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Vague generalized pre continuous mappings

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Abstract
The aim of this paper is to introduce and investigate a new class of continuous mapping in vague topological spaces namely vague generalized pre continuous mapping, vague generalized pre irresolute mapping and also studied some of its properties.

Keywords: Vague topology, vague generalized pre continuous mappings, vague generalized pre irresolute mappings

1. Introduction

In 1970, Levine [11] initiated the study of generalized closed sets. The concept of fuzzy sets was introduced by Zadeh [21] in 1965. The theory of fuzzy topology was introduced by C.L.Chang [7] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [3] as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre [9] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions defined by [6, 9]. In this paper we introduce the concept of vague generalized pre continuous mapping and vague generalized pre irresolute mappings and we also obtain their properties and relations with counter examples.

2. Preliminaries

2.1 Definition [5]

A vague set A in the universe of discourse X is characterized by two membership functions given by:

i) A true membership function $t_A : X \to [0,1]$ and

ii) A false membership function $f_A : X \to [0,1]$.

where $t_A(x)$ islower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x" and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \le \mu(x) \le 1 - f_A(x)$. The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the "vague value" of x in A and is denoted by $V_A(x)$.

2.2 Definition [5]

Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$. Then

a) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all $x \in X$.

b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

c) $A^c = \{\langle x, [f_A(x), 1 - t_A(x)]\rangle / x \in X\}$.

d) $A \cap B = \{\langle x, [(t_A(x) \wedge t_B(x)), ((1 - f_A(x)) \wedge (1 - f_B(x)))]\rangle / x \in X\}$.

e) $A \cup B = \{\langle x, [(t_A(x) \vee t_B(x)), ((1 - f_A(x)) \vee (1 - f_B(x)))]\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notion $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle\}$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$.

2.3 Definition

Let (X, τ) be a topological space. A subset *A* of *X* is called:

i) *semi closed set* (SCS in short)[12] if $\operatorname{int}(cl(A)) \subseteq A$.

ii) *pre- closed set* (PCS in short)[16] if $cl(\text{int}(A)) \subseteq A$.
iii) α *-closed set* (α CS in short)[18] if $cl(\text{int}(cl(A))) \subseteq A$.
iv) *regular closed set* (RCS in short)[20] if $A = cl(\text{int}(A))$.

2.4 Definition
Let (X, τ) be a topological space. A subset *A* of *X* is called:
i) *generalized closed* (briefly, *g-closed*) [11] if $cl(A) \subseteq U$, whenever *A* ⊆ *U* and *U* is open in X .
ii) *generalized semi closed* (briefly, *gs-closed*) [2] if $scl(A) \subseteq U$ whenever *A* ⊆ *U* and *U* is open in X .
iii) *α-generalized closed* (briefly, *αg-closed*) [13] if $\alpha\, cl(A) \subseteq U$ whenever *A* ⊆ *U* and *U* is open in X .
iv) *generalized pre closed* (briefly, *gp-closed*) [14] if $pcl(A) \subseteq U$ whenever *A* ⊆ *U* and *U* is open in X .

2.5 Definition
Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be

- *semi-continuous* [12] if $f^{-1}(V)$ is semi-closed set in (X, τ) for every closed set V of (Y, σ).
- *pre-continuous* [16] if $f^{-1}(V)$ is pre-closed set in (X, τ) for every closed set V of (Y, σ).
- α *-continuous* [17] if $f^{-1}(V)$ is α -closed set in (X, τ) for every closed set V of (Y, σ).
- *generalized continuous* [4] if $f^{-1}(V)$ is generalized closed set in (X, τ) for every closed set V of (Y, σ).
- *generalized semi-continuous* [8] if $f^{-1}(V)$ is generalized semi-closed set in (X, τ) for every closed set V of (Y, σ).
- *generalized pre-continuous* [19] if $f^{-1}(V)$ is generalized pre-closed set in (X, τ) for every closed set V of (Y, σ).
- α *-generalized continuous* [10] if $f^{-1}(V)$ is α -generalized closed set in (X, τ) for every closed set V of (Y, σ).
- *generalized pre-irresolute* [1] if $f^{-1}(V)$ is generalized pre-closed set in (X, τ) for every generalized pre-closed set V of (Y, σ).

