



The application of mind map in the teaching of maths problem solving in senior middle school

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Abstract

Mind map has been widely used in education and teaching industry since its birth in the last century. It can visualize thinking and fully express the thinking situation in the human brain. Because the abstraction and logic of high school mathematics are strong, students often can't find the solution to the problem, and it is of great significance to apply the mind map to the mathematics problem solving teaching. Mathematics teachers should follow the appropriate conditions and principles, use the mind map to solve the problem according to certain requirements, and fully utilize the advantages of mind mapping to improve students' ability to solve problems.

Keywords: mind map, problem solving ideas, divergent thinking, analysis of problems

1. Introduction

In the current middle school mathematics problem solving teaching, the phenomenon of sea tactics and the combination of explanation and practice is common. Especially in the review process, the most basic review mode is the student's practice and the teacher's explanation. Teachers always pay too much attention to the standardization of the problem-solving steps and neglect the guidance of the problem-solving ideas. In addition, there are so many points of mathematics and types of questions that the students only get some pieces of knowledge, but their mathematical thinking is not developed, and their ability to solve problems is not exercised. Mind maps can turn a long list of boring information into colored, easy-to-remember, highly organized graphs that match the natural way our brains handle things (Tony Buzan, 2005) ^[1]. Applying the mind map reasonably to the mathematics problem solving teaching can effectively help students to exercise mathematics thinking, clear up the problem solving ideas, and improve the ability to solve problems.

The current research is not perfect enough to apply the mind map to mathematics problem solving. When is the mind map used? What principles do you follow to solve problems? How to use mind map when solving problems? These are all questions worth considering.

2. The meaning and characteristics of mind map

The mind map was originally proposed by the British brain expert Tony Buzan in the 1970s to simplify the study notes. The mind map is mainly to effectively simulate the actual working mode of the human brain (Lin, Y. J., 2018) ^[2], externalizing the human thinking process through the map, showing the knowledge structure in the human brain more intuitively and visually, and maximize the potential of the brain. It requires us to focus on the middle of the graphics; Its branch consists of a key graphic or a keyword written on the associated line; Each branch forms a connected node structure (Tony Buzan, 1995) ^[3].

There are many advantages to using mind maps in mathematics teaching. (1) It has the function of refining and compressing. It only retains some key information with

keywords, graphics and connections, removes less important content, reduces the memory burden, and enables people to quickly acquire the knowledge they need. (2) It can make the thinking process explicit. The mind map reveals the implicit thinking process with simple graphics, which is conducive to reflection, communication, feedback and correction. (3) It is good for diverging thinking. Mind mapping can make people's thinking start from the central problem and spread to the surrounding, which helps students to cultivate creative thinking and is conducive to seeking solutions. (4) It can fully tap the brain's potential. The mind map combines graphics and texts, which can make full use of the left brain's language, words, numbers, logic and other functions and the right brain's imagination, creation, inspiration, memory and other functions to improve the learning efficiency.

However, the use of mind maps also has certain limitations. (1) It does not solve the problem of "how to stimulate thinking" and "how to organize thinking" (Zhao, G. Q., 2012) ^[4]. (2) Students who use the mind map initially may be difficult to adapt and will quit halfway when they have not realized the benefits. (3) Not all mathematical content can be sorted and memorized by means of mind maps, and it can never replace the traditional blackboard derivation.

3. The current requirements for mathematics problem solving teaching in high school

About the mastering of the mathematical knowledge and the solving of the mathematical problems, the "Standards of Mathematics Curriculum for General Senior High Schools" pointed out that high school mathematics teaching should be guided by the development of students' core literacy of mathematics, inspire students to think, guide students to grasp the essence of mathematics content; Mathematics teaching should highlight the main line of mathematics, the inherent logic and the thinking methods; Students should improve their ability to discover, ask, analyze and solve problems, and learn to analyze the world with mathematical thinking (Ministry of Education of P.R. China, 2017) ^[5]. In "How to Solve the Questionnaire", Polya pointed out that students should solve the problem in the following four

