

Effect of multiple stenoses and post dilatation on blood flow through an artery

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Abstract

An attempt has been made to study the fully developed one dimensional non Newtonian flow of blood through an arterial tube having multiple stenoses and post stenotic dilatation. In the present analysis I have considered the steady flow of blood through an axially symmetric but radially non-symmetric multiple stenosed artery. The variation of resistance to flow ratio has been incorporated here for different yield stresses, flux and blood viscosities.

Keywords: Herschel-Bulkley fluid, Yield Stress, Stenosis, Resistance to flow, Flux

1. Introduction

As human system is very complicated in nature, so the knowledge of blood flow through arteries is very much important for better understanding the anatomy and physiology of an organic system. For over centuries it has been noticed that cardiovascular diseases are the major cause of death in the industrialised World. Among the cardiovascular diseases, the familiar ones such as stroke and atherosclerosis are closely related to abnormality, disorder and malfunction of blood flow characteristics in human body. Owing to this, blood flow related problems have shown significant interest for biomedical researchers.

Blood flow characteristics in arteries can be altered significantly by the arterial disease, such as stenosis and aneurysm. Stenosis is most widely spread cardiovascular disease which is caused due to the unnatural growth in the lumen of the artery. Though actual formation of stenosis is unknown to us, it is believed that stenosis is formed by the deposition of various substances like cholesterol or fats/lipids in the inner wall of the artery and unnatural growth of the connective tissue (Young and Tsai ^[1]). Once arterial stenosis present in the lumen of the artery, resistance to flow is increased and as a result blood flow is insufficient to reach every cell and this resists nutrient supplement and so various types of health disorders or various types of illness occur.

In view of this many researchers have considered various types of mathematical models for blood flows through stenosed arteries (Young ^[2], Lee and Fung ^[3], Shukla *et al.*, ^[4], Chaturani and Samy ^[5]). In all these above discussions they have considered blood as a Newtonian fluid. But since blood consists of cells in an aqueous solution, blood behaves like a non-Newtonian fluid under certain conditions (Majhi and Nair ^[6]). It is also realized that Herschel-Bulkley fluid model is better than other type of fluid model like power law fluid model and Casson type fluid models (Blair and Spanner ^[7], Chaturani and Samy ^[8]). However in all of the above models, effect of single stenosis has been shown. But the stenosis may be developed in series or in irregular shape or overlapping. Iqbal *et al.*, ^[9] have studied a mathematical model to characterize the non-Newtonian flow of blood through stenosed artery in presence of transverse magnetic field. Mishra *et al.*, ^[10] have presented a mathematical model to study the effect of permeability of the wall through an artery with composite stenosis. Shah ^[11] has studied the effect of peripheral layer on blood flow characteristics in presence of stenosis. Yadav and Kumar ^[12] have presented a power law fluid model to study the non-Newtonian behaviour of blood flow through stenosed artery. Chan *et al.*, ^[13] have studied the structural response on blood flow through stenosed artery. Sung *et al.*, ^[14] have performed a mathematical model to study the effect of rotation on blood flow characteristics with four different angular velocities. Sing and Sing ^[15] have presented a mathematical model to study the non-Newtonian behaviour of blood in presence of multiple stenosis and post dilatation.

In the present analysis I proposed a Herschel-Bulkley fluid model to study the effect of multiple stenosis and post dilatation on blood flow.

2. Mathematical Formulation

Let us consider the steady flow of blood through a straight, rigid axisymmetric tube containing multiple abnormal segments. The geometry of the wall of the artery is given by

$$\frac{R}{R_0} = 1 - \frac{\delta_j}{2R_0} [1 + \cos \frac{2\pi}{l_j} (z - \alpha_j - \frac{l_j}{2})]; \alpha_j \leq z \leq \beta_j$$

$$= 1; \text{ otherwise} \quad (1)$$

The distance from the origin to the start of the j^{th} abnormal segment is given by

