

Path related extended mean cordial graphs

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Abstract

Let $G = (V, E)$ be a graphs with p vertices and q edges. A Extended Mean Cordial Labeling of a Graph G with vertex set V is a bijection from V to $\{-1, 0, 1\}$ such that each edge uv is assigned the label $(|f(u)+f(v)|)/2$ where $|x|$ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits an Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that path related graphs $(P_n \cdot S_m)$, $(z-p_n)$, $P_n \odot P_2$, $JD_2(C_2)$, $(P_n \odot K_1) + v$ are Extended Mean Cordial Graphs. 2000 Mathematics Subject classification 05C78.

Keywords: Extended Mean Cordial Graph, Extended Mean Cordial Labeling

1. Introduction

A graph G is finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{u, v\}$ of vertices in E is called edges or a line of G . In this paper, we proved that path related graphs $(P_n \cdot S_m)$, $(z-p_n)$, $P_n \odot P_2$, $JD_2(C_2)$, $(P_n \odot K_1) + v$ are Extended Mean Cordial Graphs. For graph theory terminology we follow [2].

2. Priliminaries

Let $G = (V, E)$ be a graphs with p vertices and q edges. A Extended Mean Cordial Labeling of a Graph G with vertex set V is a bijection from V to $\{-1, 0, 1\}$ such that each edge uv is assigned the label $(|f(u)+f(v)|)/2$ where $|x|$ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits an Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that path related graphs $(P_n \cdot S_m)$, $(z-p_n)$, $P_n \odot P_2$, $JD_2(C_2)$, $(P_n \odot K_1) + v$ are Extended Mean Cordial Graphs.

Definition 2.1.1

Star of length one is joined with every vertex of a path P_n through an edge. It is denoted by $(P_n \cdot S_m)$.

Definition 2.1.2

In pair of path P_n with vertex of a path P_1 is joined with $i+1$ th vertex of a path P_2 . It is denoted by $(z-p_n)$.

Definition 2.1.3

$P_n \odot P_2$ is a graph obtained from path of length n by joining one end of P_2 at each and every vertex of P_n .

Definition 2.1.4

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex through a vertex w in G_2 . The resulting graph is known as join shadow graph and it is denoted by $JD_2(C_2)$.

Definition 2.1.5

It is a graph obtained from comb by joining a pendent vertex at one end of path P_n . It is denoted by $P_n^+ + v$ (or) $(P_n \odot K_1) + v$.

3. Main Result

Theorem 3.1

Graphs $(P_n \cdot S_m)$ is an extended mean cordial graphs.

Proof:

$$\begin{aligned} \text{Let } v(P_n \cdot S_m) &= \{(u_i v_i) : 1 \leq i \leq n\} \cup \{(v_{i1} v_{i2}) : 1 \leq i \leq n\} \\ \text{Let } E(P_n \cdot S_m) &= \{(u_i u_{i+1}) : 1 \leq i \leq n\} \cup \{(u_i v_i) : 1 \leq i \leq n\} \cup \{(v_i v_{i1}) : 1 \leq i \leq n\} \\ &\quad \cup \{(v_i v_{i2}) : 1 \leq i \leq n\} \end{aligned}$$

Define $f: v(P_n: S_m) \rightarrow \{-1, 0, 1\}$ by

The vertex labeling are

$$\begin{aligned} f(u_i) &= -1, 1 \leq i \leq n \\ f(v_i) &= 1, 1 \leq i \leq n \\ f(v_{i1}) &= 0, 1 \leq i \leq n \\ f(v_{i2}) &= 1, 1 \leq i \leq n \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_i u_{i+1}) &= 1, 1 \leq i \leq n - 1 \\ f^*(u_i v_i) &= 0, 1 \leq i \leq n \\ f^*(v_i v_{i1}) &= 0, 1 \leq i \leq n \\ f^*(v_i v_{i2}) &= 0, 1 \leq i \leq n \end{aligned}$$

Here $ef(0)=2n, ef(1)=2n-1$

Hence, $(P_n: S_m)$ is satisfies the condition $|ef(0)-ef(1)| \leq 1$.

