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A new class of vague generalized alpha mapping

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Abstract
The purpose of this paper is to introduce and study the concept of vague generalized alpha continuous mappings and vague generalized irresolute mappings in vague topological spaces.

Keywords: Vague generalized alpha continuous mappings and Vague generalized irresolute mappings

1. Introduction

Zadeh [14] introduced the concept of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [2] introduced the notion of intuitionistic fuzzy sets. The theory of vague sets was first proposed by Gau and Buehrer [7]. Vague set theory is actually an extension of fuzzy set theory and vague sets are regarded as a special case of context-depending the fuzzy sets. In this paper we introduce the notion of vague generalized alpha continuous mappings and vague generalized alpha irresolute mappings and some of their properties in vague topological spaces.

2. Preliminaries

Definition 2.1 [4]

A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and
(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the “evidence for x”, $f_A(x)$ is a lower bound on the negation of x derived from the “evidence for x”, and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0, 1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$.The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2 [7]

Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$ Then

(a) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all $x \in X$
(b) A=B if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \max(t_A(x), t_B(x)), \max(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A\rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$.

Result 2.3 [10]

Every CS, GCS, RCS, αCS is an GαCS but the converses are not true in general.

Definition 2.4

A mapping $f : X \to Y$ is called

(i) a semi- continuous [9], if the inverse image of each open set in Y is semi- open in X.
(ii) a precontinuous [11], if the inverse image of each open set in Y is preopen in X.
(iii) α-continuous [12], if the inverse image of each open set in Y is α-open in X.

Definition 2.5

A mapping $f : X \to Y$ is called

(i) a g-continuous [3] if $f^{-1}(B)$ is a g-closed set of (X, τ) for every closed set B of (Y, σ)

(ii) a gp-continuous [1] if $f^{-1}(B)$ is a gp-closed set of(X, τ) for every closed set B of (Y, σ)

(iii) a gs-continuous [6] if $f^{-1}(B)$ is a gs-closed set of (X, τ) for every closed set B of (Y, σ)

(iv) a αg-continuous [8] if $f^{-1}(B)$ is a αg-closed set of (X, τ) for every closed set B of (Y, σ)

(v) a gα-continuous [10] if $f^{-1}(B)$ is a gα-closed set of (X, τ) for every closed set B of (Y, σ)

Definition 2.6 [3]

Let f be a mapping from (X,τ) into (Y,σ). Then f is said to be vague generalized irresolute $f^{-1}(B) \in GCS(X)$ for every GCS B in Y.

3. Vague continuous mapping

Definition 3.1

A mapping $f : (X,\tau) \to (Y,\sigma)$ is called

(i) vague continuous (V continuous in short) if $f^{-1}(B) \in VOS(X)$ for every $B \in \sigma$.

(ii) vague semi- continuous(VS continuous in short) if $f^{-1}(B) \in VSOS(X)$ for every $B \in \sigma$.

(iii) vague regular- continuous(VR continuous in short) if $f^{-1}(B) \in VROS(X)$ for every $B \in \sigma$.

(iv) vague precontinuous(VP continuous in short) if $f^{-1}(B) \in VPOS(X)$ for every $B \in \sigma$.

(v) vague irresolute (V irresolute in short) if $f^{-1}(B) \in VCS(X)$ for every VCS $B \in \sigma$.

Definition 3.2

A mapping $f : (X,\tau) \to (Y,\sigma)$ is called

(i) vague generalized continuous if $f^{-1}(B)$ is a VGCS in (X,τ) for every VCS B in (Y,σ).

(ii) vague generalized semi continuous if $f^{-1}(B)$ is a VGSCS in (X,τ) for every VCS B in (Y,σ).

(iii) vague generalized Pre continuous if $f^{-1}(B)$ is a VGPCS in (X,τ) for every VCS B in (Y,σ).