$$\alpha_j = \sum_{i=1}^j (d_i - l_i) - l_j$$

The distance from the origin to the end of j^{th} abnormal segment is

$$\beta_j = \sum_{i=1}^j (d_i - l_i)$$

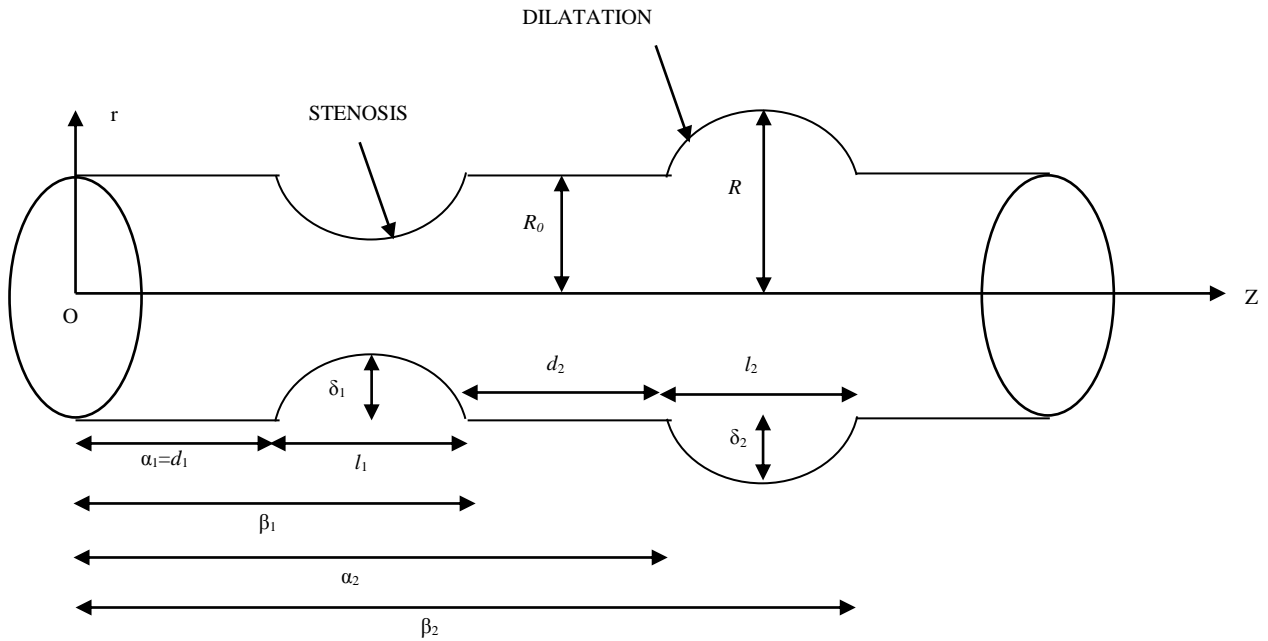


Fig 1: Geometry of arterial wall with multiple stenoses and post dilatation

In case of Herschel-Bulkley flow, the relationship between shear stress and shear rate is given by

$$\begin{aligned} \tau &= \mu \left(-\frac{\partial w}{\partial r}\right)^n + \tau_0, \tau \geq \tau_0 \\ -\frac{\partial w}{\partial r} &= 0, \tau < \tau_0 \end{aligned} \tag{2}$$

For simplicity we can take the value of the Herschel-Bulkley index “n” as 1/2
From (2) we can write

$$\left(-\frac{\partial w}{\partial r}\right)^{1/2} = \frac{\tau - \tau_0}{\mu} \tag{3}$$

The volumetric flow rate i.e. the flux is given by

$$Q = \int_0^R 2\pi r w \, dr \tag{4}$$

Integrating (4) and using no slip boundary condition i.e., $w = 0$ at $r = R$ we get

$$\begin{aligned} Q &= 2\pi \int_0^R \left(-\frac{\partial w}{\partial r}\right) \frac{r^2}{2} \, dr \\ &= \pi \int_0^R \frac{(\tau - \tau_0)^2}{\mu^2} r^2 \, dr \end{aligned} \tag{5}$$

Since

$$\begin{aligned} \tau &= -\frac{r}{2} \frac{dp}{dz} \quad \text{and} \quad \tau_w = -\frac{R}{2} \frac{dp}{dz}, \\ dr &= \frac{R}{\tau_w} d\tau \end{aligned} \tag{6}$$

