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On minimal rgwα-locally open sets and maximal rgwα-locally closed sets in topological spaces

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Abstract

In this paper, a new class of sets called minimal rgwα-locally open sets and maximal rgwα-closed sets in topological spaces are introduced, which are sub classes of rgwα-locally open sets and rgwα-locally closed sets. Also we introduce and study maximal rgwα-locally open sets and minimal rgwα-locally closed sets in topological spaces and also study $\tau_{rgw\alpha lc}$-min and $\tau_{rgw\alpha lc}$-max spaces.

Keywords: minimal rgwα-locally open sets, maximal rgwα-locally closed sets, maximal rgwα-locally open sets, minimal rgwα-locally closed set, $\tau_{rgw\alpha lc}$-min space and $\tau_{rgw\alpha lc}$-max space

1. Introduction

In the year 2001 and 2003, F. Nakaoka and N.oda [1, 2, 3] introduced and studied minimal open (resp. minimal closed) sets which are sub classes of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2006, S S Benchalli and R.S Wali [4, 6] introduced and studied rw-closed sets, rw-open sets, maximal rw-open sets, minimal rw-closed sets, maximal rw-closed sets, minimal rw-open sets in topological spaces and R.S Wali *et all* [5] introduced and studied rgwα-locally closed sets in topological spaces.

2. Preliminaries

Definition 2.1 A proper non-empty open subset U of a topological space X is said to be minimal open set [1] if any open set which is contained in U is ϕ or U.

Definition 2.2 A proper non-empty open subset U of a topological space X is said to be maximal open [2] set if any open set which contains U is either X or U.

Definition 2.3 A proper non-empty closed subset F of a topological space X is said to be minimal closed set [3] if any closed set which is contained in F is ϕ or F.

Definition 2.4 A proper non-empty closed subset F of a topological space X is said to be maximal closed set [3] if any closed set which contains F is either X or F.

Definition 2.5 A subset A of (X, τ) is called rw-closed [6] set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X.

Definition 2.6 A subset A of (X, τ) is called rw-open set [6] in X if Ac is rw-closed set in X.

Definition 2.7 A proper non-empty rw-closed subset U of X is said to be maximal rw-closed [4] set if any rw-closed set which contains F is either X or F.

Definition 2.8 A proper non-empty rw-open subset F of X is said to be Minimal rw-open [4] set if any rw-open set which is contained in U is ϕ or U.

Definition 2.9 A proper non-empty rw-open subset U of X is said to be maximal rw-open set [4] if any rw-open set which contains F is either X or F.

Definition 2.10 A proper non-empty rw-closed subset F of X is said to be Minimal rw-closed set [4] if any rw-closed set which is contained in U is ϕ or U.

Lemma 2.11 [4] A proper non-empty subset F of X is Minimal rw-open set iff X-F is a maximal rw-closed set.

Lemma 2.12 [4] a proper non-empty subset F of X is Minimal rw-closed set iff X-F is a maximal rw-open set.

Definition 2.13 A topological space X is said to be Tmin space if every nonempty proper open subset of X is minimal open set.

Definition 2.14 A topological space X is said to be Tmax space if every nonempty proper open subset of X is maximal open set.

Definition 2.15 regular generalized weakly α-locally closed [5] (briefly rgwα-locally closed) if $A=U\cap F$ where U is rgwα-open in (X, τ) and F is rgwα-closed in (X, τ).

Definition 2.16 regular generalized weakly α-locally open [5] (briefly rgwα-locally open) if A^c is rgwα-locally closed.

Definition 2.17 A topological space (X, τ) is called the

i) door space if every subset of (X, τ) is either open or closed in (X, τ).
ii) T½ space if every g-closed subset of (X, τ) is closed in (X, τ).
iii) α- space if every α-closed subset of (X, τ) is closed in (X, τ).
iv) Tw space if every w-closed subset of (X, τ) is closed in (X, τ).
v) Trw space if every rw-closed subset of (X, τ) is closed in (X, τ).
vi) T1 (or Frechet) space iff each point in X, singleton set {x} is closed in (X, τ) or for every pair of distinct points x, y in X there is an open set U such that $x\in U$ and $y\in V$.
vii) T0 space if for every pair of distinct points x, y in X there is an open set U containg one of the point but not other.
viii) T2 (or Hausdorff) space iff for every pair of distinct points x, y in X there exist disjoint open sets U, V such that $x\in U$ and $y\notin V$.