steps: reviewing the meaning of the question, drafting the plan preliminary, implementing and operating, reviewing and summarizing (Tu, R. B., 2002) ^[6]. (1) The purpose of reviewing the questions requires students to find out the conditions and the problem, and to find the core conditions or keywords, key quantities and implied conditions that can break through the problems in many conditions. The problem-solving teaching in this section requires students to understand the meaning of the question, not only to have a surface understanding of the problem, but also to have a deep understanding, and learn to re-characterize the problem. (2) The second step requires students to find the general path to solve the problem according to the core conditions. In this process, students need to spread the thinking from the conditions, contact the learned knowledge, and find a route that can lead to the problem. At this time, students should be encouraged to find a variety of solutions to the problem and strive to find the best one. The problem-solving teaching in this part should guide students to contact existing problem-solving experiences and choose appropriate problem-solving strategies, such as specializing general problems or generalizing special problems. (3) Implementing and operating requires the students to implement the program through calculation, certification and other processes, and get the final result. The problem-solving teaching requirements of this link should pay attention to guiding students to express the proposed plan with standardized mathematical language, symbols and images, etc. (4) The last step requires students to reflect on and summarize the knowledge points and thoughts used in the problem-solving process, and strive to draw inferences about other cases from one instance, and even to change the conditions to promote the questions and answers. The problem-solving teaching in this part not only leads the students to review the process of understanding the meaning of the questions, the knowledge used, the methods of thinking, but also how to explore at the beginning, what detours have been taken, what mistakes have occurred, and why is there an error, etc. Over time, students will develop regular experiences, which will become a guide for students to solve problems in the future (Tu, R. B., 2001) ^[7]. Clearing the question is the premise of solving the problem. This part should lead the students to understand the problem thoroughly, or transform it into old problems that have been encountered before, or discover new knowledge to generate new ideas and new methods. Developing the initial plan requires students to divergent thinking and find definitions, properties, and methods of thinking related to the problem. The ultimate goal of problem-solving teaching is to let students learn to solve problems, accumulate experience in solving problems, and improve their ability to solve problems. Therefore, reviewing and summarizing is also crucial for problem-solving teaching.

4. The application strategy of mind map in the teaching of mathematics problem solving in high school

According to the characteristics of the mind map and the steps of the mathematical problem solving, I believe that the mind map can be applied to the three aspects: understanding the meaning of the question, formulating preliminary plans, reviewing and summarizing. In the teaching of high school mathematics problem solving, we should follow certain requirements and use the mind map according to certain steps.

4.1 Requirements for Constructing Problem Solving Mind Maps

1. Streamlined

Pay attention to the simplicity when using the mind map to solve problems. Whether using the mind map to review questions, or drafting a solution or reviewing and summarizing, the mind map is required to be as concise and clear as possible. Students can write the problem conditions and problem-solving ideas with keywords, without having to write detailed problem-solving steps, which can save time and make the mind map simple and easy to understand.

2. Comprehensive

When solving problems, the branch of mind map should be comprehensive. To establish a mind map during the examination, the student must find the known and implied conditions in the topic as the secondary branch of the map. When analyzing the problem-solving ideas, students look for ideas along the conditions just now, and gradually move closer to the desired ones. At this time, they need to consider comprehensive and not miss important clues.

3. Diversity

Each student's knowledge framework is different, the way of understanding is different, and the degree of divergence of thinking is different, therefore, the established mind map can be varied. You can create a mind map in a topic, use the problem as the core keyword, use the topic condition as a secondary branch, and expand the clues and ideas as subordinate branches in turn until you get the final result. It is also possible to establish a mind map on the condition of the subject during the examination, establish a mind map on the solution to the problem in the initial plan, and establish a mind map on the knowledge points and thought methods during the review. Each student's mind map should be personalized and personal.

