) now reduces to

$$\left.\begin{array}{l} z = \tfrac{1}{2} : u = u_{B_1}; \quad \frac{du}{dz} = -\alpha\eta\left(u_{B_1} - q_{B_1}\right)e^{\gamma/4};\ T = T_0 - \tfrac{1}{2} \\ z = -\tfrac{1}{2} : u = u_{B_2}; \quad \frac{du}{dz} = -\alpha\eta\left(u_{B_2} - q_{B_2}\right)e^{\lambda/4};\ T = T_0 + \tfrac{1}{2} \\ x = 0,\ z = 0 : p = 0. \end{array}\right\} \quad \text{... (17)}$$

The temperature distribution can be obtained by solving equation (13) and using the fact that $T = T_0 - ½$ at $z = ½$ and $T = T_0 + ½$ at $z = -½$, we get

$$T - T_0 = -z. \quad \text{... (18)}$$

The velocity component 'u' can be obtained by solving equation (10) and using the boundary conditions (17). It gives:

$$u = C_1e^{-N_1z} + C_2e^{N_2z} - \frac{1}{M^2}(\gamma/M^2 + z)Gr\sin\varphi e\gamma^z + P/M^2e^{\gamma z}, \quad \text{... (19)}$$

where C_1 and C_2 are constants to be determined by the equations.

$$a_{11}C_1 + a_{12}C_2 = a_{13} \quad \text{... (20)}$$

and $a_{21}C_1 + a_{22}C_2 = a_{23}$.... ... (21)

Here,

$$a_{11} = \left(\alpha\eta e^{\gamma/4} + N_1\right)e^{N1/2},$$

$$a_{12} = \left(\alpha\eta e^{\gamma/4} + N_2\right)e^{N2/2},$$

$$a_{13} = -\frac{1}{M^2}\left(\alpha\eta e^{\gamma/4} + \gamma\right)P_1e^{\gamma/2} + \frac{1}{M^2} \times \left(\alpha\eta e^{\gamma/4} + \gamma\right)(\gamma/M^2 + ½)Gr\sin\varphi e^{\gamma/2}$$

$$+\frac{1}{M^2}Gr\sin\phi c^{\gamma/2} + \alpha\eta e^{\gamma/4}q_{B_1} - \lambda\delta\gamma/M^2,$$

$$a_{21} = \left(\alpha\eta e^{\gamma/4} - N1\right)e^{-N1/2},$$

$a_{22} = \left(\alpha\eta e^{\gamma/4} - N2\right)e^{-N2/2}$, and $a_{23} = -\frac{1}{M^2}\left(\alpha\eta e^{\gamma/4} - \gamma\right)P_2e^{-\gamma/2} + \frac{1}{M^2}$

$$\times\left(\alpha\eta e^{-\lambda/4} - \gamma\right)(\gamma/M^2 - ½)Gr\sin\varphi e^{-\gamma/2}$$

$$-\frac{1}{M^2}Gr\sin\varphi e^{-\gamma/2} + \alpha\eta e^{-\gamma/4}q_{B_2} + \lambda\delta\gamma/M^2,$$

where, $P_1 = -\delta\left(\lambda e^{-\gamma/2} + sin \quad \varphi\right),$

$$P_2 = -\delta\left(\lambda e^{\gamma/2} + sin \quad \varphi\right),$$

$N_1 = \gamma/2 - \left(\gamma^{2/4} + M^2\right)½,$ and $N_2 = \gamma/2 - \left(\gamma^{2/4} + M^2\right)½.$

The slip velocities at the upper and lower beds, B_1 and B_2 are given as:

$$u_{B_1} = C_1 e^{N1/2} + C_2 e^{N2/2} - \frac{1}{M^2}(\gamma/M^2 + ½)Gr\, sin \quad \varphi e^{\gamma/2} + \frac{P_1}{M^2} e^{\gamma/2} \qquad \text{... (22)}$$

And $u_{B_2} = C_1 e^{-N1/2} + C_2 e^{-N2/2} - \frac{1}{M^2}(\gamma/M^2 + ½)Gr \times sin \quad \varphi e^{-\gamma/2} + \frac{P_2}{M^2} e^{-\gamma/2}$.... ... (23)