Now limits of (5) are

$$\tau = 0 \text{ at } r = 0 \text{ and } \tau = \tau_w \text{ at } r = R$$

From (5) we get

$$\begin{aligned} Q &= \frac{\pi R^3}{\mu^2 \tau_w^3} \int_0^{\tau_w} (\tau^4 - 2\tau_0 \tau^3 + \tau_0^2 \tau^2) d\tau \\ &= \frac{5\pi R^3 \tau_w^2}{\mu^2} \left[\frac{1}{25} - \frac{1}{10} \frac{\tau_0}{\tau_w} + \frac{\tau_0^2}{\tau_w^2} \right] \end{aligned} \tag{7}$$

When $\frac{\tau_0}{\tau_w} \ll 1$, replacing $\frac{1}{15}$ by $\frac{1}{16}$ in the last term we get

$$Q = \frac{5\pi R^3 \tau_w^2}{\mu^2} \left[\frac{1}{5} - \frac{1}{4} \frac{\tau_0}{\tau_w} \right]^2 \tag{8}$$

From which we get

$$\tau_w = \frac{5}{4} \tau_0 + \sqrt{\left(\frac{5\mu^2 Q}{\pi}\right) \frac{1}{R^{3/2}}} \tag{9}$$

Since $\tau_w = -\frac{R}{2} \frac{dp}{dz}$,

We have $\frac{dp}{dz} = -\frac{5\tau_0}{2R} - \sqrt{\left(\frac{5\mu^2 Q}{\pi}\right) \frac{2}{R^{5/2}}}$

Now $p = p_1$ at $z = 0$ and $p = p_2$ at $z = l$, so we have

$$p_1 - p_2 = \frac{5\tau_0}{2R_0} \int_0^l \left(\frac{R}{R_0}\right)^{-1} dz + \sqrt{\left(\frac{20\mu^2 Q}{\pi R_0^5}\right)} \int_0^l \left(\frac{R}{R_0}\right)^{-\frac{5}{2}} dz \tag{10}$$

Now resistance to flow λ is given by

$$\begin{aligned} \lambda &= \frac{p_1 - p_2}{Q} \\ &= -\frac{5\tau_0}{2R_0 Q} \int_0^l \left(\frac{R}{R_0}\right)^{-1} dz - \sqrt{\left(\frac{20\mu^2}{\pi R_0^5 Q}\right)} \int_0^l \left(\frac{R}{R_0}\right)^{-\frac{5}{2}} dz \\ &= -f_1 \int_0^l \left(\frac{R}{R_0}\right)^{-1} dz - f_2 \int_0^l \left(\frac{R}{R_0}\right)^{-\frac{5}{2}} dz \end{aligned} \tag{11}$$

Where

$$\begin{aligned} f_1 &= \frac{5\tau_0}{2R_0 Q}, f_2 = \sqrt{\left(\frac{20\mu^2}{\pi R_0^5 Q}\right)} \\ &= -f_1 \left[\int_0^l \alpha_j dz + \sum_{j=1}^k \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-1} dz + \sum_{j=1}^{k-1} \int_{\beta_j}^{\alpha_{j+1}} dz + \int_{\beta_k}^l dz \right] \\ &\quad - f_2 \left[\int_0^l \alpha_j dz + \sum_{j=1}^k \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-\frac{5}{2}} dz + \sum_{j=1}^{k-1} \int_{\beta_j}^{\alpha_{j+1}} dz + \int_{\beta_k}^l dz \right] \\ &= (f_1 - f_2) \sum_{j=1}^{k+1} d_j - (f_1 I_1 - f_2 I_2) \end{aligned} \tag{12}$$

Where

$$I_1 = \sum_{j=1}^k \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-1} dz$$

and

$$I_2 = \sum_{j=1}^k \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-\frac{5}{2}} dz \tag{13}$$