4. Selectivity

The mind map does not apply to all topics. Students should be guided to analyze and use the mind map before solving the problem. For problems with simple subject conditions and clear single ideas, such as the concept questions, calculation questions, etc., students can also answer smoothly without using the mind map. The mind map is mainly applicable to the following types of questions: (1) Reasoning and proof questions. Such problems cannot be directly from known conditions to results. Students need to start from the known condition, contact the relevant knowledge points, and gradually move closer to the conclusion to be proved. (2) Open questions. This kind of topic examines the students' divergent thinking and requires finding a variety of methods to solve the problem. At this time, the mind map can be used to clarify the ideas and obtain various paths to solve the problem. (3) Comprehensive questions. This kind of problem often appears in the last one of the college entrance examination questions. The topic is comprehensive, and multiple knowledge points and multiple methods can be examined. The mind map can be used to analyze the ideas.

4.2 the Specific Steps of Using the Mind Map to Solve the Problem

Mind maps can be plotted using computer software or drawn by hand. Because the current educational technology

can not let students use the computer for mind mapping, and students need to spend a certain amount of time to learn related software, so I mainly analyze how to manually draw the mind map in the problem teaching.

1. Review the Meaning of the Question and Construct a Review Map

When reviewing questions, you can create a review map based on the title conditions and questions. (1) First, extract the core words as the key words of the mind map in the required questions, and put them at the center of the figure. (2) Then, the problem conditions are decomposed, and the different conditions are sequentially used as different secondary branches of the mind map, which are simply written with pens of different colors. (3) For the core conditions closest to the conclusion of the distance, we can bold this branch so that it can be traced later when analyzing the problem. In short, when establishing a mind map in the process of reviewing, the most important thing is to find out the known conditions and implied conditions in the title as a branch of the graph, and simplify the whole topic into a clear picture, provide a concise and clear environment for the second step of the "Preliminary Project" .

2. Analyze Problem-Solving Ideas and Construct the Ideas Map

The key to solving the problem is whether the students can formulate a scientific and reasonable solution plan. Therefore, the mind map mainly completes the transformation between conditions and goals in this link (Dong, R. S., 2016) [8]. In the analysis of the problem-solving ideas, you can follow the steps below: (1) Guide students to contact relevant knowledge points or thoughts based on known conditions. This requires students to have a clear understanding of the part of the knowledge to be examined, and to find relevant knowledge and use in the existing knowledge system. If students have a variety of ideas, they can be labeled as different secondary branches of the idea map, and answer them separately when time is sufficient, and choose an optimal route finally. (2) Select a secondary branch to guide students to think independently, transform existing conditions into more concise information related to the problem, and gradually extend as a subordinate branch in the depth direction until the title condition is transformed into a conclusion. (3) Guide students to draw a complete mind map, communicate with each other and share their overall ideas for solving problems. In this process, the student's ideas are refined, magnified, and displayed in the chart, no longer ambiguous, no specific direction, which can facilitate the later review and reflection.

3. Review and Summarize, Build a Reflection Map

To establish a reflection map, students should be guided to review the entire problem-solving process, sum up the necessary knowledge points and ideas, and strive to be able infer the ability to solve problems. When constructing a reflection map, you can follow the following steps: (1) Lead the students to review the entire problem-solving process, and guide the students to extract the knowledge points from

the process of solving the problem, and use the knowledge as the key word on the secondary branches. (2) Find the connection between knowledge points, use curves to connect the relationship naturally, and describe the relationship between two or more knowledge points on the line with simple and clear words or sentences. (3) Reflect on the ideas and methods used in the topic, such as the idea of equations, the combination of numbers and shapes, the idea of classification and discussion, and the methods of matching, model, and undetermined coefficients. By constructing a reflective map, students should be able to have a deeper understanding of the problem, better understand the connections between knowledge points, become more familiar with common mathematical thinking methods, and try to apply answers in similar questions. The above-mentioned review maps, ideas maps and reflection maps can also be combined into one big picture in case of sufficient time and resources, which is conducive to deepening students' understanding of this question.