(iv) vague generalized irresolute if $f^{-1}(B)$ is a VGCS in (X,τ) for every VGCS B in (Y,σ).

4. Vague generalized alpha continuous mapping

Definition 4.1

A mapping $f : (X,\tau) \to (Y,\sigma)$ is called

(i) a vague alpha continuous (Vα continuous in short) mapping if $f^{-1}(B)$ is a VαCS(X) in (X,τ) for every VCS B of (Y,σ).

(ii) a vague alpha generalized continuous (VαG continuous in short) mapping if $f^{-1}(B)$ is a VαGCS(X) in (X,τ) for every VCS B of (Y,σ).

(iii) a vague generalized alpha continuous (VGα continuous in short) mapping if $f^{-1}(B)$ is a VGαCS(X) in (X,τ) for every VCS B of (Y,σ).

Example 4.2

Let us consider X={a,b}, Y={u,v} and $G_1 = \{\langle x, [0.2,0.7], [0.4,0.6]\rangle\}, G_2 = \{\langle x, [0.1,0.5], [0.3,0.5]\rangle\}$. Then $\tau = \{0, G_1, X\}$ and $\sigma = \{0, G_2, Y\}$ are VTS on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a)=u and f(b)=v. Then f is a VGα continuous mapping.

Theorem 4.3

For any vague continuous function $f : (X,\tau) \to (Y,\sigma)$ we have the following,

(i) Every vague continuous mapping is a VGα continuous mapping,

(ii) Every Vα continuous mapping is a VGα continuous mapping,

(iii) Every VR continuous mapping is a VGα continuous mapping,
(iv) Every VGα continuous mapping is a VαG continuous mapping,
(v) Every VGα continuous mapping is a VGP continuous mapping,
(vi) Every VGα continuous mapping is a VGS continuous mapping, but the converse need not be true.

Proof

(i) Let $f:(X,\tau)\to(Y,\sigma)$ be a vague continuous mapping. Let A be a VCS in Y. Since f is vague continuous mapping, $f^{-1}(A)$ is a VCS in X. Since every VCS is a VGαCS, $f^{-1}(A)$ is a VGαCS in X. Hence f is a VGα continuous mapping.
(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be a Vα continuous mapping. Let A be a VCS in Y. Since f is Vα continuous mapping, $f^{-1}(A)$ is a VαCS in X. Since every VαCS is a VGαCS, $f^{-1}(A)$ is a VGαCS in X. Hence f is a VGα continuous mapping.

The proof of (iii), (iv), (v) and (vi) are obvious
The converse of the above theorem need not be true is given by the following examples:

Example 4.4
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.2,0.7],[0.4,0.6]\rangle\}, G_2=\{\langle x,[0.1,0.5],[0.3,0.5]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Since $G_2{}^c=\{\langle y,[0.5,0.9],[0.5,0.7]\rangle\}$ is an VCS in Y, but $f^{-1}(G_2^c)$ is not an VCS in X. Therefore f is an VGα continuous mapping but not vague continuous mapping,

