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Various mappings via (1, 2)*-M_{Gp}-Closed sets in Bitopological settings

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Abstract

In this paper, we introduce (1, 2)*-M_{gp}-continuous and contra-(1, 2)*-M_{gp}-continuous mappings with the help of (1, 2)*-M_{gp}-closed sets. Also, we define a new function called totally-(1, 2)*-M_{gp}-continuous functions, totally-(1, 2)*-M_{gp}-open functions and almost-(1, 2)*M_{gp}-closed mappings and its relationships with other functions in (1, 2)*-bitopological spaces are investigated.

Keywords: Bitopological, (1, 2)*-MGp-Closed

1. Introduction

Topology developed as a field of study out of set theory, through analysis of such concepts as, dimension, and transformation. By the middle of the 20th century, topology had become an important area of study within mathematics.

N. Levine [8] introduced the concepts of generalized closed set in topological spaces. Donchev [1] presented a new generalization of continuity called contra continuity and he defined a weaker form of contra-continuity called contra-semi-continuity and obtained several interesting results. The generalized closed maps (briefly. g-closed maps) have been defined and their properties are investigated by Malghan [9]. The attention was mainly confined to the pairwise properties of the two topologies is Bitopological. When the research was going on towards pairwise properties, in 1990, the endeavour of Lellis Thivagar [3] brought a new idea without using pairwise properties, namely, (1, 2)-bitopological spaces. In 2005, Lellis Thivagar and O. Ravi [7] introduced yet another bitopological space, namely, (1, 2)*-bitopological space.

The purpose of the present paper is to define a new class of continuous function called (1, 2)*-M_{gp}-continuous and contra-(1, 2)*-M_{gp}-continuous mappings via (1, 2)*-M_{gp}-closed sets and compare with the existing functions in (1, 2)*-bitopological spaces. Also investigate a new function called totally-(1, 2)*-M_{gp}-continuous functions, totally-(1, 2)*-M_{gp}-open functions and almost-(1, 2)*-M_{gp}-closed mappings in (1, 2)*-bitopological spaces.

2. Prelimnaries

Throughout this paper by a space X and Y represent non-empty (1, 2)* bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned. We recall the following definitions and results which are useful in the sequel.

Definition 2.1

i) A subset of S of a bitopological space X is said to be **$\tau_{1,2}$-open [7],** if $S=A\cup B$ where $A\epsilon\tau_1$ and $B\ \epsilon\tau_2$.
ii) A subset S of X is said to be **(i) $\tau_{1,2}$-closed [7],** if the complement of S is $\tau_{1,2}$–open.
iii) **$\tau_{1,2}$–clopen [8],** if S is both $\tau_{1,2}$–open and $\tau_{1,2}$–closed.

Definition 2.2

In the space X, a subset A is said to be (1, 2)*- **regular-open [7],** if A= $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).The complement of (1, 2)*-regular-open is (1, 2)*-regular-closed.

Definition 2.3

Let S be a subset of the bitopological space X. then
i) The (1, 2)*-**pre-interior [4]** of S denoted by pre-int(S) is defined by $\cup$\{G:G$\subseteq$S and G is (1, 2)*-pre-open\}.
ii) The (1, 2)*-**pre-closure [4]** of S denoted by pre-cl(S) is defined by $\cap$\{F: S$\subseteq$F and F is (1, 2)*-pre-closed\}.

Definition 2.4 A subset A of X is said to be
i) **(1, 2)*-M_{gp}-closed set [10]** if (1, 2)*-pcl (A)$\subseteq$ U whenever A $\subseteq$ U and U is (1.2)*-g-open.
ii) **(1, 2)*-gp-closed [14],** if (1, 2)*-pcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}$-open.
iii) **(1, 2)*-gpr-closed [12],** if (1, 2)*-pcl(A)$\subseteq$U whenever A$\subseteq$U and U is (1, 2)*- regular open.
iv) **(1, 2)*-πgb-closed [13],** if (1, 2)*-bcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}$-π-open.
v) **(1, 2)*- πgp-closed [11],** if (1, 2)*-pcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}$-π-open.