Therefore, $(P_n: S_m)$ is a extended mean cordial graphs, for example, $(P_n: S_m)$ is a extended mean cordial graph as shown in the figure 3.2

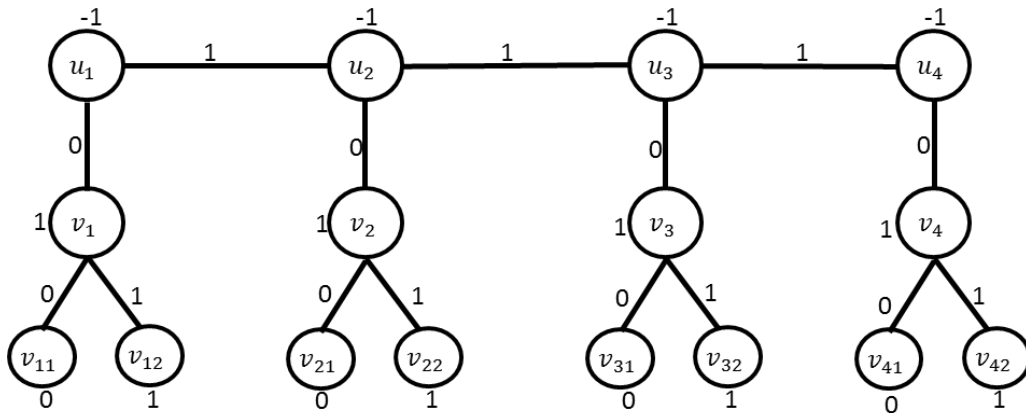


Fig 3.2($P_n: S_m$)

Theorem: 3.2

Graph $(z-p_n)$ is a extended mean cordial graphs

Proof

Let $v(z-p_n) = \{[u_i v_i: 1 \leq i \leq n]\}$

Let $E(z-p_n) = \{[(u_i u_{i+1}): 1 \leq i \leq n-1] \cup [(v_i v_{i+1}): 1 \leq i \leq n-1] \cup [(u_{i+1} v_i): 1 \leq i \leq n-1]\}$

Define $f: v(z-p_n) \rightarrow \{-1, 0, 1\}$ by

The vertex labeling are

$$\begin{aligned} f(u_i) &= \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}, 1 \leq i \leq n, \\ f(v_i) &= \begin{cases} -1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}, 1 \leq i \leq n \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_i u_{i+1}) &= 0, 1 \leq i \leq n-1, f^*(v_i v_{i+1}) = 0, 1 \leq i \leq n-1 \\ f^*(u_{i+1} v_i) &= \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}, 1 \leq i \leq n \end{aligned}$$

Here, When $n = 2m$

$$\begin{aligned} ef(0) &= m, ef(1) = m+1 \\ \text{When } n &= 2m+1 \\ ef(0) &= ef(1) = 3m \end{aligned}$$

Hence, $(z-p_n)$ satisfies the condition $|ef(0)-ef(1)| \leq 1$

Therefore, $(z-p_n)$ is an extended mean cordial graph. For example, $(z-p_n)$ is an extended mean cordial graph as shown in the figure 3.4