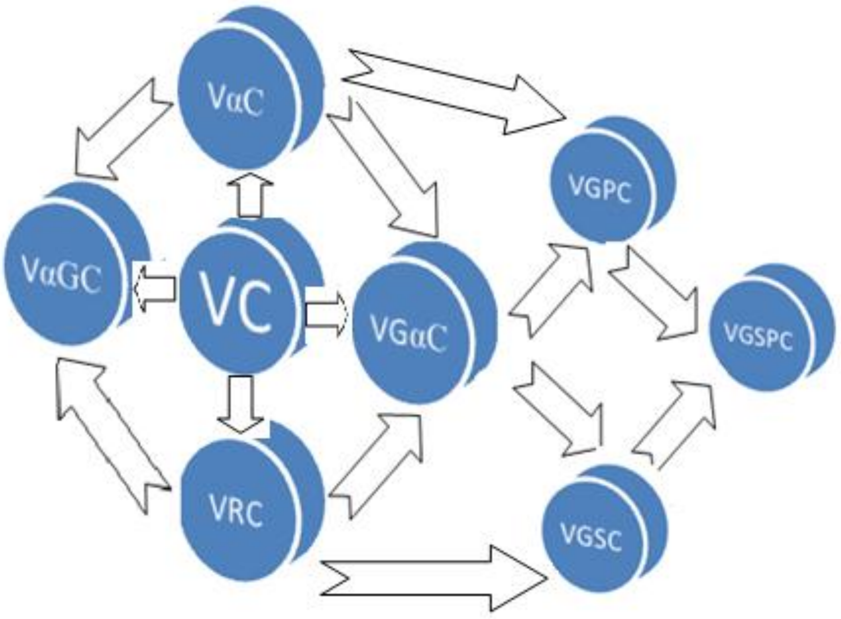
Example 4.5
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.2,0.6],[0.3,0.5]\rangle\}, G_2=\{\langle x,[0.3,0.4],[0.5,0.6]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGα continuous mapping but not Vα continuous mapping

Example: 4.6
Let us consider X={a,b}, Y={u,v} and Let X ={a,b} and $G_1=\{\langle x,[0.1,0.5],[0.3,0.7]\rangle\}, G_2=\{\langle x,[0.4,0.8],[0.2,0.6]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGα continuous mapping but not VR continuous mapping, since $f^{-1}(G_2^c)$ is not an VRCS in X.

Example 4.7
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.3,0.5],[0.2,0.5]\rangle\}, G_2=\{\langle x,[0.3,0.6],[0.3,0.5]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Since $G_2{}^c=\{\langle y,[0.4,0.7],[0.5,0.7]\rangle\}$ is an VCS in Y, but $f^{-1}(G_2^c)$ is not an VGαCS in X.

Example 4.8
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.3,0.6],[0.3,0.6]\rangle\}, G_2=\{\langle x,[0.4,0.7],[0.5,0.7]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGP continuous, Here $V\alpha clf^{-1}(G_2^c)=G_1^c\not\subset G_1$ which shows that f is not an VGα continuous mapping.

Example 4.9
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.2,0.4],[0.2,0.8]\rangle\}, G_2=\{\langle x,[0.6,0.8],[0.3,0.7]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGS continuous, Now we have $f^{-1}(G_2{}^c)=\{\langle x,[0.2,0.4],[0.3,0.7]\rangle\}$ which is VGS continuous but not VGα continuous mapping.
From the above theorems and examples we have the following implications.



Remark 4.10
An VP continuous and VGα continuous mapping is independent of each other

Example 4.11
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.4,0.7],[0.4,0,8]\rangle\}, G_2=\{\langle x,[0.4,0.7],[0.3,0.7]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VP continuous mapping, Now we have $f^{-1}(G_2{}^c)=\{\langle x,[0.3,0.6],[0.3,0.7]\rangle\}$.Here $V\alpha clf^{-1}(G_2^c)=G_1^c\not\subset G_1$,which shows that f is not an VGα continuous mapping.

Example 4.12
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.2,0.6],[0.4,0.8]\rangle\}, G_2=\{\langle x,[0.3,0.7],[0.1,0.5]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGα continuous mapping, Now we have $f^{-1}(G_2{}^c)=\{\langle x,[0.3,0.7],[0.5,0.9]\rangle\}$.Here $V\operatorname{int} f^{-1}(G_2^c)=G_1, Vcl(G_1)=G_1^c$,which shows that $Vcl(V\operatorname{int}(f^{-1}(G_2^c))\not\subset f^{-1}(G_2^c)$. Here f is not an VP continuous mapping.