Definition 2.5

A function f: X→Y is called

i) **(1, 2)*-gp-continuous** [5]**,** if $f^{-1}(V)$ is (1, 2)*-gp-closed set in X, for every $\sigma_{1,2}$ -closed set V of Y.
ii) **(1, 2)*-gpr-continuous** [6]**,** if $f^{-1}(V)$ is (1, 2)*-gpr-closed sets in X, for every $\sigma_{1,2}$ -closed set V of Y.
iii) **(1, 2)*-πgb-continuous** [13]**,** if $f^{-1}(V)$ is (1, 2)*-πgb-closed set in X, for every $\sigma_{1,2}$ -closed set V of Y.
iv) **(1, 2)*-contra-continuous** [1], if $f^{-1}(V)$ is $\tau_{1,2}$ -open set in X, for every $\sigma_{1,2}$ -closed set V of Y.
v) **(1, 2)*-continuous** [4], if the inverse image of every $\sigma_{1,2}$ –closed set of Y is $\tau_{1,2}$ -closed set in X.

Definition 2.6

A function f : (X, τ)→(Y, σ) is said to be **Totally -continuous** [2]**,** if the inverse image of every open subsets of Y is a clopen subsets of X.

3.1 (1, 2)*-M_{gp}-Continuous Functions

Definition 3.1

A function f: X→Y is called

i) (1, 2)*-M_{gp}-continuous if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*- M_{gp}-closed set in X.
ii) contra-(1, 2)*-M_{gp}-continuous if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*- M_{gp}-closed set in X

Remark 3.2

(1, 2)*-M_{gp} –O(X) denotes the set of all (1, 2)*-M_{gp}-open sets in X.

Theorem 3.3

A function f: X→Y is (1, 2)*-M_{gp} –continuous iff f^{-1} (U) is (1, 2)*-M_{gp} –open in X, for every $\sigma_{1,2}$ –open set U in Y.

Proof

Suppose U is a $\sigma_{1,2}$ –open set in Y. Then $f^{-1}(U^C)$ is (1, 2)*-M_{gp} –closed set in X. But $f^{-1}(U^C) = [f^{-1}(U)]^C$ and hence f^{-1} (U) is (1, 2)*-M_{gp} –open in X.

Conversely, $f^{-1}(U)$ is (1, 2)*-M_{gp} –open set in X, for every $\sigma_{1,2}$ –open set U in Y. U^C is $\sigma_{1,2}$ –closed set in Y. Then $[f^{-1}(U)]^C$ is (1, 2)*-M_{gp} –closed in X. Hence $f^{-1}(U^C)$ is (1, 2)*-M_{gp} –closed set in X. Therefore, f is (1, 2)*-M_{gp} –continuous function.

Proposition 3.4

If a function f: X →Y is (1, 2)*-M_{gp}-continuous, then every subset A of X, f ((1, 2)*-M_{gp} -cl(A)) $\subseteq$ $\sigma_{1,2}$ -cl(f(A)).

Proof

Let A $\subseteq$ X. Since f is (1, 2)*-M_{gp} –continuous and A $\subseteq$ $f^{-1}(\sigma_{1,2}$ -cl(f(A))).

Also, A $\subseteq$ (1, 2)*-M_{gp} -cl(A). We obtain f ((1, 2)*-M_{gp} -cl(A)) $\subseteq$ ($\sigma_{1,2}$ -cl(f(A))).

Theorem 3.5

Let f: X→Y and g: Y→Z be two functions. Then (gof) is (1, 2)*-M_{gp}-continuous, if g is (1, 2)*-continuous and f is (1, 2)*-M_{gp}-continuous.

Proof

Let V be any $\eta_{1,2}$-closed set in Z. Then g^{-1} (V) is $\sigma_{1,2}$-closed set in Y. Thus f^{-1} (g^{-1} (V)) is (1, 2)*-M_{gp}-closed set in X, since f is (1, 2)*-M_{gp}-continuous. Thus (gof) is (1, 2)*-M_{gp}-continuous.

Proposition 3.6

Suppose (1, 2)*-M_{gp} –O(X) is $\tau_{1,2}$-closed under arbitrary union, for a function f: X → Y. Then, the following are equivalent.

i) f is contra-(1, 2)*-M_{gp}-continuous.
ii) The inverse image of each $\sigma_{1,2}$-closed set in Y is (1, 2)*-M_{gp}-open set in X.
iii) For each x ϵ X and each F ϵ $\sigma_{1,2}$-C(Y, f(x)), there exists U Є (1, 2)*-M_{gp}-O(X, x) such that f (U) $\subseteq$ F.

Proof

(i) => (ii), (ii) => (i) and (ii) => (iii) are obvious.