2.6 Definition [15]
A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

- 0, 1 $\in \tau$
- $G_1 \cap G_2 \in \tau, \text{for any } G_1, G_2 \in \tau$
- $\cup G_i \in \tau \text{ for any family } \{G_i / i \in J\} \subseteq \tau.$

In this case the pair (X, τ) is called a vague topological space (VTS in short) and any VS in τ is known as a vague open set (VOS in short) in X .
The complement A^c of a VOS in a VTS (X, τ) is called a vague closed set (VCS in short) in X .

2.7 Definition [15]
A VTS (X, τ) is said to be a vague $_p T_{1/2}$ space (V $_p T_{1/2}$ in short) if every VGPCS in X is a VCS in X .

2.8 Definition [15]
A VTS (X, τ) is said to be a vague $_{gp} T_{1/2}$ space (V $_{gp} T_{1/2}$ in short) if every VGPCS in X is a VPCS in X .

3. Vague continuous mapping
3.1 Definition
Let (X, τ) and (Y, σ) be any two vague topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be

- *vague continuous* (V continuous in short) if $f^{-1}(V)$ is vague closed set in (X, τ) for every vague closed set V of (Y, σ).
- *vague semi-continuous* (VS continuous in short) if $f^{-1}(V)$ is vague semi-closed set in (X, τ) for every vague closed set V of (Y, σ).

- *vague pre-continuous* (VP continuous in short) if $f^{-1}(V)$ is vague pre-closed set in (X,τ) for every vague closed set V of (Y,σ).
- *vague α-continuous* (Vα-continuous in short) if $f^{-1}(V)$ is vague α-closed set in (X,τ) for every vague closed set V of (Y,σ).
- *vague regular continuous* (VP continuous in short) if $f^{-1}(V)$ is vague regular closed set in (X,τ) for every vague closed set V of (Y,σ).
- *vague generalized continuous* (VG continuous in short) if $f^{-1}(V)$ is vague generalized closed set in (X,τ) for every vague closed set V of (Y,σ).
- *vague generalized semi-continuous* (VGS continuous in short) if $f^{-1}(V)$ is vague generalized semi-closed set in (X,τ) for every vague closed set V of (Y,σ).
- *vague α-generalized continuous* (VαG continuous in short)if $f^{-1}(V)$ is vague α-generalized closed set in (X,τ) for every vague closed set V of (Y,σ).

4. Vague generalized pre-continuous mappings

4.1 Definition

A map $f:(X,\tau)\to(Y,\sigma)$ is said to be *vague generalized pre-continuous* (VGP continuous in short) mapping if $f^{-1}(A)$ is a VGPCS in (X,τ) for every vague closed set A of (Y,σ).

4.2 Example

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.3,0.4],[0.4,0.6]\rangle\}$, $G_2=\{\langle x,[0.2,0.4],[0.3,0.5]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VGP continuous mapping.

4.3 Theorem

Let (X,τ) and (Y,σ) be any two vague topological spaces. For any vague continuous function $f:(X,\tau)\to(Y,\sigma)$ we have the following:

i) Every V continuous mapping is a VG continuous mapping.
ii) Every V continuous mapping is a VP continuous mapping.
iii) Every V continuous mapping is a Vα continuous mapping.
iv) Every Vα continuous mapping is a VS continuous mapping.
v) Every Vα continuous mapping is a VP continuous mapping.
vi) Every VR continuous mapping is a V continuous mapping.
vii) Every V continuous mapping is a VGP continuous mapping.
viii) Every VG continuous mapping is a VGP continuous mapping.
ix) Every Vα-continuous mapping is a VGP continuous mapping.
x) Every VR continuous mapping is a VGP continuous mapping.
xi) Every VP continuous mapping is a VGP continuous mapping.
xii) Every VαG continuous mapping is a VGP continuous mapping.

Proof

(i) Let $f:(X,\tau)\to(Y,\sigma)$ be a V continuous mapping. Let A be a VCS in Y. Then $f^{-1}(A)$ is a VCS in X. Since every VCS is a VGCS, $f^{-1}(A)$ is a VGCS in X. Hence f is a VG continuous mapping.

(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be a V continuous mapping. Let A be a VCS in Y. Then $f^{-1}(A)$ is a VCS in X. Since every VCS is a VPCS, $f^{-1}(A)$ is a VPCS in X. Hence f is a VP continuous mapping.