Remark 4.13
An VS continuous and VGα continuous mapping is independent of each other

Example 4.14
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.2,0.7],[0.3,0.9]\rangle\}, G_2=\{\langle x,[0.3,0.8],[0.4,0.7]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is an VGα continuous mapping, Now we have $f^{-1}(G_2{}^c)=\{\langle x,[0.2,0.7],[0.3,0.6]\rangle\}$.Here $Vclf^{-1}(G_2^c)=1, V\operatorname{int}(Vcl(f^{-1}(G_2^c))=1\not\subset f^{-1}(G_2^c)$,which shows that f is not an VS continuous mapping

Example 4.15
Let us consider X={a,b}, Y={u,v} and $G_1=\{\langle x,[0.4,0.6],[0.3,0.7]\rangle\}, G_2=\{\langle x,[0.4,0.7],[0.4,0.8]\rangle\}$. Then $\tau=\{0,G_1,X\}$ and $\sigma=\{0,G_2,Y\}$ are VTS on X and Y respectively. Define a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=u and f(b)=v. Then f is a VS continuous mapping, Now we have $f^{-1}(G_2{}^c)=\{\langle x,[0.3,0.7],[0.2,0.6]\rangle\}$.Here

$Vclf^{-1}(G_2^c) = G_1^c, V\operatorname{int}(G_1^c) = G_1, Vcl(G_1) = G_1^c \quad V\alpha cl(f^{-1}(G_2^c) = G_1^c \not\subset G_1$ which shows that f is not an VGα continuous mapping.

Definition 4.16

An VTS (X, τ) is said to be a vague $\alpha_k T_{1/2}$ space if every VGαCS in X is a VCS in X.

Theorem 4.17

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following are equivalent if X is a $V\alpha_k T_{1/2}$ space.

(i) f is a VGα continuous mapping.

(ii) $f^{-1}(B)$ is a VGαCS in X for every VCS B in Y.

(iii) $Vcl(V\operatorname{int}(Vcl(f^{-1}(B))))\subseteq f^{-1}(Vcl(B))$ for every VS B in Y.

Proof:

(i) $\Rightarrow$(ii): is obviously true.

(ii) $\Rightarrow$(iii): Let B be a VS in Y. Then Vcl(B) is a VCS in Y. By hypothesis $f^{-1}(Vcl(B))$ is a VGαCS in X. Since X is a $V\alpha_k T_{1/2}$ space, $f^{-1}(Vcl(B))$ is a VCS in X. Therefore $Vcl(f^{-1}(Vcl(B)))= f^{-1}(Vcl(B))$. Now we have $Vcl(V\operatorname{int}(Vcl(f^{-1}(B))))\subseteq Vcl(V\operatorname{int}(Vcl(f^{-1}(Vcl(B)))))\subseteq f^{-1}(Vcl(B))$.

(iii) $\Rightarrow$(i): Let B be a VCS in Y. By hypothesis $Vcl(V\operatorname{int}(Vcl(f^{-1}(B))))\subseteq f^{-1}(Vcl(B))= f^{-1}(B)$. This implies $f^{-1}(B)$ is a VαCS in X and hence $f^{-1}(B)$ is a VGαCS in X. Therefore f is a VGα continuous mapping.

Theorem 4.18

A mapping $f:(X,\tau)\to(Y,\sigma)$ is VGα continuous mapping if and only if the inverse image of each VOS in Y is a VGαOS in X.

Proof

Necessary: Let A be a VOS in Y. This implies A^c is a VCS in Y. Since f is a VGαCS in X. $f^{-1}(A^c)=(f^{-1}(A))^c$, $f^{-1}(A)$ is a VGαOS in X.

Sufficent: It follows from the definition.

Theorem 4.19

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping and let $f^{-1}(A)$ is a VRCS in X. for every VCS A in Y. Then f is a VGα continuous mapping.

Proof: Let A be a VCS in Y. Then $f^{-1}(A)$ is a VRCS in X. Since every VRCS is a VGαCS, $f^{-1}(A)$ is a VGαCS in X. Hence f is a VGα continuous mapping.

