International Journal of Multidisciplinary Research and Development
Online ISSN: 2349-4182, Print ISSN: 2349-5979, Impact Factor: RJIF 5.72
www.allsubjectjournal.com
Volume 3; Issue 7; July 2016; Page No. 241-242

Correlation of cylinder and frustum of the cone with special numbers

[1] Pandichelvi V, [2] Sivakamasundari P

[1] Assistant Professor, Department of Mathematics, Urumu Dhanalkshmi College, Trichy, Tamil Nadu, India

[2] Guest Lecturer, Department of Mathematics, BDUCC, Lalgudi, Trichy, Tamil Nadu, India

Abstract
In this paper, we have proved that the difference between the volume of the frustum of the cone and lateral surface area of the cylinder is equal to $\pi(6Hpp_h - 6\mathrm{Th}_h - Cube_h)$.

Keywords: Pythagorean equation, ternary quadratic equation

Introduction

The enthralling branch of Mathematics is the theory of numbers where in Pythagorean triangles have been a topic of interest to different Mathematicians and to the followers of Mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-5]. In [6-19], various relations between Pythagorean triangles and polygonal numbers are studied.

In this paper, we find the relation among the geometrical figures consisting cylinder, the frustum of the cone and some special numbers such as heptagonal pyramidal number, tetrahedral number and a cubical integer.

Method of Analysis

Let r, h be the base radius and height of the cylinder respectively. Suppose a frustum of the cone with the same height h of the cylinder and the circular end at the bottom of this frustum coincides with the base of the cylinder is attached on either of its base.

This means that the base radius of the cylinder is equal to the radius of the circular end at the bottom of the frustum. Let R be the radius of the circular end at the top of the frustum.

Assume that the difference between the volume of the frustum of the cone and lateral surface area of the cylinder is equal to $\pi(6Hpp_h - 6\mathrm{Th}_h - Cube_h)$.

The mathematical statement of our assumption is

$$\left[\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right) - 2\pi r h\right] = \pi\left[h(h+1)(5h-2) - h(h+1)(h+2) - h^3\right] \quad (1)$$

which reduces to

$$R^2 + r^2 + Rr - 6r = 9h^2 - 12 \quad (2)$$

Introducing the linear transformations

$$R = u + v, r = u - v \text{ where } u \neq v \neq 0 \quad (3)$$

in (2), it becomes

$$3X^2 + Y^2 = Z^2 \quad (4)$$

where

$$X = u - 1, Y = v + 3, Z = 3h \quad (5)$$

The most cited solution to (4) is exhibited by

$$\left.\begin{aligned} X &= 2\alpha\beta \\ Y &= 3\alpha^2 - \beta^2 \\ Z &= 3\alpha^2 + \beta^2 \end{aligned}\right\} \quad (6)$$

In view of (5) and (6), we obtain

$$\left.\begin{aligned} u &= 2\alpha\beta + 1 \\ v &= 3\alpha^2 - \beta^2 - 3 \\ h &= \frac{3\alpha^2 + \beta^2}{3} \end{aligned}\right\} \quad (7)$$

Since our attention focused on finding the integer values to the height and radius, we observe that the value of h is an integer when

$$\beta = 3\delta$$

This choice leads (7) to

$$u = 6\alpha\delta + 1 \quad (8)$$

$$v = 3\alpha^2 - 9\delta^2 - 3 \quad (9)$$

$$h = \alpha^2 + 3\delta^2 \quad (10)$$

Using (8) and (9) in (3), the non-zero integral values of the radii of the circular ends at the top and bottom of the frustum and the height of the cylinder cum frustum satisfying (1) are given by

$$R = 3\alpha^2 + 6\alpha\delta - 9\delta^2 - 2$$
$$r = 6\alpha\delta - 3\alpha^2 + 9\delta^2 + 4$$

and equation (10)

Some numerical examples satisfying our assumption are tabulated as follows

α	δ	R	r	h	L.H.S of (1)	R.H.S of (1)
2	1	13	13	7	429	429
3	`1	34	4	12	1284	1284
4	2	58	40	28	7044	7044
5	2	97	25	37	12309	12309
7	3	190	64	76	51972	51972

Notations Used

$Th_h = \frac{1}{6}h(h+1)(h+2)$ be a Tetrahedral number of rank h.

$Hpp_h = \frac{1}{6}h(h+1)(5h-2)$ be a Heptagonal pyramidal number of rank h.

$Cube_h = h^3$

Conclusion

In this paper, we expose the relation between the cylinder, the frustum of the cone and some special numbers such as heptagonal pyramidal number, tetrahedral number and a cubical integer. In this approach, one may compare some other Geometrical figures with two dimensional, three dimensional and four dimensional figurate numbers.

References

1. Sierpinski W, Pythagorean triangles, Dover publications, INC, Newyork, 2003.
2. Dickson LE. History of the theory numbers, Diophantine analysis New York, Dover 2005, 2.
3. Carmichael, RD. The Theroy of Numbers and Diophantine Analysis, Dover Publication, New York, 1959.
4. Mordell LJ. Diophantine Equations, Academic press, London, 1969.
5. John H, Conway, Richard K, Guy. The Book of Numbers, Springer Verlag, New York, 1995.
6. Gopalan MA, Devibala S. On a Pythagorean problem, Acta Ciencia Indica, 2006; XXXII, M(4):1451-1452.
7. Gopalan MA, Gnanam A. Pairs of Pythagorean triangles with equal perimeters, Impact. J Sci Tech. 2007; 1(2):67-70.
8. Gopalan MA, Leelavathi S. Pythagorean triangle with 2 area/perimeter as a cubic integer, Bulletin of Pure and Applied Science, 2007; 26, E(2):197-200.
9. Gopalan MA, Gnanam A. A special Pythagorean problem, Acta Ciencia Indica, 2007; XXXIII, M(4):1435-1439.
10. Gopalan MA, Gnanam A, Janaki G. A Remarkable Pythagorean problem, Acta Ciencia Indica, 2007; XXXIII, M(4):1429-1434.
11. Gopalan MA, Janaki G. Pythagorean triangle with area/perimeter as a special polygonal number, Bulletin of Pure and Applied Science, 2008; 27, E(2):393-402.
12. Gopalan MA, Sangeetha G. Pythagorean triangle with perimeter as triangular number, GJ-AMMS, 2010; 3(1-2):93-97.
13. Gopalan MA, Gnanam A. Pythagorean triangles and Polygonal numbers, International Journal of Mathematical Sciences, 2010; 9(1-2):211-215.
14. Gopalan MA, Sivakami B. Pythagorean triangle with hypotenuse minus 2(area/perimeter) as a square integer, Archimedes. J Math. 2012; 2(2):153-166.
15. Gopalan MA, Manjusomanath, Geetha K. Pythagorean triangle with area/perimeter as a Special polygonal number", IOSR-JM, 2013; 7(3):52-62.
16. Gopalan MA, Geetha V. Pythagorean triangle with area/perimeter as a special polygonal number, IRJES, 2013; 2(7):28-34.
17. Gopalan MA, Sangeetha V, Manjusomanath. Pythagorean triangle and Polygonal number, Cayley J. Math., 2013; 2(2):151-156.
18. Meena K, Vidhyalakshmi S, Geetha B, Vijayasankar A, Gopalan MA. Relations between special polygonal numbers generated through the solutions of Pythagorean equation, IJISM, 2014: 2(2):257-258.
19. Gopalan MA, Vidhyalakshmi S, Thiruniraselvi N, Presenna R. On Pairs of Pythagorean Triangles-I, IOSR Journal of Mathematics, 11, 2015; 1(IV):15-17.