(iii) Let $f:(X,\tau)\to(Y,\sigma)$ be a Vα continuous mapping. Let A be a VCS in Y. Then $f^{-1}(A)$ is a Vα CS in X. Since every VCS is a Vα CS, $f^{-1}(A)$ is a Vα CS in X. Hence f is a Vα continuous mapping.

The proof of (iv) to (xii) are similar.

4.4 Remark
However the converse of the Theorem 4.3 need not be true.

Counter Example for Theorem 4.3
4.5 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.4,0.8],[0.3,0.6]\rangle\}$, $G_2=\{\langle y,[0.5,0.9],[0.4,0.6]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VG continuous mapping but not V continuous mapping, since $G_2^c=\{\langle y,[0.1,0.5],[0.4,0.6]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not VCS in X.

4.6 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.2,0.7],[0.3,0.4]\rangle\}$, $G_2=\{\langle y,[0.4,0.5],[0.3,0.6]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VP continuous mapping but not V continuous mapping, since $G_2^c=\{\langle y,[0.5,0.6],[0.4,0.7]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not VCS in X.

4.7 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$, $G_2=\{\langle y,[0.7,0.8],[0.6,0.8]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a Vα continuous mapping but not V continuous mapping, since $G_2^c=\{\langle y,[0.2,0.3],[0.2,0.4]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not VCS in X.

4.8 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.3,0.4],[0.3,0.6]\rangle\}$, $G_2=\{\langle y,[0.5,0.6],[0.4,0.6]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VS continuous mapping but not Vα continuous mapping, since $G_2^c=\{\langle y,[0.4,0.5],[0.4,0.6]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a Vα CS in X.

4.9 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.4,0.6],[0.3,0.6]\rangle\}$, $G_2=\{\langle y,[0.3,0.4],[0.2,0.5]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VP continuous mapping but not Vα continuous mapping, since $G_2^c=\{\langle y,[0.6,0.7],[0.5,0.8]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a Vα CS in X.

4.10 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.6,0.8],[0.5,0.8]\rangle\}$, $G_2=\{\langle y,[0.6,0.8],[0.5,0.8]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a V continuous mapping but not VR continuous mapping, since $G_2^c=\{\langle y,[0.2,0.4],[0.2,0.5]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a VRCS in X.

4.11 Example
Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.6,0.7],[0.5,0.8]\rangle\}$, $G_2=\{\langle y,[0.3,0.5],[0.4,0.7]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$.

Then f is a VGP continuous mapping but not V continuous mapping, since $G_2^c = \{\langle y, [0.5,0.7], [0.3,0.6] \rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a VCS in X.

4.12 Example

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x, [0.3,0.6], [0.2,0.7] \rangle\}$, $G_2 = \{\langle y, [0.5,0.7], [0.4,0.8] \rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGP continuous mapping but not VG continuous mapping, since $G_2^c = \{\langle y, [0.3,0.5], [0.2,0.6] \rangle\}$ is a VCS in Y, but $v\alpha cl(f^{-1}(G_2^c)) \not\subset G_1$ in X.

4.13 Example

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x, [0.3,0.5], [0.4,0.8] \rangle\}$, $G_2 = \{\langle y, [0.6,0.7], [0.5,0.7] \rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGP continuous mapping but not Vα continuous mapping, since $G_2^c = \{\langle y, [0.3,0.4], [0.3,0.5] \rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a Vα CS in X.

4.14 Example

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x, [0.2,0.3], [0.4,0.5] \rangle\}$, $G_2 = \{\langle y, [0.1,0.3], [0.2,0.4] \rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGP continuous mapping but not VR continuous mapping, since $G_2^c = \{\langle y, [0.7,0.9], [0.6,0.8] \rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a VRCS in X.