Theorem 4.20

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following are equivalent if X is a $V\alpha_k T_{1/2}$ space.

(i) f is a VGα continuous mapping.

(ii) $f^{-1}(A)$ is a VGαOS in X for every VOS A in Y.

(iii) $f^{-1}(V\operatorname{int}(A))\subseteq V\operatorname{int}(Vcl(V\operatorname{int}(f^{-1}(A))))$ for every VS A in Y.

Proof

(i) $\Rightarrow$(ii): is obviously true.

(ii) $\Rightarrow$(iii): Let A be a VS in Y. Then Vint(A) is a VOS in Y. By hypothesis $f^{-1}(V\operatorname{int}(A))$ is a VGαCS in X. Since X is a $V\alpha_k T_{1/2}$ space, $f^{-1}(V\operatorname{int}(A))$ is a VOS in X. Therefore $f^{-1}(V\operatorname{int}(A))=V\operatorname{int}(f^{-1}(V\operatorname{int}(A)))\subseteq V\operatorname{int}(Vcl(V\operatorname{int}(f^{-1}(A))))$

(iii) $\Rightarrow$(i): Let A be a VCS in Y. Then its complement A^c is a VOS in Y. By hypothesis $f^{-1}(V\operatorname{int}(A^c))\subseteq V\operatorname{int}(Vcl(V\operatorname{int}(f^{-1}(A^c))))$. Hence $f^{-1}(A^c)$ is a VGαOS in X. Since every VαOS is a VGαOS, $f^{-1}(A^c)$ is a VGαOS in X. Hence f is a VGα continuous mapping.

Theorem 4.21
Let $f:(X,\tau)\to(Y,\sigma)$ be a VGα continuous mapping, then f is a vague continuous mapping if X is a $V\alpha_k T_{1/2}$ space.

Proof

Let A be a VCS in Y. Then $f^{-1}(A)$ is a VGαCS in X, by hypothesis. Since X is a $V\alpha_k T_{1/2}$ space, $f^{-1}(A)$ is a VCS in X. Hence f is a vague continuous mapping.

Theorem 4.22
Let $f:(X,\tau)\to(Y,\sigma)$ be a VGα continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ is vague continuous mapping then $g\circ f:(X,\tau)\to(Z,\delta)$ is VGα continuous mapping.

Proof

Let A be a VCS in Z. Then $g^{-1}(A)$ is a VCS in Y, by hypothesis. Since f is a VGα continuous mapping, $f^{-1}(g^{-1}(A))$ is a VGαCS in X. Since X is a $V\alpha_k T_{1/2}$ space, $(f^{-1}(g^{-1}(A)))^{-1}$ is a VCS in X. Since we know that $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$ and hence $g\circ f$ is a VGα continuous mapping.

Remark 4.23
The composition of two VGα- continuous mapping may not be VGα- continuous.

Example 4.24
Let X={a,b}, Y={x,y} and Z={p,q} and vague sets U,V and W defined as follows: $U=\{\langle x,[0.1,0.7],[0.3,0.8]\rangle\}$, $V=\{\langle x,[0.3,0.7],[0.3,0.8]\rangle\}$, $W=\{\langle x,[0.4,0.9],[0.4,0.8]\rangle\}$. Let $\tau=\{0,U,X\}$, $\sigma=\{0,V,Y\}$ and $\mu=\{0,W,Z\}$ be Vague topologies on X, Y and Z respectively. Let the mapping $f:(X,\tau)\to(Y,\sigma)$ defined by f(a)=x and f(b)=y and $g:(Y,\sigma)\to(Z,\mu)$ defined by g(x)=p and g(y)=q. Then the mappings f and g are VGα continuous mappings but the mapping $g\circ f:(X,\tau)\to(Z,\mu)$ is not VGα- continuous mapping.