2. Minimal rgwα-Locally Open sets.

Definition 2.1 A proper nonempty rgwα-locally open subset U of X is said to be a minimal rgwα-locally open set if and only if any rgwα-locally open set which is contained in U is ϕ or U.

Theorem 2.2 Every minimal open set is minimal rgwα-locally open set but not conversely.
Proof: Proof follows from the fact that every open set is rgwα-locally open set. But converse is not true as seen from the following example.

Example 2.3 Let X= {a,b,c} be a set with topology τ={X, ϕ, {a}, {b}, {a,b}}. Here minimal open sets are {a}, {b}. rgwα-locally open sets are X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}. Minimal rgwα-locally open sets are {a}, {b}, {c}. Therefore set {c} is minimal rgwα-locally open set but not minimal open set.

Remark 2.4 From the known results and by the above example 2.3 we have following implications:

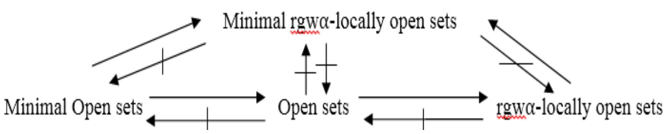

Fig 1

Theorem 2.5 (i) Let U be a minimal rgwα-locally open set and W be a rgwα-locally open set, then U∩W= ϕ or U⊂W.
(ii) Let U and V be minimal rgwα-locally open sets, then U∩V= ϕ or U=V.
Proof: (i) Let U be a minimal rgwα-locally open set and W be a rgwα-locally open set. If U∩W= ϕ, then there is nothing to prove but if U∩W≠ ϕ then we have to prove that U⊂W. Suppose U∩W≠ ϕ then U∩W⊂U and U∩W is rgwα-locally open as the finite intersection of rgwα-locally open sets is a rgwα-locally open set. Since U is a minimal rgwα-locally open set, we have U∩W=U therefore U⊂W.
(ii) Let U and V be minimal rgwα-locally open sets. Suppose U∩V≠ ϕ then we see that U⊂V and V⊂U by (i) Therefore U=V.

Theorem 2.6 Let U be a minimal rgwα-locally open set. If x is an element of U, then U⊂W for any open neighbourhood Wof x.
Proof: Let U be a minimal rgwα-locally open set and x be an element of U. Suppose there exits an open neighbourhood W of x such that U⊄W then U∩W is a rgwα-locally open set such that U∩W⊂U and U∩W≠ ϕ. Since U is a minimal rgwα-locally open set, we have U∩W=U that is U⊂W. This contradicts our assumption that U⊄W. Therefore U⊂W for any open neighbourhood W of x.

Theorem 2.7 Let U be a minimal rgwα-locally open set, if x is an element of U then U⊂W for any rgwα-locally open set W containing x.
Proof: Let U be a minimal rgwα-locally open set containing an element x. Suppose there exists an rgwα-locally open set W containing x such that U⊄ W then U∩W is an rgwα-locally open set such that U∩W⊂U and U∩W≠ ϕ Since U is a minimal rgwα-locally open set, we have U∩W=U that is U⊂W. This contradicts our assumption that U⊄W. Therefore U⊂W for any rgwα-locally open set W containing x.

Theorem 2.8 Let U be a minimal rgwα-locally open set then U= ∩{W: W is any rgwα-locally open set containing x} for any element x of U
Proof: By theorem 2.7 and from the fact that U is a rgwα-locally open set containing x, We have U⊂∩ {W: W is any rgwα-locally open set containing x} ⊂U. Therefore we have the result.