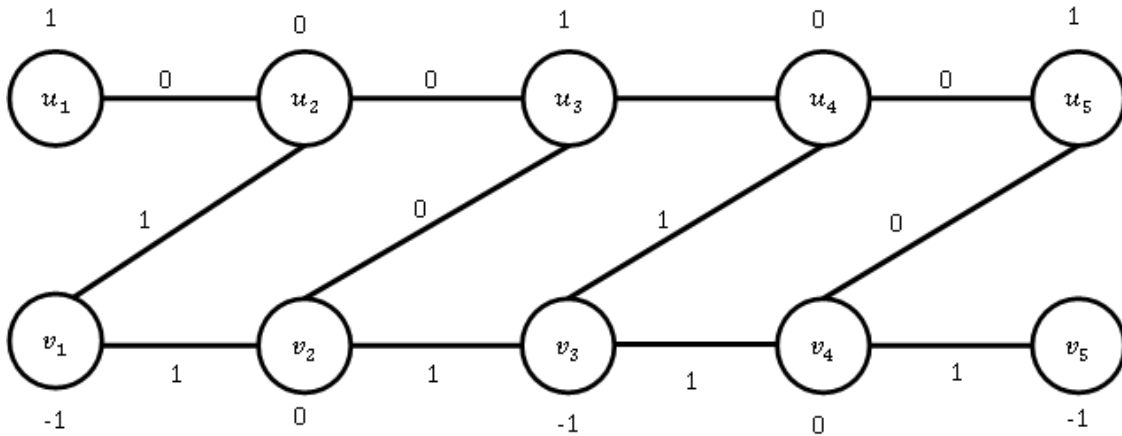


Fig 3.4 $(z-p_n)$

Theorem 3.5

Graph $P_n \odot P_2$ is an extended mean cordial graph

Proof

$$\begin{aligned} V(P_n \odot P_2) &= \{(v_i): 1 \leq i \leq n, (u_{i1}u_{i2}): 1 \leq i \leq n\} \\ E(P_n \odot P_2) &= \{(u_{i1}u_{i2}): 1 \leq i \leq n\} \cup \{(v_i v_{i+1}): 1 \leq i \leq n-1\} \cup \{(v_i u_{i1}): 1 \leq i \leq n\} \\ \text{Define } f: V(P_n \odot P_2) &\rightarrow \{-1, 0, 1\} \text{ by} \end{aligned}$$

The vertex labeling are

$$\begin{aligned} f(u_{i1}) &= 0, 1 \leq i \leq n \\ f(u_{i2}) &= 0, 1 \leq i \leq n \\ f(v_i) &= \begin{cases} -1 & i \equiv 1 \pmod{2} \\ 0 & i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(v_i v_{i+1}) &= 1, 1 \leq i \leq n-1 \\ f^*(u_{i1}u_{i2}) &= 0, 1 \leq i \leq n \\ f^*(v_i u_{i1}) &= \begin{cases} 1 & i \equiv 1 \pmod{2} \\ 0 & i \equiv 0 \pmod{2} \end{cases}, 1 \leq i \leq n \end{aligned}$$

Here, When $n = 2m$

$$\begin{aligned} ef(0) &= 3m \\ ef(1) &= 3m-1 \end{aligned}$$

When $n = 2m-1$

$$ef(0) = ef(1) = 3m+1$$

Hence, $P_n \odot P_2$ satisfies the condition $|ef(0)-ef(1)| \leq 1$

Therefore, $P_n \odot P_2$ is an extended mean cordial graph. For example, $P_n \odot P_2$ is an extended mean cordial graph as shown in figure 3.6

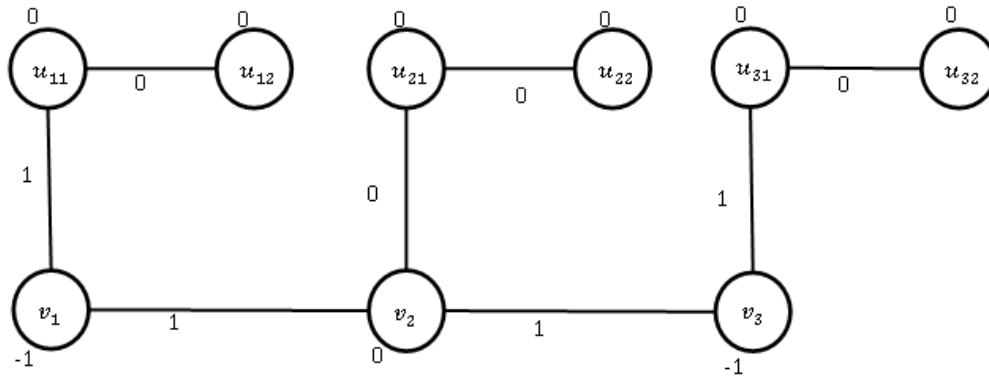


Fig 3.6 $P_n \odot P_2$

Theorem 3.7

Graph $JD_2(C_2)$ is an extended mean cordial graph

Proof

Let $v(JD_2(C_2)) = \{(u_i v_i) : 1 \leq i \leq n, (w_i) : 1 \leq i \leq n\}$
 Let $E(JD_2(C_2)) = \{(u_i u_{i+1}) U (w_i w_{i+1}) U (u_i v_i) U (u_{i+1} v_i) U (w_i v_i) U (w_{i+1} v_i) : 1 \leq i \leq n-1\}$
 Define $f : v(JD_2(C_2)) \rightarrow \{-1, 0, 1\}$ by

The vertex labeling are

$$\begin{aligned} f(u_i) &= 1, 1 \leq i \leq n \\ f(w_i) &= 0, 1 \leq i \leq n \\ f(v_i) &= -1, 1 \leq i \leq n \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_i u_{i+1}) &= 1, 1 \leq i \leq n-1 \\ f^*(w_i w_{i+1}) &= 0, 1 \leq i \leq n-1 \\ f^*(u_i v_i) &= 0, 1 \leq i \leq n-1 \\ f^*(u_{i+1} v_i) &= 0, 1 \leq i \leq n-1 \\ f^*(w_i v_i) &= 1, 1 \leq i \leq n-1 \\ f^*(w_{i+1} v_i) &= 1, 1 \leq i \leq n-1 \end{aligned}$$