(iii)=> (ii). Let F be any $\sigma_{1,2}$-closed set of Y and x $\in f^{-1}(F)$. Then f(x) $\in$ F and there exists $U_x \in$ (1, 2)*-M_{gp}-O(X, x) such that f $(U_x) \subset$ F.

Hence we obtain $f^{-1}(F) = \bigcup \{U_x / x \in f^{-1}(F)\}$ Є (1, 2)*-M_{gp}-O(X). Thus the inverse image of each $\sigma_{1,2}$-closed set in Y is (1, 2)*-M_{gp}-open set in X.

Remark 3.7

The concepts of (1, 2)*-M_{gp}-continuity and contra-(1, 2)*-M_{gp}-continuity are independent as shown in the following example.

Example 3.8

Let X = {a, b, c, d} =Y, τ_1 = {Φ, X, {a, c, d}}, τ_2 = {Φ, X, {b}, {d}, {b, d}}σ_1= {Φ, Y, {b} }, σ_2 = {Φ, Y, {d}} and f: X → Y be the identity function.

Clearly, f is (1, 2)*-M_{gp}-continuous function, but not contra-(1, 2)*-M_{gp}-continuous.

Because, f^{-1}({b}, {d}, {b, d}) = ({b}, {d}, {b, d}) are not (1, 2)*-M_{gp}-closed set in X.

Example 3. 9

Let X = {a, b, c, d } =Y; τ_1 = {Φ, X, {b, d}}, τ_2 = {Φ, X, {d}}, σ_1= {Φ, Y, {a}, {b}, {a, b}}, σ_2 = {Φ, Y, {a, b, d}}, and f: X → Y be defined by f (a) = a, f (b) = c, f(c) = b, f(d)=d. Clearly, f is contra-(1, 2)*-M_{gp}-continuous function, but f is not (1, 2)*-M_{gp}-Continuous. Because, f^{-1}({a, c, d}, {c, d}) = ({a, b, d}, {b, d}) are not (1, 2)*-M_{gp}-closed set in X.

Preposition 3.10

A function f: X → Y is (1, 2)*-M_{gp}-continuous if for each x ∈ X and each $\sigma_{1, 2}$-open set V of Y containing f(x), there exists U∈ (1, 2)*-M_{gp}-O(X, x) such that f (U) ⊂ V.

Preposition 3.11

Suppose (1, 2)*-M_{gp} –O(X) is $\tau_{1, 2}$-closed under arbitrary unions. If a function f: X→Y is contra-(1, 2)*-M_{gp}-continuous and Y is (1, 2)*-regular open then f is (1, 2)*-M_{gp}-continuous.

Proof

Let x be an arbitrary point of X and V be an $\sigma_{1, 2}$-open set of Y containing f(x). Since Y is (1, 2)*-regular open, there exists an $\sigma_{1, 2}$-open set W of Y containing f(x) such that $\sigma_{1, 2}$-cl (W) ⊂ V. Since f is contra-(1, 2)*-M_{gp}-continuous, by theorem 3.3, there exists U∈(1, 2)*-M_{gp}-O(X, x) such that f (U) ⊂ $\sigma_{1, 2}$-cl (W). Then f (U) ⊂ $\sigma_{1, 2}$-cl (W) ⊂ V. Hence by theorem 3. 7, f is (1, 2)*-M_{gp}-continuous function.

Definition 3.12

A function f: X→Y is called

i) contra-(1, 2)*-gp-continuous if the inverse image of every $\sigma_{1, 2}$-open set of Y (1, 2)*-gp closed set in X.
ii) contra-(1, 2)*- gpr- continuous if the inverse image of every $\sigma_{1, 2}$-open set of Y (1, 2)*- gpr-closed set in X.
iii) contra-(1, 2)*- πgp - continuous if the inverse image of every $\sigma_{1, 2}$-open set of Y (1, 2)*- πgp-closed set in X.
iv) contra-(1, 2)*-πgb- continuous if the inverse image of every $\sigma_{1, 2}$-open set of Y (1, 2)*- πgb-closed set in X.

Preposition 3.13

i) Every contra-(1, 2)*-M_{gp}-continuous function is contra-(1, 2)*-gp-continuous function.
ii) Every contra-(1, 2)*- M_{gp}-continuous function is contra-(1, 2)*-gpr-continuous function.
iii) Every contra-(1, 2)*-M_{gp}-continuous function is contra-(1, 2)*-πgp-continuous function.
iv) Every contra (1, 2)*- M_{gp}-continuous function is contra-(1, 2)*-πgb-continuous function.
v) Every contra-(1, 2)*-continuous function is contra-(1, 2)*M_{gp}-continuous.