Theorem 2.9 Let U be a non-empty rgwα-locally open set then the following three conditions are equivalent.
(i) U is a minimal rgwαlc open set.
(ii) U⊂ rgwαlc-cl(S) for any non-empty subset S of U.
(iii) rgwαlc-cl (U)= rgwαlc-cl(S) for any non-empty subset S of U.
Proof: (i) => (ii) Let U be a minimal rgwα-locally open set and S be a non-empty subset of U. Let x∈U by theorem 2.7 for any rgwα-locally open set W containing x, S⊂U⊂W which implies S⊂W. Now S=S∩U⊂S∩W. Since S is non-empty therefore S∩W≠ ϕ. Since W is any rgwα-locally open set containing x, we know that, for an x∈X, x∈ rgwαlc-cl(A) iff V∩A ≠ ϕ for any every rgwα-locally open set V Containing x, that is x∈U implies x∈ rgwαlc-cl(S) which implies U⊂ rgwαlc-cl(S) for any non-empty subset S of U.
(ii) => (iii) Let S be a non-empty subset of U, that is S⊂U which implies rgwαlc-cl(S) ⊂ rgwαlc-cl (U) -- (a)
Again from (ii) U⊂ rgwαlc-cl(S) for any non-empty subset S of U. Which implies rgwαlc-cl (
U) ⊂ rgwαlc-cl (rgwαlc-cl(S)) = rgwαlc -cl(S) i.e., rgwαlc -cl (U) ⊂ rgwαlc -cl(S) -- (b), from (a) and (b), rgwαlc -cl (U) = rgwαlc -cl(S) for any non-empty subset S of U.
(iii) => (i) From (iii) we have rgwαlc -cl(U) = rgwαlc -cl(S) for any non-empty subset S of U. Suppose U is not a minimal rgwα-locally open set then there exist a non-empty rgwα-locally open set V such that V⊂U and V ≠ U. Now there exists an element a∈U such that a∉V which implies a∈V^c that is rgwαlc- cl ({a}) ⊂ rgwαlc -cl{(V^c)} = V^c, as V^c is a rgwα-locally closed set in X. It follows that rgwαlc –cl ({a}) ≠ rgwαlc –cl (U). This is contradiction to fact that rgwαlc –cl ({a}) = rgwαlc –cl (U) for any non-empty subset {a} of U. Therefore U is a minimal rgwα-locally open set.

Theorem 2.10 Let V be a non-empty finite rgwα-locally open set, then there exists at least one (finite) minimal rgwα-locally open set U such that U⊂V.
Proof: Let V be a non-empty finite rgwα-locally open set. If V is a minimal rgwα-locally open set, we may set U=V. If V is not a minimal rgwα-locally open set, then there exists a (finite) rgwα-locally open set V_1 such that ϕ≠V_1⊂ V. If V_1 is a minimal rgwα-locally open set, we may set U=V_1. If V_1 is not a minimal rgwα-locally open set then there exists a (finite) rgwα-locally open set V_2 such that ϕ≠V_2⊂ V_1. Continuing this process we have a sequence of rgwα-locally open sets V_k …⊂V_3⊂ V_2⊂V_1⊂ V. Since V is a finite set, this process repeats only finitely then finally we get a minimal rgwα-locally open set U=V_n for some positive integer n.

Corollary 2.11 Let X be a locally finite space and V be a non-empty rgwα-locally open set then there exists at least one(finite) minimal rgwα-locally open set such that U⊂V.
Proof: Let X be a locally finite space and V be a non empty rgwα-locally open set. Let x∈V since X is a locally finite

space we have a finite open set V_x such that x∈V_x then V∩V_x is a finite rgwα-locally open set. By theorem 2.10 there exist at least one (finite) minimal rgwα-locally open set U such that U⊂V∩ V_x that is U⊂V∩V_x⊂V. Hence there exists at least one (finite) minimal rgwα-locally open set U such that U ⊂V.