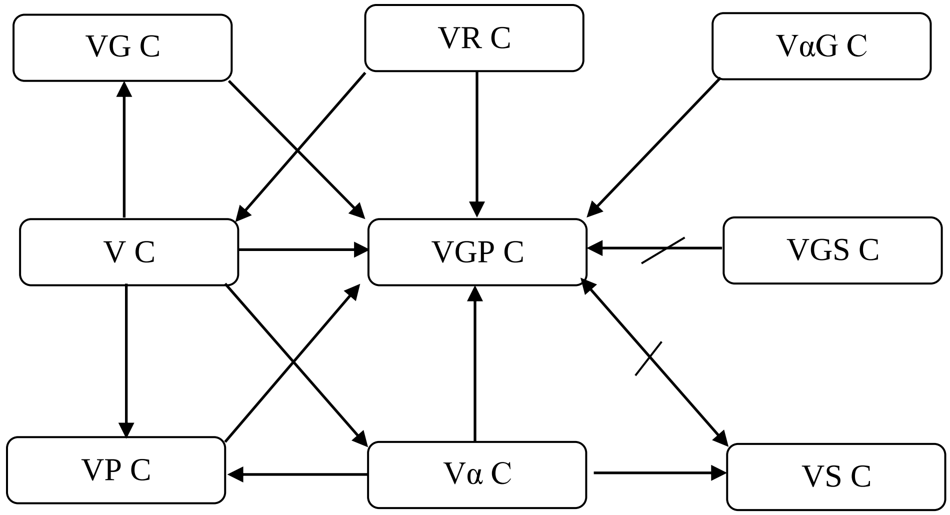
4.15 Example

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x, [0.5,0.7], [0.6,0.8] \rangle\}$, $G_2 = \{\langle y, [0.2,0.4], [0.1,0.4] \rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGP continuous mapping but not VP continuous mapping, since $G_2^c = \{\langle y, [0.6,0.8], [0.6,0.9] \rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a VPCS in X.

4.16 Example

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x, [0.7,0.8], [0.5,0.7] \rangle\}$, $G_2 = \{\langle y, [0.3,0.4], [0.4,0.5] \rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGP continuous mapping but not VαG continuous mapping, since $G_2^c = \{\langle y, [0.6,0.7], [0.5,0.6] \rangle\}$ is a VCS in Y, but $v\alpha cl(f^{-1}(G_2^c)) \not\subset G_1$ in X.

The relations between various types of vague continuity are given in the following diagram.



In this diagram by "A → B" we mean A implies B but not conversely and "A ↮ B" means A and B are independent of each other. None of them is reversible.

4.17 Proposition
VS continuous mapping and VGP continuous mapping are independent to each other.

4.18 Example
Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{\langle x,[0.2,0.3],[0.2,0.4]\rangle\}$, $G_2 = \{\langle y,[0.7,0.8],[0.6,0.8]\rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VS continuous mapping but not VGP continuous mapping, since $G_2^c = \{\langle y,[0.2,0.3],[0.2,0.4]\rangle\}$ is a VCS in Y, but $vpcl(f^{-1}(G_2^c)) \not\subset G_1$ in X.

4.19 Example
Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{\langle x,[0.5,0.6],[0.4,0.7]\rangle\}$, $G_2 = \{\langle y,[0.3,0.5],[0.5,0.6]\rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VGP continuous mapping but not VS continuous mapping, since $G_2^c = \{\langle y,[0.5,0.7],[0.4,0.5]\rangle\}$ is a VCS in Y, but $f^{-1}(G_2^c)$ is not a VSCS in X.

4.20 Proposition
VGS continuous mapping and VGP continuous mapping are independent to each other.

4.21 Example
Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.6],[0.4,0.5]\rangle\}$, $G_2 = \{\langle y,[0.4,0.7],[0.5,0.6]\rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VGS continuous mapping but not VGP continuous mapping, since $G_2^c = \{\langle y,[0.3,0.6],[0.4,0.5]\rangle\}$ is a VCS in Y, but $vpcl(f^{-1}(G_2^c)) \not\subset G_1$ in X.

4.22 Example
Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.4],[0.3,0.6]\rangle\}$, $G_2 = \{\langle y,[0.7,0.8],[0.5,0.7]\rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by

$f(a)=u$ and $f(b)=v$. Then f is a VGP continuous mapping but not VGS continuous mapping, since $G_2^c=\{\langle y,[0.2,0.3],[0.3,0.5]\rangle\}$ is a VCS in Y, but $vscl(f^{-1}(G_2^c))\not\subset G_1$ in X.

4.23 Theorem
A mapping $f:(X,\tau)\to(Y,\sigma)$ is a VGP continuous mapping if and only if the inverse image of each VOS in Y is a VGPOS in X.

Proof: Necessity
Let A be a VOS in Y. This implies A^c is a VCS in Y. Since f is a VGP continuous mapping, $f^{-1}(A^c)$ is a VGPCS in X. Since $f^{-1}(A^c)=(f^{-1}(A))^c$, $f^{-1}(A)$ is a VGPOS in X.

Sufficiency
The proof is obvious from the Definition 4.1.