Proof

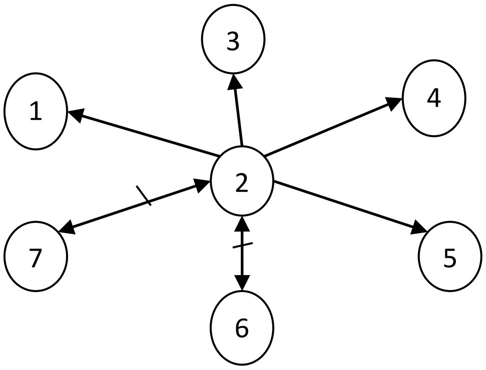
The proof follows from definitions. The converse of the preposition 3.10 need not be true as shown in the following example.

Example 3. 14

i) Let X = {a, b, c, d} =Y;, τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {c}, {b, c, d}}, σ_1= {Φ, Y, {a}, {d}, {a, d}}, σ_2 = {Φ, Y, {a, c}, {c, d}, {a, c, d}, {c}}Define f: X → Y be a function f (a) = b, f (b) = c, f(c) = d, f (d) =a. Then, f is contra-(1, 2)*-gp-continuous function, but not contra-(1, 2)*- M_{gp} -continuous. Since, f^{-1}({c, d}) = ({b, c}) is not (1, 2)*- M_{gp} -closed set in X.
ii) Let X = {a, b, c, d} =Y;τ_1 = {Φ, X, {a}, {d}, {a, d}}, τ_2 = {Φ, Y, {a, c}, {c, d}, {a, c, d}, {c}} σ_1= {Φ, Y, {c}, {a, b, d}}, σ_2 = {Φ, Y, {a, c, d}, {c, d}}and f: X → Y be a function defined by f (a) = a, f (b) = c, f(c) = b, f (d) = d. Then, f is contra-(1, 2)*-gpr-continuous function, but not contra-(1, 2)*-M_{gp}-continuous. Since, f^{-1}({a, b, d}, = ({a, c, d}) is not (1, 2)*-M_{gp}-closed set in X.
iii) Let X = {a, b, c, d} =Y, τ_1 = {Φ, X, {a}, {b}, {a, b}, τ_2 = {Φ, X, {a, b, d}}, σ_1= {Φ, Y, {a}, {b, c}, {a, b, c}}, σ_2 = {Φ, Y, { c, d}} and f: X → Y be a function defined by f (a) = c, f (b) = a, f(c) = d, f (d) =b.Then, f is a contra-(1, 2)*- πgp-continuous function, but not contra-(1, 2)*-M_{gp}-continuous function. Since, f^{-1}({{b, c}) = ({a, d}) is not (1, 2)*-M_{gp}-closed set in X.
iv) Let X = {a, b, c, d} =Y, τ_1 = {Φ, X, {b}}, τ_2 = {Φ, X, {a, d}}, σ_1= {Φ, Y, {b}, {d}, {b, d}}, σ_2 = {Φ, Y, {c}} and f: X → Y be a function defined by f (a) = c, f (b) = b, f(c) = a, f (d) =d Then, f is a contra-(1, 2)*- πgb-continuous function, but not contra-(1, 2)*-M_{gp}-continuous function. Since, f^{-1}({d}, {b, c, d}) = ({d}, {a, b, d}) are not (1, 2)*-M_{gp}-closed set in X.
v) Let X = {a, b, c, d} =Y;, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {a, b, d}}, σ_1= {Φ, Y, {b}, {a, b}}, σ_2 = {Φ, Y, {b, c}} and f: X → Y be an function f(a)=b, f(b)=c, f(c)=d, f(d)=a.Then, f is contra-(1, 2)*-M_{gp}-continuous function, but not contra-(1, 2)*-continuous. Because, f^{1}({a, b}, {b, c}) = ({a, d}, {a, b}) are not $\tau_{1, 2}$-closed set in X.

Example 3. 15

The above results and examples are summarized in the following diagram.



1. contra-(1, 2)*-gp-continuous
2. contra-(1, 2)*-M_{gp}-continuous
3. contra-(1, 2)*-gpr continuous
4. contra (1, 2)*-πgb-continuous
5. contra-(1, 2)*-πgp-continuous
6. (1, 2)*-M_{gp}-continuous,
7. contra-(1, 2)*-continuous

Definition 3.16

i) A function f: X→Y is called (**1, 2)*-M_{gp}-irresolute,** if the inverse image of (1, 2)*-M_{gp}-closed set in Y is (1, 2)*- M_{gp}-closed set in X.

ii) A function f: X→Y is called **Contra-(1, 2)*-M_{gp}-irresolute,** if the inverse image of (1, 2)*-M_{gp}-open set in Y is (1, 2)*- M_{gp}-closed set in X.