If there is no abnormal segments

$$\lambda_N = -(f_1 - f_2)L$$

Therefore the non-dimensional resistance to flow is given by

$$\begin{aligned} \bar{\lambda} &= \frac{\lambda}{\lambda_N} \\ &= \sum_{j=1}^{k+1} \frac{d_j}{L} + \frac{(f_1 I_1 + f_2 I_2)}{(f_1 + f_2)L} \end{aligned} \tag{14}$$

If we put

$$a_j = 1 - \frac{\delta_j}{2R_0}, b_j = \frac{\delta_j}{2R_0}, \theta = \pi - \frac{2\pi}{l_j} (z - \alpha_j - \frac{l_j}{2})$$

In equation (1) we get

$$\frac{R}{R_0} = 1 - b_j (1 - \cos\theta)$$

As $z = \alpha_j$ implies that $\theta = 2\pi$ and $z = \beta_j$ implies $\theta = 0$.

Then (13) reduces to

$$I_1 = \sum_{j=1}^k \frac{l_j}{2\pi} \int_0^{2\pi} \frac{d\theta}{(a_j + b_j \cos\theta)}$$

and

$$I_2 = \sum_{j=1}^k \frac{l_j}{2\pi} \int_0^{2\pi} \frac{d\theta}{(a_j + b_j \cos\theta)^{5/2}}$$

3. Numerical Discussions

Here we have considered the influence of yield stress, blood viscosity and flux on resistance to flow for Herschel-Bulkley flow of blood through abnormal arterial segments. It is clear from the figure that variation of resistance to flow ratio $\bar{\lambda}$ is symmetrical with respect to $\frac{\delta_1}{R_0}$ and $\frac{\delta_2}{R_0}$ variation. For any constant value of $\frac{\delta_1}{R_0}$, $\bar{\lambda}$ increases with the increase of $\frac{\delta_2}{R_0}$. Same phenomenon occurs when $\frac{\delta_1}{R_0}$ increases. If both $\frac{\delta_1}{R_0}$ and $\frac{\delta_2}{R_0}$ are negative then $\bar{\lambda}$ increases with the increase of τ_0 , but the reverse effect occurs when both $\frac{\delta_1}{R_0}$ and $\frac{\delta_2}{R_0}$ are positive. Rate of change of $\bar{\lambda}$ gradually decreases with the increase of τ_0 . $\bar{\lambda}$ increases with the increase of Q, but there is no significant variation with the variation of μ .

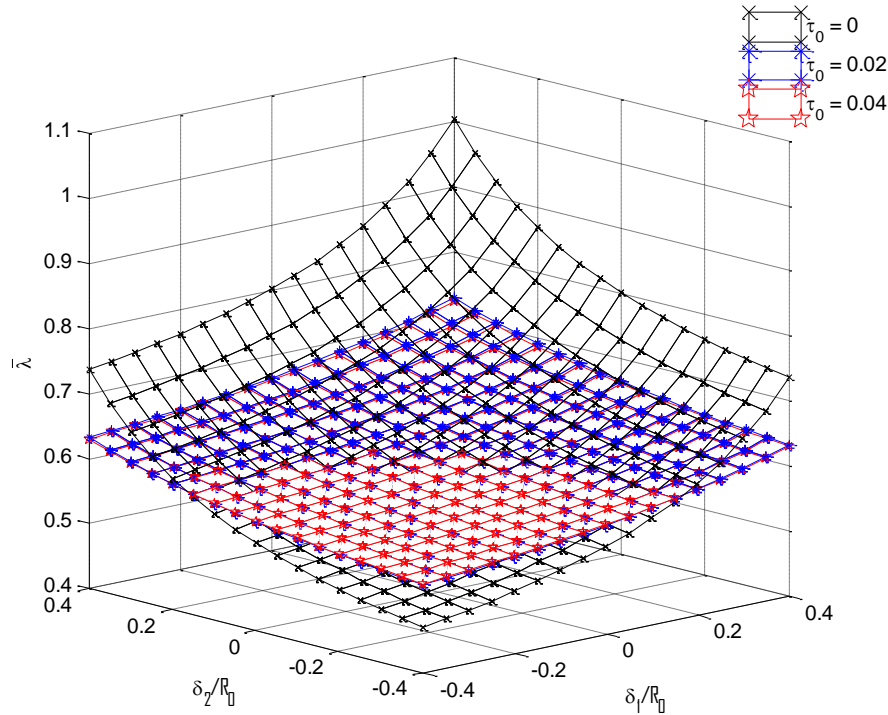


Fig 2: Variation in resistance to flow for different yield stress ($Q = 30; \mu = 0.000345$).