4.3 Explain how to use the mind map in solving the problem teaching through specific questions

Below I will use a college entrance examination question about conic curves as an example to illustrate how to construct a mind map in a specific problem explanation.

Title: It is known that the center of the hyperbola is at the origin, the right vertex is $A(1,0)$, the points P and Q are on the right branch of the hyperbola, and the distance from the point $M(m,0)$ to the straight line AP is 1.

(1) If c is k , and $k \in \left[-\frac{\sqrt{3}}{3}, \sqrt{3}\right]$, find the value range of the real number m ;

(2) At that time $m = \sqrt{2} + 1$, the inner heart of the triangle APQ happened to be point M , please request the equation for this hyperbola.

1. Construction of the Review Map

1. Leading students to read the questions, they can easily find that the main point of knowledge in this question is hyperbolic, so the "hyper curve" is taken as the core word, and the mind map will be built around it.
2. Guide students to find the conditions directly related to the hyperbola as the secondary branch. There are three: "Center is the origin", "Right vertex is $A(1,0)$ " and "Point P, Q are on the right branch of the hyperbola". Next, the student is guided to judge the relationship between other conditions and the existing conditions, and they can find that the "straight line AP " and the "inner heart" can directly contact the secondary branch, and then use it as a three-level branch. Once again, the student is guided to judge the conditions related to the third-level branch: "slope k of the straight line AP ", "distance of the point M to the straight line AP ", and use them as four-level branches.
3. Ask the students to draw the map according to the above analysis, and mark the links between the branches at the line. The general review map is shown below.

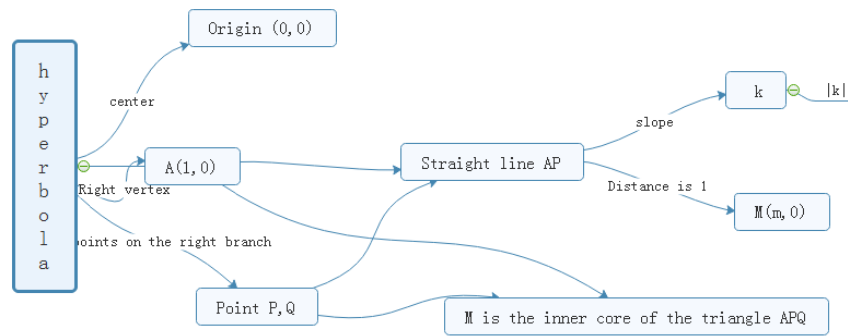


Fig 1

The review map

2. Construction of the Ideas Map

1. Guide students to think according to the problem. The first question asks the value range of m , and what are the relevant conditions? Which condition can be used as the main condition for solving the problem? According to the above review map, the student can quickly find out that “the distance from point M to the line AP is 1” should be the core condition for solving the problem, so the problem is taken as a key word, and the condition is taken as the second branch of the idea map.
2. (2) At this time, the student can associate the point-to-line distance formula, but they need to find the expression of the line AP , which becomes a three-level branch; According to the examination map, the student finds that “the straight line AP passes the point $A(1,0)$ ” and the “slope is k ”, and the expression of the straight line AP can be obtained by the undetermined coefficient method, so these two conditions are taken as the four-level branch. And the condition “ $|k| \in [\frac{\sqrt{3}}{3}, \sqrt{3}]$ ” naturally becomes the last level branch to use for the scope.
3. The second question asks for the equation of the hyperbolic, which can be used as a key word; The student associates the method to find the equation, and they can think of the undetermined coefficient method naturally. Since the hyperbola passes the point $A(1,0)$,