Remark 3. 17

The concepts of contra-(1, 2)*-M_{gp}-irresolute function and (1, 2)*-M_{gp}-irresolute function are independent of each other as shown in the following example.

Example 3. 18

Let X = {a, b, c, d} =Y;, τ_1 = {Φ, X, {a, }, {d}, {a, d}}, τ_2 = {Φ, X, {c}} σ_1= {Φ, Y, {a}, {c}, {a, c}}, σ_2 = {Φ, Y, {a, d}, {d}} and f: X → Y be an identity function. Then, f is (1, 2)*-M_{gp}-irresolute function, but not contra-(1, 2)*-M_{gp}-irresolute, because f^{-1}({a, c, d}, {c, d}, {a, d}, {a, c}, {d}, {a}) = ({a, c, d}, {c, d}, {a, d}, {a, c}, {d}, {a}) are not (1, 2)*-M_{gp}-closed set in X.

Example 3. 19

Let X = {a, b, c, d} =Y;, τ_1 = {Φ, X, {a, b}, {b, c}, {b}, {a, b, c}}, τ_2 = {Φ, X, {d}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, b}, {c}, {a, b, c}}. Define a function f: X → Y be an identity function. Then, f is contra-(1, 2)*-M_{gp}-irresolute function, but not (1, 2)*-M_{gp}-irresolute. Because, f^{-1}({b, c, d}, {a, b, d}) = ({b, c, d}, {a, b, d}) are not (1, 2)*-M_{gp}-closed set in X.

Remark 3.20

The composition of two contra-(1, 2)*-M_{gp}-continuous functions need not be contra-(1, 2)*-M_{gp}-continuous as the following example shows.

Example 3. 21

Let X = {a, b, c, d} =Y=Z, τ_1 = { ϕ , X, {a}}, τ_2 = { ϕ , X, {a, b, d}}, σ_1= { ϕ , Y, {a}, {a, b, d}}, σ_2 = { ϕ , Y, {c}, {b, c, d}}, η_1= { ϕ , Z, {a}}, η_2 = { ϕ , Z, {b}}. Define a function f: X → Y and g:Y→Z be an identity function.Then, both f and g are contra-(1, 2)*-M_{gp}-continuous function, but gof :X→Z is not contra-(1, 2)*-M_{gp}-continuous. Because, $(gof)^{-1}$({a}, {a, b}) = {a}, {a, b}) are not (1, 2)*-M_{gp}-closed set in X.

Theorem 3.22

Suppose (1, 2)*-M_{gp} –O(X) is $\tau_{1,2}$-closed under arbitrary unions. Let f: X→ Y is a function, and then the following are equivalent.

i) f is contra-(1, 2)*-M_{gp}-irresolute.

ii) For $x \in X$ and any (1, 2)*-M_{gp}-open set V of Y containing f(x), there exists an (1, 2)*-M_{gp}-closed set U such that $x \in X$ and f (U) $\subset$ V.

iii) Inverse image of every (1, 2)*-M_{gp}-closed set in Y is (1, 2)*-M_{gp}-open in X.

Proof

(i) ⇒ (ii). Let V be an (1, 2)*-M_{gp}-open set in Y and f(x) Є V. Since f is contra-(1, 2)*-M_{gp}-irresolute, $f^{-1}(V)$ is (1, 2)*-M_{gp}-closed set in X and x Є $f^{-1}(V)$. Put U= $f^{-1}(V)$. Then x Є U and f (U) $\subset$ V.

(ii)⇒ (i). Let V be an (1, 2)*-M_{gp}-open set in Y and x Є $f^{-1}(V)$. Then f(x) Є V. Hence by (ii), there exists an (1, 2)*-M_{gp}-closed set U_x such that x Є U_x and f (U_x) $\subset$ V. Thus $x \in U_x \subset f^{-1}(V)$. This implies that $f^{-1}(V)$ is a union of (1, 2)*-M_{gp}-closed sets of X. Thus $f^{-1}(V)$ is (1, 2)*-M_{gp}-closed set of X. Hence, f is contra-(1, 2)*-M_{gp}-irresolute.