Corollary 2.12 Let V be a finite minimal open set then there exist at least one (finite) minimal rgwα-locally open set U such that U⊂V.
Proof: Let V be a finite minimal open set then V is a non-empty finite rgwα-locally open set, by
Theorem 2.10 there exist at least one (finite) minimal rgwα-locally open set U such that U⊂V.

Theorem 2.13 Let U and U_λ be minimal rgwα-locally open sets for any element λ of Λ. If U ⊂ ⋃□□□□U_λ, then there exists λ element λ ∈ Λ such that U= U_λ.
Proof: Let U⊂⋃□□□□U_λ,, then U ∩ (⋃□□□□U_λ,)= U. That is□⋃□□□□□U⋂U_λ,) = U, also by Theorem 2.5 (ii)□U⋂U_λ= ϕ or U=U_λ for any λ ∈ Λ. It follows that there exist an element λ ∈ Λ such that U=U_λ.

Theorem 2.14 Let U and U_λ be minimal rgwα-locally open sets for any element λ of Λ. If U = U_λ for any element λ ∈ Λ then (⋃□□□□U_λ□□∩ U= ϕ.
Proof: Suppose that (⋃□□□□U_λ) ⋂U ≠ ϕ. That is□⋃□□□ (U $_\lambda$ ∩U) ≠ ϕ. Then there exists an element□λ ∈ Λ such that U ∩U_λ ≠ ϕ. By theorem 2.5 (ii), we have U= U_λ, which contradicts the fact that□U≠ U_λ for any λ ∈ Λ. Hence (⋃□□□□U_λ□□∩U = ϕ.

Theorem 2.15 Let U_λ be a minimal rgwα-locally open set for any element λ of Λ and U_λ ≠ U_μ for any element λ and μ of Λ with λ≠ μ assume that |Λ| ≥2. Let μ be any element of Λ, then (⋃□□□□□□□□□U_λ□□∩ U_μ = ϕ.
Proof: Put U= U_μ in theorem 2.14, then we have the result.

Corollary 2.16 Let U_λ be a minimal rgwα-locally open set for any element λ of Λ and U_λ ≠ U_μ for any elements λ and μ of Λ with λ≠ μ. If Γ a proper non-empty subset of Λ, then (⋃□□□□□□U□) ∩ (⋃□□□U□□□= ϕ.

Theorem 2.17 Let U_λ and U_γ be minimal rgwα-locally open sets for any element λ ∈ Λ and γ∈ Γ. If there exists an element γ of Γ such that U_λ≠ U_γ for any element λ of Λ, then ⋃□□□ U□□⊄ ⋃□□□□ U□□
Proof: Suppose that an element γ[1] of Γ satisfies U_λ = $U_{\gamma 1}$ for any element λ of Λ. If ⋃□□□U□□
⊂ ⋃□□□U□ then we see $U_{\gamma 1}$ ⊂⋃□□□ U□. By theorem 2.13, there exists an element λ of Λ such that
$U_{\gamma 1}$ = U_λ which is a contradiction. It follows that ⋃□□□ U□□⊄ ⋃□□□□ U□□

Theorem 2.18 Let U_λ be a minimal rgwα-locally open set for any element λ of Λ and U_λ≠ U_γ for any elements λ and μ of Λ with λ≠ μ. If Γ a proper non-empty subset of Λ, then ⋃□□□ U□□⊂⋃□□□□ U□□
Proof: Let k be any element of Λ- Γ. Then U_k ⋂(⋃□□□ U□□□□⋃□□□□U_k⋂U□□□□ϕ and U_k⋂ (⋃□□□□ U□□□□⋃□□□□ U_k⋂ U□□□ U_k. If ⋃□□□ U□□= ⋃□□□□ U□□□then we have ϕ= U_k. This contradicts our assumption that U_k is a minimal rgwα-locally open set. Therefore we have the result.

3. Maximal rgwα-Locally Closed sets.

Definition 3.1 A proper nonempty rgwα-locally closed subset F of X is said to be a **maximal rgwα-locally closed** set if and only if any rgwα-locally closed set which is contains F is either X or F.