Definition 4.25
Let (X,τ) be a VTS. The generalized alpha closure (Vgαcl(A) in short) for any A is defined as follows.
Vgαcl(A)=$\bigcap$ {K/K is a VGαCS in X and $A\subseteq K$ }. If A is a VGαCS, then Vgαcl(A)=A.

Remark 4.26
It is clear that $A\subseteq Vg\alpha cl(A)\subseteq Vcl(A)$.

Theorem 4.27
Let $f:(X,\tau)\to(Y,\sigma)$ be a VGαContinuous mapping. Then the following ststements hold.

(i) $f(Vg\alpha cl(A))\subseteq Vcl(f(A))$ for every VS A in X.
(ii) $Vg\alpha cl(f^{-1}(B))\subseteq f^{-1}(Vcl(B))$ for every VS B in X.

Proof

(i) Let $A\subseteq X$. Then Vcl(f(A)) is VCS in Y. Since f is a VGα continuous mapping, $f^{-1}(Vcl(f(A)))$ is a VGαCS in X. (That is $Vgcl(A)\subseteq f^{-1}(Vcl(f(A)))$. Since $A\subseteq f^{-1}(f(A))\subseteq f^{-1}(Vcl(f(A)))$ and $f^{-1}(Vcl(f(A)))$ is a VGα-closed, implies $Vg\alpha cl(A)\subseteq f^{-1}(Vcl(f(A)))$. Hence $f(Vg\alpha cl(A))\subseteq Vcl(f(A))$.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(Vg\alpha cl(f^{-1}(B)))\subseteq Vcl(f(f^{-1}(B)))\subseteq Vcl(B)$. Hence $Vg\alpha cl(f^{-1}(B))\subseteq f^{-1}(Vcl(B))$, for every Vs B in Y.

Definition 4.28
A VTS (X,τ) is said to be a vague $\alpha_l T_{1/2}$ space if every VGαCS in X is a VαCS in X.

Theorem 4.29

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. If X is a $V\alpha_l T_{1/2}$ space then f is VGα- continuous mapping if and only if it is VGα continuous mapping.

Proof

Let f be a VGα- continuous mapping and let A be a VCS in Y. Then by definition, $f^{-1}(A)$ is a VGαCS in X. Since X is a $V\alpha_l T_{1/2}$ space, $f^{-1}(A)$ is a VαCS in X. Hence f is Vα- continuous mapping.
conversely, assume that f is Vα- continuous mapping by theorem 3.5, f is a VGα- continuous mapping.

5. Vague generalized alpha irresolute mapping:

Definition 5.1

A mapping $f:(X,\tau)\to(Y,\sigma)$ is called

(i) a vague generalized alpha irresolute (VGα irresolute in short) mapping if $f^{-1}(B)$ is a VαGCS in (X,τ) for every VαGCS B of (Y,σ).

(ii) a vague generalized alpha irresolute (VGα irresolute in short) mapping if $f^{-1}(B)$ is a VGαCS in (X,τ) for every VGαCS B of (Y,σ).

Theorem 5.2

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then every VGα- irresolute mapping is a VGα continuous mapping.

Proof

Let A be a VCS in Y. We know that every VCS is a VGαCS. Therefore A is a VGαCS in Y. Since f is a VGα irresolute mapping, by definition $f^{-1}(A)$ is VGαCS in X. Hence f is a VGα continuous mapping.

Theorem 5.3

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following are equivalent.

(i) f is a VGα irresolute mapping.

(ii) $f^{-1}(B)$ is a VGαOS in X for every VGαOS B in Y.

(iii) $Vg\alpha cl\left(f^{-1}(B)\right)\subseteq f^{-1}(Vg\alpha cl(B))$, for every VS B in Y.

(iv) $f^{-1}(Vg\alpha\operatorname{int}(B))\subseteq Vg\alpha\operatorname{int}\left(f^{-1}(B)\right)$, for every VS B in Y.

Proof

(i) $\Rightarrow$(ii): it can be proved by taking the complement of definition 4.1.