4.24 Theorem
Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping and let $f^{-1}(A)$ is a VRCS in X for every VCS A in Y. Then f is a VGP continuous mapping but not conversely.

Proof
Let A be a VCS in Y. Then $f^{-1}(A)$ is a VRCS in X. Since every VRCS is a VGPCS, $f^{-1}(A)$ is a VGPCS in X. Hence f is a VGP continuous mapping.

4.25 Theorem
Let $f:(X,\tau)\to(Y,\sigma)$ is a VGP continuous mapping, then f is a vague continuous mapping if X is a $V_pT_{1/2}$ space.

Proof
Let A be a VCS in Y. Then $f^{-1}(A)$ is a VGPCS in X, by hypothesis. Since X is a $V_pT_{1/2}$ space, $f^{-1}(A)$ is a VCS in X. Hence f is a vague continuous mapping.

4.26 Theorem
Let $f:(X,\tau)\to(Y,\sigma)$ is a VGP continuous mapping, then f is a VP continuous mapping if X is a $V_{gp}T_{1/2}$ space.

Proof
Let A be a VCS in Y. Then $f^{-1}(A)$ is a VGPCS in X, by hypothesis. Since X is a $V_{gp}T_{1/2}$ space, $f^{-1}(A)$ is a VPCS in X. Hence f is a VP continuous mapping.

4.27 Theorem
Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following conditions are equivalent if X is a $V_{gp}T_{1/2}$ space.

i) f is a VGP continuous mapping.
ii) $f^{-1}(B)$ is a VGPCS in X for every VCS B in Y.
iii) $vcl(v\operatorname{int}(f^{-1}(A)))\subseteq f^{-1}(vcl(A))$ for every VS A in Y.

Proof
(i) $\Rightarrow$ **(ii):** is obvious from the Definition 4.1.
(ii) $\Rightarrow$ **(iii):** Let A be a VS in Y. Then $vcl(A)$ is a VCS in Y. By hypothesis, $f^{-1}(vcl(A))$ is a VGPCS in X. Since X is a $V^{gp}T_{1/2}$ space, $f^{-1}(vcl(A))$ is a VPCS. Therefore $vcl(v\operatorname{int}(f^{-1}(vcl(A))))\subseteq f^{-1}(vcl(A))$. Now $vcl(v\operatorname{int}(f^{-1}(A)))\subseteq vcl(v\operatorname{int}(f^{-1}(vcl(A))))\subseteq f^{-1}(vcl(A))$.

(iii) $\Rightarrow$ **(i):** Let A be a VCS in Y. By hypothesis $vcl(v\operatorname{int}(f^{-1}(A))) \subseteq f^{-1}(vcl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is a VPCS in X and hence it is a VGPCS. Thus f is a VGP continuous mapping.

4.28 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following conditions are equivalent if X is a $V_{gp}T_{1/2}$ space.

i) f is a VGP continuous mapping.

ii) $f^{-1}(A)$ is a VGPOS in X for every VOS A in Y.

iii) $f^{-1}(v\operatorname{int}(A)) \subseteq v\operatorname{int}(vcl(f^{-1}(A)))$ for every VS A in Y.

Proof

(i) $\Rightarrow$ **(ii):** is obvious.

(ii) $\Rightarrow$ **(iii):** Let A be a VS in Y. Then $v\operatorname{int}(A)$ is a VOS in Y. By hypothesis, $f^{-1}(v\operatorname{int}(A))$ is a VGPOS in X. Since X is a $V_{gp}T_{1/2}$ space, $f^{-1}(v\operatorname{int}(A))$ is a VPOS in X. Therefore $f^{-1}(v\operatorname{int}(B)) \subseteq v\operatorname{int}(vcl(f^{-1}(v\operatorname{int}(B)))) \subseteq v\operatorname{int}(vcl(f^{-1}(B)))$.

(iii) $\Rightarrow$ **(i):** Let A be VCS in Y. Then its complement, say A^c is a VOS in Y, then $v\operatorname{int}(A^c) = A^c$. Now by hypothesis $f^{-1}(v\operatorname{int}(A^c)) \subseteq v\operatorname{int}(vcl(f^{-1}(A^c)))$. This implies $f^{-1}(A^c) \subseteq v\operatorname{int}(vcl(f^{-1}(A^c)))$. Hence $f^{-1}(A^c)$ is a VPOS in X. Since every VPOS is VGPOS, $f^{-1}(A^c)$ is a VGPOS in X. Thus $f^{-1}(A)$ is a VGPCS in X. Hence f is a VGP continuous mapping.