Theorem 3.2 Every maximal closed set is maximal rgwα-locally closed set, but not conversely.
Proof: Proof follows from the fact that every closed set is rgwα-locally closed set. Converse of this is not true as seen from the following example.

Example 3.3 Let X= {a,b,c}, be a set with topology τ={X, ϕ, {a}, {b}, {a,b}}. Here maximal closed sets are {a,c}, {b,c}. Rgwα-locally closed sets are X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}. Maximal rgwα-locally closed sets are {a,b}, {a,c}, {b,c}. Therefore set {a,b} is maximal rgwα-locally closed set but not maximal closed set.

Remark 3.4 From the known results and by the above example 3.3 we have the following implications:

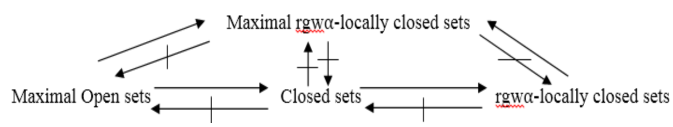

Fig 2

Theorem 3.5 A proper non empty subset F of X is maximal rgwα-locally closed set if and only if X-F is a minimal rgwα-locally open set.
Proof: Let F be a maximal rgwα-locally closed set. Suppose X-F is not a minimal rgwα-locally open set. Then there exists an rgwα-locally open set U≠X-F such that ϕ≠ U ⊂ X-F.That is F⊂ X-U and X-U is a rgwα-locally closed set. This contradicts our assumption that F is a minimal rgwα-locally open set.
Conversely let X-F be a minimal rgwα-locally open set. Suppose F is not maximal rgwα-locally closed set. Then there exists a rgwα-locally closed set E≠F such that F⊂E ≠X. That is ϕ≠ X-E ⊂ X-F and X-E is a rgwα-locally open set. This contradicts our assumption that X-F is a minimal rgwα-locally open set. Therefore F is a maximal rgwα-locally closed set.

Theorem 3.6 (i) Let F be a Maximal rgwα-locally closed set and W be a rgwα-locally closed set Then F∪W=X or W⊂F.
(ii) Let F and S be Maximal rgwα-locally closed sets then F∪S=X or F=S
Proof: (i): Let F be a Maximal rgwα-locally closed set and W be a rgwα-locally closed set if F∪W=X then there is nothing to prove, but if F∪W≠ X, then we have to prove that W⊂F. Suppose F∪W≠ X, then F ⊂F∪W and F∪W is rgwα-locally closed as the finite union of rgwα-locally closed set is a rgwα-locally closed set we have F∪W=X or F∪W=F. Therefore F∪W=F which implies W⊂F.
(ii): Let F and S be Maximal rgwα-locally closed sets. Suppose F∪S≠ X then we see that F⊂S and S⊂F by (i). Therefore F=S.

Theorem 3.7 Let F be a Maximal rgwα-locally closed set. If x is an element of F then for any rgwα-locally closed set S containing x, F∪S=X or S⊂F.

Proof: Let F be maximal rgwα-locally closed set and x is an element of F. Suppose there exists a rgwα-locally closed set containing x such that F∪S≠X. Then F⊂ F∪S and F∪S is a rgwα-locally closed set, as the finite union of rgwα-locally closed sets is a rgwα-locally closed set. Since F is a rgwα-locally closed set, we have F∪S=F. Therefore S⊂F.

Theorem 3.8 Let F_α, F_β,F_γ be Maximal rgwα-locally closed sets such that $F_\alpha \neq F_\beta$ if $F_\alpha \cap F_\beta \subset F_\gamma$, then either $F_\alpha = F_\gamma$ or $F_\beta = F_\gamma$.

Proof: Given that $F_\alpha \cap F_\beta \subset F_\gamma$, if $F_\alpha = F_\gamma$ then there is nothing to prove but if $F_\alpha \neq F_\gamma$ then We have to prove $F_\beta = F_\gamma$.