The pressure distribution is obtained by solving equation (11) with the help of boundary conditions (17) and is given as:

$$p = -\left(z + \frac{Gr}{2\delta} z^2\right) cos \quad \varphi + \lambda \times e^{-\gamma z}. \qquad \text{... (24)}$$

The average Darcy velocities q_{B_1} and q_{B_2} in the upper and lower beds can be obtained by solving equations (15) and (16). Thus,

$$q_{B_1} = -\frac{1}{\eta^2}(-P_1 + ½Gr\, sin \quad \varphi) \qquad \text{... (25)}$$

and $q_{B2} = -\frac{1}{\eta^2}(-P_2 - ½Gr\, sin \quad \varphi).$ ... (26)

4. Results and Discussions

We now discuss the important flow characteristics of the problem.

(i) Mass Flow Rate

The dimensionless mass flow rate 'M*' per unit width of the channel bounded by two inclined permeable porous beds is defined as:

$$M^* = \int_{-½}^{½} e^{-\gamma z} u dz. \qquad \text{... (27)}$$

Therefore, on substituting for u and on evaluating the definite integral, we get

$$M^* = \frac{2C_1}{N_2} sin \quad h\frac{N_2}{2} + \frac{2C_2}{N_1} sin \quad h\frac{N_1}{2} - \frac{2\lambda\delta}{\gamma M^2} sin\, h\, \gamma/2 - \frac{1}{M^2}\left(\delta + \frac{\gamma Gr}{M^2}\right) sin \quad \varphi. \qquad \text{... (28)}$$

Now, taken $\eta \rightarrow \infty$ in equation (28), the mass flow rate per unit width of the channel bounded by rigid walls is given as:

$$M_\infty = \frac{2A_1}{N_2} sin\, h\frac{N_2}{2} + \frac{2A_2}{N_1} sin \quad \frac{N_1}{2} - \frac{2\lambda\delta}{\gamma M^2} sin\, h\frac{\gamma}{2} - \frac{1}{M^2} \times \left(\delta^2 + \frac{\gamma Gr}{M^2}\right) sin\, \varphi, \qquad \text{... (29)}$$

where, $A_1 = \left\{Gr\, sin \quad \varphi\, cos\, h\frac{N_1}{2} + 2\left(\delta + \frac{\gamma Gr}{M^2}\right) sin \quad \varphi\, sin \quad h\frac{N_1}{2} \; -2\delta\lambda\, sin \quad \frac{N_2}{2}\right\} / \{2M^2\, sin \quad h(\gamma^2/4 + M^2)^{½}\},$

and $A_2 = \left\{Gr\, sin \quad \varphi\, cos\, h\frac{N_2}{2} + 2\left(\delta + \frac{\gamma Gr}{M^2}\right) sin \quad \varphi \times sin \quad h\frac{N_2}{2} - 2\delta\lambda\, sin\, h\frac{N_1}{2}\right\} / \{2M^2\, sin \quad h(\gamma^2/4 + M^2)^{½}\}.$

(ii) Fractional Increase in Mass Flow Rate

The fractional increase 'F' in mass flow rate through the inclined porous channel over the inclined flow with rigid wall is given as:

$$F = \frac{M^*}{M_\infty} - 1. \qquad \text{... (30)}$$

On substituting the values of M^* and M∞ in equation (30), we get

$$\begin{aligned} F = &\left\{2\gamma N_1 M^4 (C_1 - A_1) sin \quad h\frac{N_2}{2} + 2\gamma N_2 M^4 (C_2 - A_2)\right. \\ &\left.\times sin\, h\frac{N_1}{2}\right\} / \left\{2\gamma A_1 N_1 M^4\, sin \quad h\frac{N_2}{2} + 2\gamma A_2 N_2 M^4\right. \\ &\times sin\, h\frac{N_1}{2} - 2N_1 N_2 \lambda\delta M^2\, sin \quad h\gamma/2 - N_1 N_2 \gamma \\ &\left.\times (\delta M^2 + \gamma Gr)\, sin \quad \varphi\right\}. \end{aligned} \qquad \text{... (31)}$$

(iii) The velocity distribution (19) and the fractional increase (31) in mass flow rate are numerically evaluated for different values of δ, λ, η and M with α = 0.1, γ = 1, Gr sin φ = 25 and $\delta \, sin \quad \varphi = 0.5$. The velocity profiles are shown in Fig. 2 and Fig. 3.