4.29 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ be a VGP continuous mapping and $g:(Y,\sigma)\to(Z,\mu)$ is a vague continuous mapping, then $g\circ f:(X,\tau)\to(Z,\mu)$ is a VGP continuous mapping.

Proof

Let A be VCS in Z. Then $g^{-1}(A)$ is a VCS in Y, by hypothesis. Since f is a VGP continuous mapping, $f^{-1}(g^{-1}(A))$ is a VGPCS in X. Hence $g\circ f$ is a VGP continuous mapping.

4.30 Remark

The composition of two VGP continuous mapping need not be VGP continuous mapping.

4.31 Example

Let $X=\{a,b\}, Y=\{x,y\}$ and $Z=\{p,q\}$ vague sets G_1, G_2 and G_3 defined as follows: $G_1=\{\langle x,[0.3,0.6],[0.4,0.5]\rangle\}$, $G_2=\{\langle y,[0.6,0.8],[0.5,0.8]\rangle\}$ and $G_3=\{\langle z,[0.4,0.7],[0.5,0.6]\rangle\}$. Let $\tau=\{0,G_1,1\}$, $\sigma=\{0,G_2,1\}$ and $\mu=\{0,G_3,1\}$ be vague topologies on X, Y and Z respectively. Let the mapping $f:(X,\tau)\to(Y,\sigma)$ defined by $f(a)=x$ and $f(b)=y$, $g:(Y,\sigma)\to(Z,\mu)$ defined by $g(x)=p$ and $f(y)=q$. Then the mapping f and g are VGP continuous mapping but the mapping $g\circ f:(X,\tau)\longrightarrow(Z,\mu)$ is not a VGP continuous mapping.

4.32 Definition

Let (X,τ) be a VTS. The vague generalized pre closure ($vgpcl(A)$ in short) for any VS A is defined as follows,

$vgpcl(A)=\cap\{K / K \text{ is a VGPCS in } X \text{ and } A\subseteq K\}$.

If A is VGPCS, then $vgpcl(A)=A$.

4.33 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ be a VGP continuous mapping. Then the following conditions hold.

i) $f(vgpcl(A)) \subseteq vcl(f(A))$, for every VS A in X.

ii) $vgpcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$, for every VS B in Y.

Proof

(i) Since $vcl(f(A))$ is a VCS in Y and f is a VGP continuous mapping, then $f^{-1}(vcl(f(A)))$ is a VGPCS in X. That is $vgpcl(A) \subseteq f^{-1}(vcl(f(A)))$. Therefore $f(vgpcl(A)) \subseteq vcl(f(A))$, for every VS A in X.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(vgpcl(f^{-1}(B))) \subseteq vcl(f(f^{-1}(B))) \subseteq vcl(B)$. Hence $vgpcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$, for every VS B in Y.

5. Vague generalized pre irresolute mapping

5.1 Definition

A map $f:(X,\tau)\to(Y,\sigma)$ is said to be *vague generalized pre irresolute* (VGP irresolute in short) mapping if $f^{-1}(A)$ is a VGPCS in (X,τ) for every VGPCS A in (Y,σ).

5.2 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ be a VGP irresolute mapping, then f is a VGP continuous mapping but not conversely.

Proof

Let f be a VGP irresolute mapping. Let A be any VCS in Y. Since every VCS is a VGPCS, A is a VGPCS in Y. Since f is a VGP irresolute mapping, by definition $f^{-1}(A)$ is a VGPCS in X. Hence f is a VGP continuous mapping.

5.3 Example

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\{\langle x,[0.5,0.6],[0.4,0.7]\rangle\}$, $G_2=\{\langle y,[0.3,0.6],[0.2,0.6]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Then f is a VGP continuous mapping. Let $B=\{\langle y,[0.6,0.7],[0.5,0.8]\rangle\}$ is a VGPCS in Y. But $f^{-1}(B)$ is not a VGPCS in X. Therefore f is not a VGP irresolute mapping.

5.4 Theorem

A mapping $f:(X,\tau)\to(Y,\sigma)$ is a VGP irresolute mapping if and only if the inverse image of each VGPOS in Y is a VGPOS in X.