Now we have $F_\beta \cap F_\gamma = F_\beta \cap (F_\gamma \cap X)$

$= F_\beta \cap (F_\gamma \cap (F_\alpha \cup F_\beta))$ (by theorem 3.6 (ii))

$= F_\beta \cap ((F_\gamma \cap F_\alpha) \cup (F_\gamma \cap F_\beta))$

$= (F_\beta \cap F_\gamma \cap F_\alpha) \cup (F_\beta \cap F_\gamma \cap F_\beta)$

$= (F_\alpha \cap F_\beta) \cup (F_\gamma \cap F_\beta)$ (by $F_\beta \cap F_\gamma \subset F_\beta$)

$= (F_\alpha \cup F_\gamma) \cap F_\beta$

$= X \cap F_\beta$ (since F_α, and F_γ are Maximal rgwα-locally closed sets by thm 3.6 (ii) $F_\alpha \cup F_\gamma = X$)

$= F_\beta$

That is $F_\beta \cap F_\gamma = F_\beta$ implies $F_\beta \subset F_\gamma$, since F_β, F_γ are maximal rgwα-locally closed sets, we have

$F_\beta = F_\gamma$.

Theorem 3.9 Let Let F_α, F_β,F_γ be Maximal rgwα-locally closed sets which are different from each other then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\gamma)$.

Proof: Let $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\gamma)$ which implies $(F_\alpha \cap F_\beta) \cup (F_\gamma \cap F_\beta) \subset (F_\alpha \cap F_\gamma) \cup (F_\gamma \cap F_\beta)$ which implies $(F_\alpha \cup F_\gamma) \cap F_\beta \subset F_\gamma \cap (F_\alpha \cup F_\beta)$ since by theorem 3.6 (ii) $F_\alpha \cap F_\gamma = X$ and $F_\alpha \cap F_\beta = X$ which implies $X \cap F_\beta \subset F_\gamma \cap X$ which implies $F_\beta \subset F_\gamma$. From the definition of Maximal rgwα-locally closed set it follows that $F_\beta = F_\gamma$. This is contradiction to the fact that Let F_α, F_β,F_γ are different from each other. Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\gamma)$.

Theorem 3.10 Let F be a Maximal rgwα-locally closed set and x be an element of F then F=∪{S: S is a rgwα-locally closed set containing x such that F∪S ≠ X}.

Proof: By theorem 3.7 and from the fact that F is a rgwα-locally closed set containing x we have F⊂∪{S:S is a rgwα-locally closed set containing x such that F∪S ≠ X}⊂ F therefore we have the result.

Theorem 3.11 Let F be a Proper non-empty co-finite rgwα-locally closed subset then there exists (co-finite) Maximal rgwα-locally closed set E such that F ⊂ E.

Proof: Let F be a non-empty co-finite rgwα-locally closed set. If F is a Maximal rgwα-locally closed set, we may set E=F. If F is not a Maximal rgwα-locally closed set, then there exists a (co-finite) rgwα-locally closed set F_1 such that $F \subset F_1 \neq X$. If F_1 is a Maximal rgwα-locally closed set, we may set $E = F_1$. If F_1 is not a Maximal rgwα-locally closed set, then there exists a (co-finite) rgwα-locally closed set F2 such that $F \subset F_1 \subset F_2 \neq X$ continuing this process we have a sequence of rgwα-locally closed sets $F \subset F_1 \subset F_2 \subset \ldots \subset F_k \subset \ldots$Since F is a co-finite set, this process repeats only finitely. Then finally we get a maximal rgwα-locally open set $E=E_n$ for some positive integer n.

Theorem 3.12 Let F be a Maximal rgwα-locally closed set. If x is an element of X-F then X-F⊂ E for any rgwα-locally closed set containing set E containing x

Proof: Let F be a Maximal rgwα-locally closed set and x∈ X–F. E⊄F for any rgwα-locally closed set E containing x, Then E∪F=X by theorem 3.6(ii).Therefore X–F ⊂ E.