Here, When $n = m+1$

$$ef(0) = ef(1) = 3m$$

Hence, $JD_2(C_2)$ satisfies the condition $|ef(0) - ef(1)| \leq 1$

Therefore, $JD_2(C_2)$ is an extended mean cordial graph

For example, $JD_2(C_2)$ is an extended mean cordial graph as shown in figure 3.8

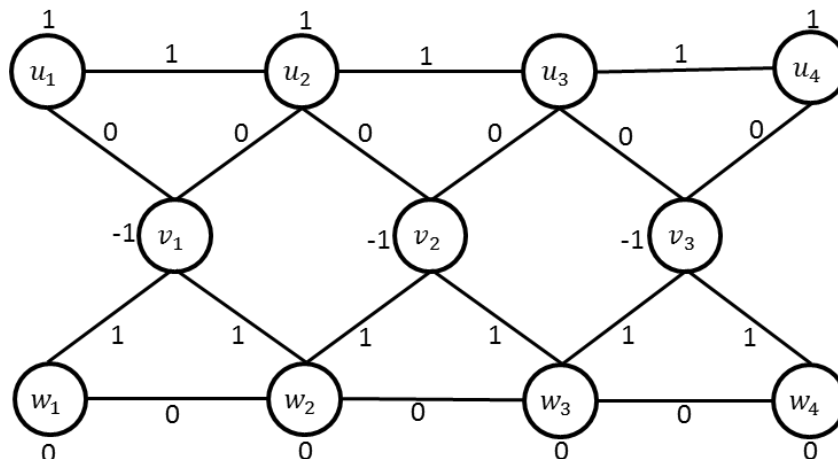


Fig 3.8 $JD_2(C_2)$

Theorem 3.9

Graph $(P_n \odot K_1) + v$ is an extended mean cordial graph

Proof

Let $v((P_n \odot K_1) + v) = \{(u_i v_i) : 1 \leq i \leq n\}$
 Let $E((P_n \odot K_1) + v) = \{(u_i u_{i+1}) : 1 \leq i \leq n\} \cup \{(v_i u_{i+1}) : 1 \leq i \leq n\}$
 Define $f: v((P_n \odot K_1) + v) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$$f(u_i) = 1, 1 \leq i \leq n \text{ and } f(v_i) = -1, 1 \leq i \leq n$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n$$

$$f^*(v_i u_{i+1}) = 0, 1 \leq i \leq n$$

Here, When $n = 2m$
 $ef(0) = ef(1) = 2m - 1$
 When $n = 2m + 1$
 $ef(0) = ef(1) = 2m$

Hence, $(P_n \odot K_1) + v$ satisfies the condition $|ef(0) - ef(1)| \leq 1$

Therefore, $(P_n \odot K_1) + v$ is an extended mean cordial graph. For example, $(P_n \odot K_1) + v$ is an extended mean cordial graph as shown in figure 3.10

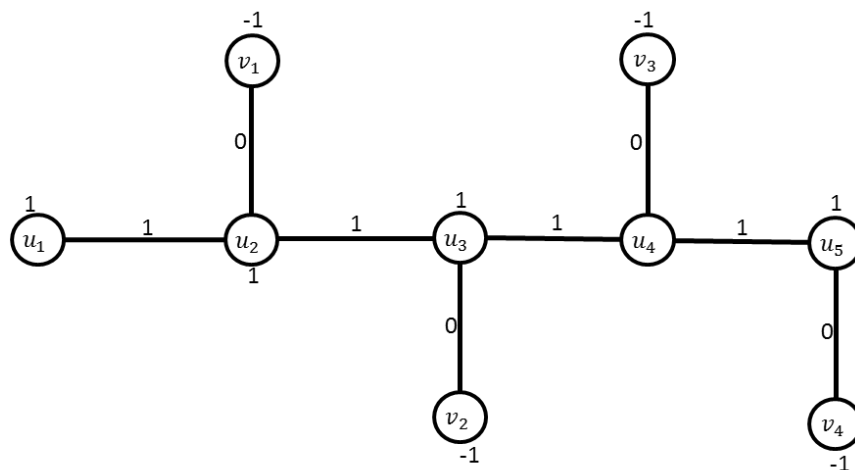


Fig 3.10 $(P_n \odot K_1) + v$

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