(i)<=> (iii). Let V be an (1, 2)*-M_{gp}-closed in Y. Then Y-V is (1, 2)*-M_{gp}-open set in Y. Since f is contra (1, 2)*-M_{gp}-irresolute. $f^{-1}(Y\text{-}V)$ is (1, 2)*-M_{gp}-closed set in X. Also $f^{-1}(Y\text{-}V) = X - f^{-1}(V)$. Therefore, X- $f^{-1}(V)$ is (1, 2)*-M_{gp}-closed set in X. Hence, $f^{-1}(V)$ is (1, 2)*-M_{gp}-open set in X.

Theorem 3.23

Let f: X→Y and g: Y→Z be two functions. Then (gof) is (1, 2)*-M_{gp}-irresolute, if g is (1, 2)*-M_{gp}-irresolute and f is (1, 2)*-M_{gp}-irresolute.

Proof

Let U be any (1, 2)*-M_{gp}-closed set in Z. Since g is (1, 2)*-M_{gp}-irresolute, $g^{-1}(U)$ is (1, 2)*-M_{gp}-closed in Y. Then $f^{-1}(g^{-1}(U))=(gof)^{-1}(U)$ is (1, 2)*-M_{gp}-closed set in X. Therefore, (gof) is (1, 2)*-M_{gp}-irresolute.

Preposition 3.24

Let f: X→ Y and g: Y→ Z be two functions. Then, the composition gof : X→Z may be studied in the following several cases for different choices of f and g:

S. No	Mapping f :X→Y	Mapping g:Y→Z	g o f :X→Z
1.	contra-(1, 2)*-M_{gp}-continuous	(1, 2)*-continuous	contra-(1, 2)*-M_{gp}-continuous.
2.	(1, 2)*-M_{gp}-irresolute	contra-(1, 2)*-M_{gp}-continuous	contra-(1, 2)*-M_{gp}-continuous
3.	contra-(1, 2)*-M_{gp}-irresolute	(1, 2)*-M_{gp}-continuous	contra-(1, 2)*-M_{gp}-continuous
4.	(1, 2)*-M_{gp}-irresolute	contra-(1, 2)*-M_{gp}-irresolute	contra-(1, 2)*-M_{gp}-irresolute

Proof

The proof follows from the definitions.

4. Totally-(1, 2)*-M_{gp}-Continuous Functions

Definition: 4.1

A function f: X→Y is said to be

i) Totally-(1, 2)*-M_{gp}-continuous if the inverse image of every $\sigma_{1,2}$-open set in Y is (1, 2)*- M_{gp}-clopen set in X.

ii) Totally-(1, 2)*-M_{gp}-open (resp. (1, 2)*-open) if the image of every $\tau_{1,2}$ –open set in X is (1, 2)*-M_{gp}-clopen(resp. $\tau_{1,2}$ -open) set in Y.

Preposition: 4. 2

If a bijective function f: X→Y is totally-(1, 2)*-M_{gp}-open, then the image of each $\tau_{1,2}$ -closed set in X is (1, 2)*-M_{gp}-clopen in Y.

Preposition: 4.3

For any bijective function f: X→Y, the following statements are equivalent:

i) Inverse of f is totally-(1, 2)*-M_{gp}-continuous.

ii) f is totally-(1, 2)*-M_{gp}-open functions.

Proof

(i) => (ii): Let U be a $\tau_{1,2}$ -open set of X. By assumption $(f^{-1})^{-1}(U) = f(U)$ is (1, 2)*-M_{gp}-clopen in Y. So f is totally-(1, 2)*-M_{gp}-open.

(ii) => (i): Let V be $\tau_{1,2}$ -open in X. Then f(V) is (1, 2)*-M_{gp}-clopen in Y. That is $(f^{-1})^{-1}(V)$ is (1, 2)*-M_{gp}-clopen in Y. Therefore f^{-1} is totally-(1, 2)*-M_{gp}-continuous function.

Remark: 4.4

The composition of two totally-(1, 2)*-M_{gp}-open functions need not be totally-(1, 2)*-M_{gp}-open. The following example shows this result.