4. Minimal rgwα-Locally Closed sets and Maximal rgwα-Locally Open sets.

Definition 4.1 A proper non empty rgwα-locally closed subset F of X is said to be a minimal

rgwα - locally closed set if and only if any rgwα-locally closed set which is contained in F is ϕ or F.

Remark 4.2 Minimal closed sets and minimal rgwα-locally closed sets are independent of each other as seen from the following example.

Example 4.3 Let X= **{**a, b,c,d} be any set with topology τ= {X, ϕ, {a}, {a,b}}. Closed sets are X, ϕ, {c,d}, {b,c,d}. Minimal closed set is {c,d}. Rgwα-locally closed sets are P(X) and minimal rgwα-locally closed sets are {a}, {b}, {c}, {d}. Here {a}, {b}, {c}, {d} are minimal rgwα-locally closed sets but not minimal closed set and set {c,d} is Minimal closed set but not minimal rgwα-locally closed set.

Definition 4.4 A proper non empty rgwα-locally open subset U of X is said to be a maximal

rgwα-locally open set if and only if any rgwα-locally open set which is contains U is X or U.

Remark 4.5 Maximal open sets and Maximal rgwα-locally open sets are independent of each other as seen from the following example.

Example 4.6 Let X= **{**a, b,c,d} be any set with topology τ= {X, ϕ, {a}, {a,b}}. Open sets are X, ϕ, {a}, {a,b}. Maximal open set is {a,b}. rgwα-locally closed sets are P(X) and maximal rgwα-locally open sets are {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}. Here {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, are maximal rgwα-locally closed sets but not maximal closed set and set {a,b} is maximal open set but not maximal rgwα-locally closed set.

Theorem 4.7 A proper non empty subset U of X is maximal rgwα-locally open set if and only if X-U is a minimal rgwα-locally closed set.

Proof: Let U be a maximal rgwα-locally open set. Suppose X-U is not a minimal rgwα-locally closed set. Then there exists an rgwα-locally closed set V≠X-Usuch that ϕ≠ V ⊂ X-U. That is U⊂ X-V and X-V is an rgwα-locally open set. This contradicts our assumption that F is a minimal rgwα-locally closed set.

Conversely let X-U be a minimal rgwα-locally closed set. Suppose U is not maximal rgwα-locally open set. Then there exists a rgwα-locally open set E≠U such that U⊂E ≠X. That is ϕ≠ X-E ⊂ X-U and X-E is a rgwα-locally closed set. This contradicts our assumption that X-U is a minimal rgwα-locally closed set. Therefore U is a maximal rgwα-locally open set.

Lemma 4.8 A proper non empty subset F of X is maximal rgwα-locally closed set if and only if X-F is a maximal rgwα-locally open set.

5. $\tau_{rgw\alpha Lc}$-Min and $\tau_{rgw\alpha Lc}$-Max Spaces.

Definition 5.1 A topological space X is said to be $\tau_{rgw\alpha lc}$-min space if every nonempty proper rgwα-locally open subset of X is minimal rgwα-locally open set.

Definition 5.2 A topological space X is said to be $\tau_{rgw\alpha lc}$-max space if every nonempty proper rgwα-locally open subset of X is maximal rgwα-locally open set.

Remark 5.3 The concepts of $\tau_{rgw\alpha lc}$-min and $\tau_{rgw\alpha lc}$-max spaces are not enough general. $\tau_{rgw\alpha lc}$-min or $\tau_{rgw\alpha lc}$-max topological space X will be either indiscrete space or {X, ϕ, A} or {X, ϕ, A, Ac}.

Theorem 5.4 A space X is $\tau_{rgw\alpha lc}$-min if and only if it is $\tau_{rgw\alpha lc}$-max

Proof: Let X is $\tau_{rgw\alpha lc}$-min space. Suppose that X is not $\tau_{rgw\alpha lc}$-max, so there is a proper rgwα-locally open subset K of X which is not maximal, this mean there exist a rgwα-locally open subset of X with K⊂H ≠ ϕ. Thus we get that H is not minimal which is contradict of being X is $\tau_{rgw\alpha lc}$ -min.