it can be directly set as $x^2 - \frac{y^2}{b^2} = 1 (b \neq 0)$. According to

the condition of the subject, the student knows that “at that time $m = \sqrt{2} + 1$, the point M just happened to be the heart of the triangle APQ ” is the core condition for solving the problem, and it can be used as the secondary branch; Next, the teacher should guide the students to think about the information that requires to find the expression of the equation. Students can think of the coordinates of one of the points P ; At this time, draw a general image according to the condition of the subject, and combine the number and shape. Because of “ $m = \sqrt{2} + 1$ ”, $AM = \sqrt{2}$ is available, and the point M is the inner heart of the triangle APQ , through the definition of inner heart, they can know that the distance from point M to line AP , AQ , PQ is all 1. According to the image students can draw $\angle MAP = 45^\circ$, so the straight line PQ : $x = 2 + \sqrt{2}$; Continue to extend the idea downward,

students can get $k_{AP} = 1, k_{AQ} = -1$, then get the line AP : $y = x - 1$, and finally get the required coordinate of point P .

4. Let the students express the above ideas in the order of the first level and one level on the schematic map, and exchange and analyze the process of solving the problem.

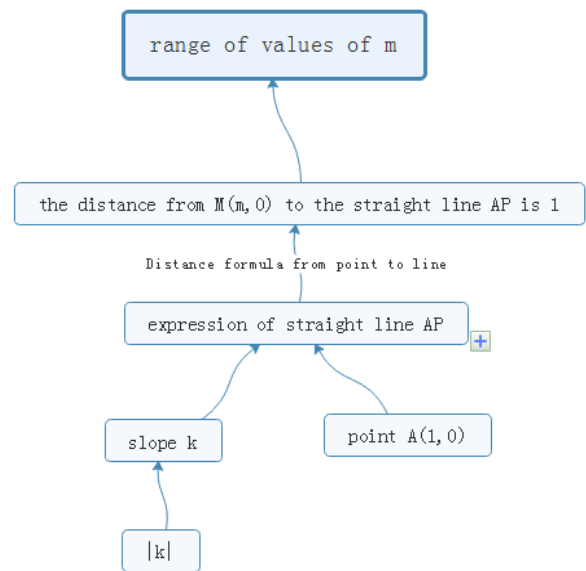


Fig 2

The ideas map 1

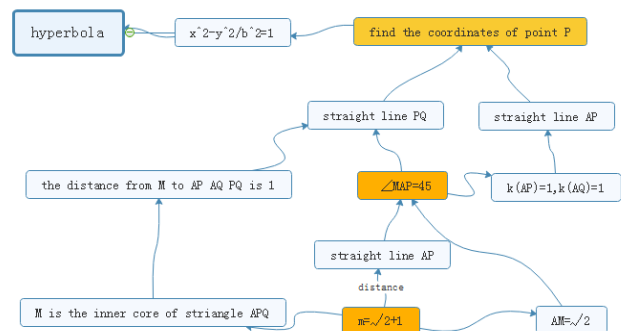


Fig 3

The ideas map 2

3. Construction of the Reflection Map

1. Lead students to review the knowledge points

encountered during the process of resolving problems: the standard equations, images and properties of hyperbolic curves, distance formulas from point to line, linear equations, solution of inequalities, and the inner heart of triangles.

2. Guide students to think about the connections between these knowledge points, and connect the related knowledge with lines to mark them with words. In this process, let the students reflect on the entire problem-solving process: What is the problem? What definitions are associated with the problem? What is the nature of each definition? How to relate the problem conditions to the problem? Where is my idea stopped? How is it solved? and many more.
3. Guide students to review the main thoughts and methods used in the process of solving problems: undetermined coefficient method and the combination of numbers and shapes.
4. Draw the reflection process with the mind map according to the actual situation.

5. Conclusion

Using mind maps in the process of solving problems can help students to clear their minds, deepen their understanding of the relationship between knowledge, and improve their ability to solve problems. Teachers should guide students to establish reasonable review maps, ideas maps and reflection maps according to certain principles and steps in solving problems. Of course, how to construct a more scientific mind map is also an important learning process for students, and to establish a complete solution map, students need to have a well-organized mathematical knowledge structure. Therefore, in the problem-solving teaching, it is necessary to continuously strive for both teachers and students in order to truly exert the advantages of mind mapping.

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