Proof: Necessity

Let A be a VGPOS in Y. This implies A^c is a VGPCS in Y. Since f is a VGP irresolute mapping, $f^{-1}(A^c)$ is a VGPCS in X. Since $f^{-1}(A^c)=(f^{-1}(A))^c$, $f^{-1}(A)$ is a VGPOS in X.

Sufficiency

The proof is obvious from the Definition 5.1.

5.5 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ is a VGP irresolute mapping, then f is a vague irresolute mapping if X is a $V_pT_{1/2}$ space.

Proof

Let A be a VCS in Y. Then A is a VGPCS in Y. Since f is a VGP irresolute mapping, $f^{-1}(A)$ is a VGPCS in X, by hypothesis. Since X is a $V_pT_{1/2}$ space, $f^{-1}(A)$ is a VCS in X. Hence f is a vague irresolute mapping.

5.6 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ is a VGP irresolute mapping, then f is a VP irresolute mapping if X is a $V_{gp}T_{1/2}$ space.

Proof

Let A be a VPCS in Y. Then A is a VGPCS in Y. Since f is a VGP irresolute mapping, $f^{-1}(A)$ is a VGPCS in X, by hypothesis. Since X is a $V_{gp}T_{1/2}$ space, $f^{-1}(A)$ is a VPCS in X. Hence f is a VP irresolute mapping.

5.7 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ is a VGP irresolute mapping, where X, Y and Z are VTS, then $g\circ f$ is a VGP irresolute mapping.

Proof

Let A be VGPCS in Z. Since g is a VGP irresolute mapping, $g^{-1}(A)$ is a VGPCS in Y. Since f is a VGP irresolute mapping, $f^{-1}(g^{-1}(A))$ is a VGPCS in X. Hence $(g\circ f)^{-1}(A)$ is a VGPCS in X. Therefore $g\circ f$ is a VGP irresolute mapping.

5.8 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ is a VGP irresolute mapping and $g:(Y,\sigma)\to(Z,\mu)$ is a VGP continuous mapping, where X, Y and Z are VTS, then $g\circ f$ is a VGP continuous mapping.

Proof

Let A be VCS in Z. Since g is a VGP continuous mapping, $g^{-1}(A)$ is a VGPCS in Y. Since f is a VGP irresolute mapping, $f^{-1}(g^{-1}(A))$ is a VGPCS in X. Hence $(g\circ f)^{-1}(A)$ is a VGPCS in X. Therefore $g\circ f$ is a VGP continuous mapping.

5.9 Theorem

Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a VTS X into a VTS Y. Then the following conditions are equivalent if X and Y are $V_{gp}T_{1/2}$ space.

i) f is a VGP irresolute mapping.
ii) $f^{-1}(B)$ is a VGPOS in X for each VGPOS B in Y.
iii) $f^{-1}(vp\mathrm{int}(B))\subseteq vp\mathrm{int}(f^{-1}(B))$ for each VS B of Y.
iv) $vpcl(f^{-1}(B))\subseteq f^{-1}(vpcl(B))$ for each VS B of Y.

Proof

(i) $\Rightarrow$ **(ii):** is obviously true.

(ii) $\Rightarrow$ **(iii):** Let B be a VS in Y and $vp\mathrm{int}(B)\subseteq B$. Also $f^{-1}(vp\mathrm{int}(B))\subseteq f^{-1}(B)$. Since $vp\mathrm{int}(B)$ is a VPOS in Y, it is a VGPOS in Y. Therefore $f^{-1}(vp\mathrm{int}(B))$ is a VGPOS in X, by hypothesis. Since X is a $V_{gp}T_{1/2}$ space $f^{-1}(vp\mathrm{int}(B))$ is a VPOS in X. Hence $f^{-1}(vp\mathrm{int}(B)) = vp\mathrm{int}(f^{-1}(vp\mathrm{int}(B)))\subseteq vp\mathrm{int}(f^{-1}(B))$.

(iii) $\Rightarrow$ **(iv):** is obvious by taking complement in (iii).

(iv) $\Rightarrow$ **(i):** Let B be a VGPCS in Y. Since Y is a $V_{gp}T_{1/2}$ space, B is a VPCS in Y and $vpcl(B)=B$. Hence $f^{-1}(B) = f^{-1}(vpcl(B))\supseteq vpcl(f^{-1}(B))$. Therefore $vpcl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is a VPCS and hence it is a VGPCS in X. Thus f is a VGP irresolute mapping.

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