Conversely, Let X is $\tau_{rgw\alpha lc}$-max space. Suppose that X is not $\tau_{rgw\alpha lc}$-min, so there is a proper rgwα-locally open subset K of X which is not minimal, this mean there exist an rgwα-locally open subset of X with ϕ ≠ H□K. Thus we get that H is not maximal which is contradict of being X is $\tau_{rgw\alpha lc}$ -max.

Theorem 5.5 A topological space X is $\tau_{rgw\alpha lc}$-min space if and only if every nonempty proper rgwα-locally closed subset of X is maximal rgwα-locally closed set in X.

Proof: Let F be a proper rgwα-locally closed subset of X and suppose F is not maximal, then there exists an rgwα-locally closed subset K of X with K≠X such that F□K. Thus X-K □X-F. Hence X-F is a proper rgwα-locally open which is not minimal and this contradicts of being X is $\tau_{rgw\alpha lc}$-min space.

Conversely, Suppose U is a proper rgwα-locally open subset of X. thus X-U is a proper rgwα-locally closed subset of X, so X-U is maximal rgwα-locally closed subset of X. and from Theorem 4.7 U is minimal rgwα-locally open. Thus X is $\tau_{rgw\alpha lc}$-min space.

Theorem 5.6 A topological space X is $\tau_{rgw\alpha lc}$-max space if and only if every nonempty proper $\tau_{rgw\alpha lc}$-closed subset of X is minimal rgwα-locally closed set in X.

Proof: let F be a proper rgwα-locally closed subset of X, suppose F is not minimal rgwα-locally closed in X, so there is a proper rgwα-locally closed subset of X such that K□F Thus X-F□X-K but X-K is proper rgwα-locally open in X. so X-F is not maximal in X. Contradiction to the fact X-F is maximal rgwα-locally open.

Conversely, let U be a proper rgwα-locally open subset of X, then X-U is a proper rgwα-locally closed subset of X and so it is minimal rgwα-locally closed set. Theorem 4.7 we get that U is maximal rgwα-locally open. Thus X is rgwα-locally max space.

Theorem 5.7 Every pair of different minimal rgwα-locally open sets of $\tau_{rgw\alpha lc}$-min space are disjoint.

Proof: Let U and V be minimal rgwα-locally open subsets of $\tau_{rgw\alpha lc}$-min space X such that U≠V to show that U∩V=ϕ suppose not i.e. U∩ V≠ϕ. So U∩V □U and U∩V □V. Since U∩V□U and U is minimal rgwα-locally open then U∩V=U or U∩V=ϕ. Thus U∩V=U. Now since U∩V□V and V is minimal rgwα-locally open then U∩V=V or U∩V=ϕ. Thus U∩ V=V. Hence we get that U=V this result contradicts the fact that U and V are different. Therefore U∩ V=ϕ.

Theorem 5.8 Union of every pair of different maximal rgwα-locally open sets in $\tau_{rgw\alpha lc}$-max space is X.

Proof: Let U and V be maximal rgwα-locally open subsets of $\tau_{rgw\alpha lc}$-max space X such that U≠V to show that U∪V=X suppose not i.e. U∪V=X. So U□□U∪V and V□□U∪V. Since U□U∪V and U is maximal rgwα-locally open then U∪V=U or U∪V=X. Thus U∪V=U… (1). Now since V□U∪V and V is maximal rgwα-locally open then U∪V=V or U∪ V=X Thus U∪V=V… (2) Hence from (1) and (2) we get that U=V this result contradicts the fact that U and V are different. Therefore U∪V=X.

6. Conclusion

In this paper we have introduced and studied the properties of Minimal rgwα-Locally Open Sets and Maximalrgwα-Locally Closed Sets in Topological Spaces. Our future extension is to study rgwα-Locally Separation Axioms in Topological Spaces.

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