(ii) $\Rightarrow$(iii): Let B be any VS in Y. Then $B\subseteq Vcl(B)$ also $f^{-1}(B)\subseteq f^{-1}(Vg\alpha cl(B))$. Since $Vg\alpha cl(B)$ is a VGαCS in Y, $f^{-1}(Vg\alpha cl(B))$ is a VGαCS in X. Therefore $Vg\alpha cl(f^{-1}(B))\subseteq f^{-1}(Vg\alpha cl(B))$.

(iii) $\Rightarrow$(iv): Let B be any VS in Y. Then Vint(B) is a VOS in Y. Then $f^{-1}(V\operatorname{int}(B))$ is a VGαOS in X. Since $(Vg\alpha\operatorname{int}(B))$ is a VGαOS in X, $f^{-1}(Vg\alpha\operatorname{int}(B))$ is a VGαOS in X. Therefore $(Vg\alpha cl(B))$ is a VGαCS in Y, $f^{-1}(Vg\alpha cl(B))$ is a VGαCS in X. Therefore $Vg\alpha cl(f^{-1}(B))\subseteq f^{-1}(Vg\alpha cl(B))$.

(iv) $\Rightarrow$(i): Let B be any VGαOS in Y. Then $(Vg\alpha\operatorname{int}(B))=B$. By our assumption we have $f^{-1}(B)=f^{-1}(Vg\alpha\operatorname{int}(B))\subseteq Vg\alpha\operatorname{int} f^{-1}(B)$, so $f^{-1}(B)$ is a VGαOS in X. Hence f is a VGα irresolute mapping.

Theorem 5.4

Let $f:(X,\tau)\to(Y,\sigma)$ be a VGα irresolute mapping, then f is a vague irresolute mapping if X is a $V\alpha_k T_{1/2}$ space.

Proof

Let A be a VCS in Y. Then A is a VGαCS in Y.Therefore $f^{-1}(A)$ is a VGαCS in X, by hypothesis. Since X is a $V\alpha_k T_{1/2}$ space, $f^{-1}(A)$ is a VCS in X. Hence f is a vague irresolute mapping.

Theorem 5.5

Let $f:(X,\tau)\to(Y,\sigma)$ be a VGα- irresolute mapping, then f is a VGα irresolute mapping if X is a Vα_lT$_{1/2}$ space.

Proof

Let B be a VαCS in Y. Then B is a VGαCS in Y. Since f is a VGα- irresolute, $f^{-1}(B)$ is a VGαCS in X, by hypothesis. Since X is a Vα_lT$_{1/2}$ space, $f^{-1}(B)$ is a VαCS in X. Hence f is a Vα- irresolute mapping.

Theorem 5.6

Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ is vague irresolute mapping, where X, Y, Z are VTS. then $g\circ f$ is VGα irresolute mapping.

Proof: Let A be a VGαCS in Z. Then g is a VGα irresolute mapping $g^{-1}(A)$ is a VGαCS in Y, Also since f is an VGα irresolute mapping, $f^{-1}(g^{-1}(A))$ is a VGαCS in X. $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$ for each A in Z. Hence $(g\circ f)^{-1}(A)$ is a VGαCS in X. Therefore $g\circ f$ is a VGα irresolute mapping.

Theorem 5.7

Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ is vague irresolute mapping and vague continuous mapping respectively, where X, Y, Z are VTS. then $g\circ f$ is VGα continuous mapping.

Proof

Let A be a VCS in Z. Then g is a vague continuous mapping, $g^{-1}(A)$ is a VGαCS in Y, Also since f is an VGα irresolute mapping, $f^{-1}(g^{-1}(A))$ is a VGαCS in X.Since $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$ is a VGαCS in X for each A in Z. Hence $(g\circ f)^{-1}(A)$ is a VGαCS in X. Therefore $g\circ f$ is a VGα irresolute mapping.

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