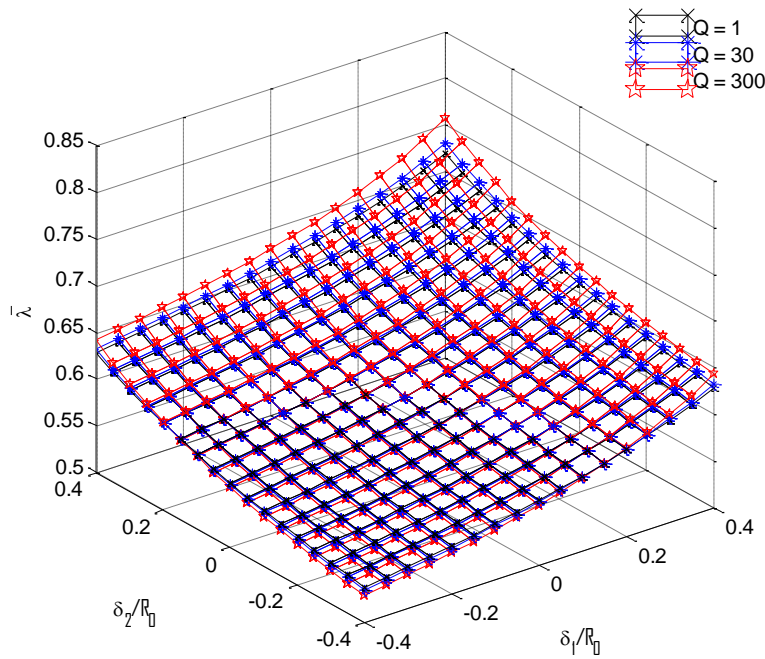


Fig 3: Variation in resistance to flow for different Flux ($\tau = 0.02; \mu = 0.000345$).

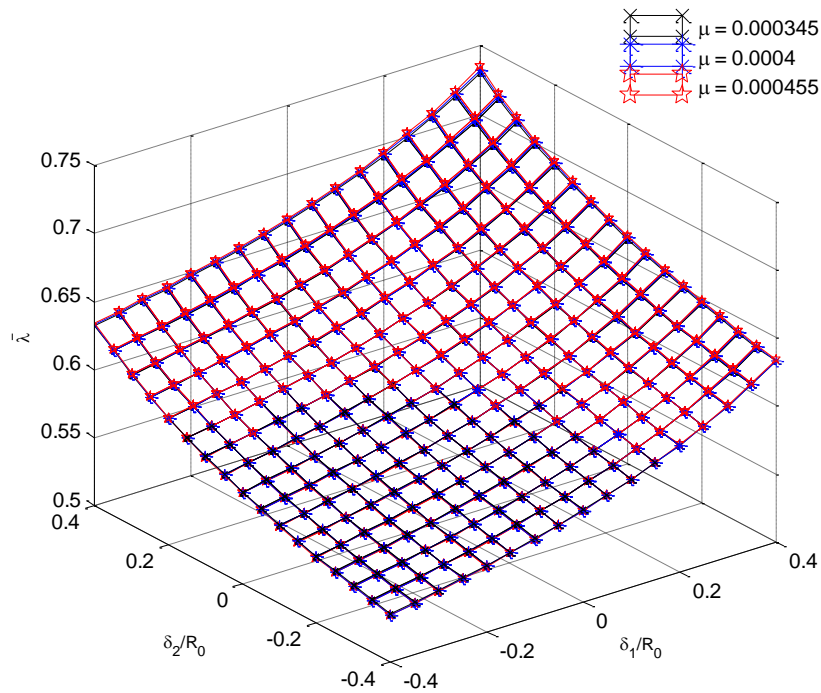


Fig 4: Variation in resistance to flow for different blood viscosities, ($\tau = 0.02; Q = 30$)

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