Example: 4.5

Let X =Y= {a, b, c, d}=Z, τ_1 = {X, ϕ, {a}, {a, b, d}}, τ_2 = {X, ϕ, {b, c, d}}, σ_1 = {Y, ϕ, {a}};σ_2 = {Y, ϕ, {b, c}}, η_1 = {Z, ϕ, {a}, {b, d}, {a, b, d}}, η_2 = {Z, ϕ, {b, c}}.
Let f: X→Y be the function defined by f(a) = c, f(b) = a, f(c) = b, f(d) = d and g : Y → Z be defined by g(a) = a, g(b) = d, g(c) = c, g(d) = b. Then f and g are totally-(1, 2)*-M_{gp}-open functions, but gof is not totally-(1, 2)*-M_{gp}-open function.

Preposition: 4.6

A function f: X→Y is totally-(1, 2)*-M_{gp}-continuous if and only if the inverse image of every $\sigma_{1,2}$ closed subset of Y is (1, 2)*- M_{gp}-clopen in X.

Proof

The proof is obvious.

Preposition: 4.7

Every totally-(1, 2)*-M_{gp}-continuous function is (1, 2)*-M_{gp}-continuous function.

Proof

Let V be $\sigma_{1,\,2}$ closed set in Y.Since f is totally-(1, 2)*-M_{gp}-continuous function, $f^{-1}(V)$ is (1, 2)*-M_{gp}-clopen set in X.

The converse of the above preposition need not be true, as the following example shows this result.

Example: 4.8

Let X= {a,b,c,d}=Y, τ_1 = {X, ϕ, {c}}, , τ_2 = {X, ϕ, {a}, { b, c}, {a, b, c}}, σ_1 = {Y, ϕ, {d}}, σ_2 = {Y, ϕ, {b}, {a}, {d}, {a, b}, {a, d}, {b, d}, {a, b, d}}. Define a function f: X→Y by f(a)=b, f(b)=a, f(c)=c, f(c)=d. Then f is a (1, 2)*-M_{gp}-continuous function, but not totally-(1, 2)*-M_{gp}-continuous function.

Remark: 4.9

The composition of two totally- (1, 2)*-M_{gp}-continuous functions need not be totally- (1, 2)*-M_{gp}-continuous.

Example: 4.10

Let X =Y= {a, b, c, d}=Z, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b, d}}, σ_1 = {Y, ϕ, {a}};σ_2 = {Y, ϕ, {b, d}}, , η_1 = {Z, ϕ, {b}}, η_2 = {Z, ϕ, {b, c}}. Let f: X→Y be a function defined by f(a)= b, f(b) =a, f(c) = d, f(d) = c and g : Y → Z be the identity function. Then f and g are totally-(1, 2)*-M_{gp}-open function, but gof is not totally-(1, 2)*-M_{gp}-open function.

Preposition: 4.11

Let f: X→Y and g: Y→Z be two maps. Then, the composition gof : X→Z may be studied in the following several cases for different choices of f and g:

S. No	Mapping f:X→Y	Mapping g:Y→Z	gof:X→Z
1.	(1, 2)*-open	totally-(1, 2)*-M_{gp}-open	totally-(1, 2)*-M_{gp}-open
2.	(1, 2)*-continuous	totally- (1, 2)*-M_{gp}-continuous	totally- (1, 2)*-M_{gp}-continuous

Proof

The proof follows from the definitions.

5. Almost-(1, 2)*-M_{gp}-Closed Mappings

Definition: 5.1

A map f: X→Y is called

i) (1, 2)*-M_{gp}-closed map if the image of each $\tau_{1,\,2}$- closed set in X is (1, 2)*-M_{gp}-closed set in Y.

ii) Almost-(1, 2)*-M_{gp}-closed map if the image of each (1, 2)*-regular closed set in X is (1, 2)*-M_{gp}-closed set in Y.

iii) (1, 2)*-g-closed map if the image of each $\tau_{1,\,2}$- closed set in X is (1, 2)*-g-closed set in Y.

iv) Almost-(1, 2)*-g-closed map if the image of each (1, 2)*-regular closed set in X is (1, 2)*-g-closed set in Y.

Theorem: 5.2

Let f: X→Y and g: Y→Z be two maps and gof: X→Z be an (1, 2)*-M_{gp}-closed map if f is (1, 2)*-continuous, then g is (1, 2)*-M_{gp}-closed map.

Proof

Let V be $\sigma_{1,\,2}$-closed set in Y. Since f is (1, 2)*-continuous, $f^{-1}(V)$ is $\tau_{1,\,2}$-closed set in X. Since gof is (1, 2)*-M_{gp}-closed set. So $(gof)(f^{-1}(V))$ is (1, 2)*-M_{gp}-closed set in Z.(i.e) g (V) is (1, 2)*-M_{gp}-closed set in Z. Hence g is (1, 2)*-M_{gp}-closed map.