It is observed that the velocity profiles between -0.5 and 0.5 are in the shape of a wave for fixed M, η and λ S. These resemble the velocity profiles of Rudraiah and Wilfred [5] and Ramakrishna *et al.* [4] for a fixed γ. For fixed M and η, the velocity deceases with increase in the values of $\lambda\delta$.

For the fixed value of M and $\lambda\delta$, the velocity decreases in the lower half of the channel with the increase in η whereas the velocity increases in the upper half of the channel with the increase in η. From the Fig. 3, we observe that for the fixed values of η and $\lambda\delta$, the velocity decreases in the lower half of the channel with increase in M whereas the velocity increases in the upper half of the channel with the increase in M.

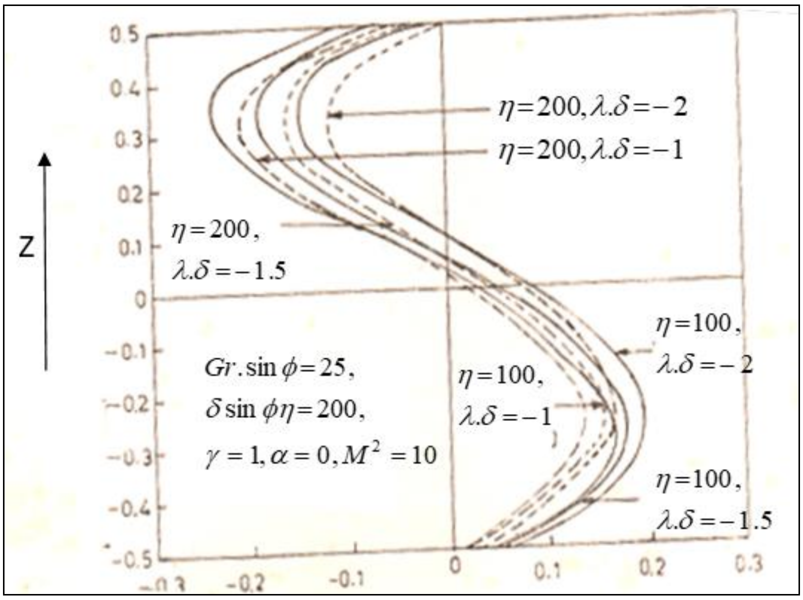

Fig 2: "Velocity Profiles"

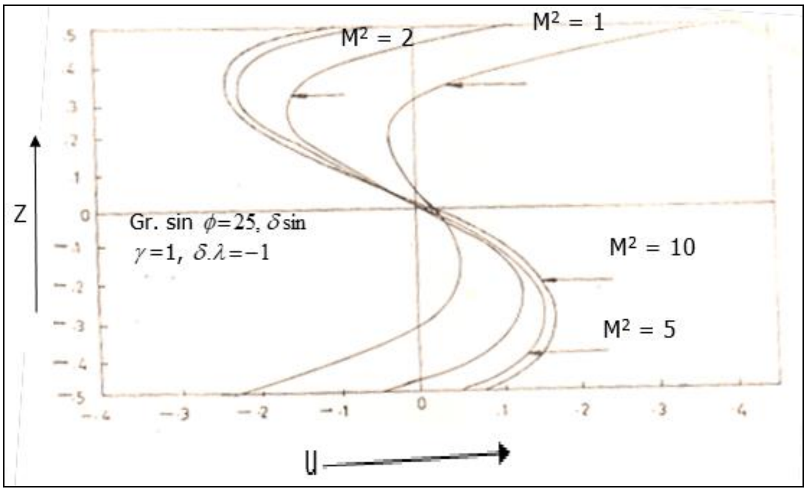

Fig 3: "velocity profiles

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