Theorem: 5.3

A bijection f: X→Y is (1, 2)*-M_{gp}-closed map iff f(U) is (1, 2)*-M_{gp}-open set in Y, for every $\tau_{1,\,2}$-open set U in X.

Proof

The proof is obvious.

Theorem: 5.4

i) Every almost-(1, 2)*-g-closed map is almost-(1, 2)*-M_{gp}-closed map.

ii) Every (1, 2)*-M_{gp}-closed map is almost-(1, 2)*-M_{gp}-closed map.

iii) Every (1, 2)*-g-closed map is almost-(1, 2)*-M_{gp}-closed map.

Proof

The Proof follows from the Definition.

The converse of the above theorem need not be true, as the following example shows this result.

Example: 5.5

Let X= {a,b,c,d}=Y ;

i) τ_1 = {Φ, X, {a, b}, {c, d}}, τ_2 = {Φ, X, {a, b}{a, b, c}} σ_1= {Φ, Y, {a}, {d}, {a, d}}, σ_2 = {Φ, Y, {a, c}, {c, d}, {a, c, d}} and f: X → Y be f(a)=b, f(b)=d, f(c)=a, f(d)=c. Then, f is almost-(1, 2)*-M_{gp}-closed map, but not almost-(1, 2)*-g-closed map, because f ({a, c}) = ({a, c}) are not almost-(1, 2)*-g-closed map.

ii) τ_1 = {Φ, X, {a}, {d}, {a, d}}, τ_2 = {Φ, X, {c}} σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, b, d}}and f: X → Y be f(a)=b, f(b)=d, f(c)=c, f(d)=a. Then, f is almost-(1, 2)*-M_{gp}-closed map, but not (1, 2)*-M_{gp}-closed map, because f ({b, d}) = ({a, d}) are not (1, 2)*-M_{gp}-closed map.

iii) τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {b}} σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, b, d}}and f: X → Y be f(a)=a, f(b)=c, f(c)=b, f(d)=d. Then, f is almost-(1, 2)*-M_{gp}-closed map, but not (1, 2)*-g-closed map, because f ({a, c, d}, {c, d}) = ({a, b, d}, {b, d}) are not (1, 2)*-g-closed map.

Theorem: 5.6

A surjective map f:X→Y is almost-(1, 2)*-M_{gp}-closed map iff for each subset S of Y and each (1, 2)*-regular open set U of X containing $f^{-1}(S)$, there exist (1, 2)*-M_{gp}-open sets V of Y such that S⊂V and $f^{-1}(V)\subset U$.

Proof

Necessity

Suppose f is almost-(1, 2)*-M_{gp}-closed map and let S be an subset of Y and U be an (1, 2)*-regular-open set of X containing $f^{-1}(S)$.Then X-U is (1, 2)*-regular closed subset of X. Then f(X-U) is (1, 2)*-M_{gp}-closed set in Y and Y-f(X-U) is (1, 2)*-M_{gp}-open set in Y. Take V=Y-f(X-U). Then V is (1, 2)*-M_{gp}-open subset of Y containing S such that $f^{-1}(V)\subset U$

Sufficient

Let F be an (1, 2)*-regular subset of X. Then $f^{-1}(Y-f(F)) \subset X-F$ and X-F is (1, 2)*-regular open in X. Put B=Y-f(F). Then $f^{-1}(B) \subset X-F$. There exist an (1, 2)*-M_{gp}-open set V of Y, such that B=Y-f(F) ⊂ V and $f^{-1}(V)\subset X-F$. Therefore, f(F) =Y-V and hence f(F) is (1, 2)*-M_{gp}-closed set in Y. Thus, f is almost-(1, 2)*-M_{gp}-closed map.

Theorem: 5.7

Let f: X→Y and g: Y→Z be two maps. Then the composition gof:X→Z may be studied in the following several cases for different choices of f and g:

S. No	Mapping f:X→Y	Mapping g:Y→Z	gof:X→Z
1.	(1, 2)*-closed map	(1, 2)*-M_{gp}-closed map	(1, 2)*-M_{gp}-closed map
2.	(1, 2)*-continuous	(1, 2)*-M_{gp}-closed map	(1, 2)*-M_{gp}-closed map
3.	(1, 2)*-closed map	(1, 2)*-M_{gp}-closed	Almost-(1, 2)*-M_{gp}-closed

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