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Construction of euler square involving some figurate numbers

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Abstract
In this communication, we determine the triple (a,b,c) involving some figurate numbers such that the sum of any two of them is a perfect square.

Keywords: Diophantine m -tuples, cubic equation

Introduction
The problem of constructing sets with property that product of any two of its distinct elements added with an integer is a perfect square has a very long history and such sets were studied by Diophantus [1]. In this context one may refer [2-10].
In this communication, we evaluate the triple

$$\left([6(f_k)-3]^3,[4(g_k)+2]^3,\frac{\left[[6(f_k)-3]^3-[4(g_k)+2]^3-1\right]^2-4[4(g_k)+2]^3}{4}\right)$$

Where

$$f_k=\left(280k^4+752k^3+900k^2+540k+122\right)\text{and}$$
$$g_k=\left(280k^4+1032k^3+1512k^2+1134k+364\right)$$

such that sum of any two of them is a perfect square.

Notations

Let $RD_p=(2p-1)(2p^2-2p+1)$ be a Rhombic Dodecahedral number of rank p.

$SP_q=\frac{1}{6}q(q+1)(2q+1)$ be a Square Pyramidal number of rank q.

$Gno_n=(2n-1)$ be a Gnomonic number of rank n.

$Th_p=\frac{1}{6}p(p+1)(p+2)$ be a Tetrahedral number of rank p.

$Hp_q=\frac{1}{6}q(q+1)(4q-1)$ be a Hexagonal pyramidal number of rank q.

Method of analysis

Let $a(p)=54RD_p-27Gno_p$
$b(q)=192SP_q+8Gno_{q+1}$

which are equivalent to the following two equations

$$a(p)=(6p-3)^3,b(q)=(4q+2)^3 \text{ where } p,q\in z-\{0\}$$

Now, we assume that

$$a(p)+b(q)=\alpha^2$$

The above equation can be written as

$$x^3+y^3=\alpha^2 \tag{1}$$

Where

$$x=6p-3 \tag{2}$$

$$y=4q+2 \tag{3}$$

The general solutions to (1) are exhibited by

$$x=m(m^3+n^3) \tag{4}$$

$$y=n(m^3+n^3) \tag{5}$$

$$\alpha=(m^3+n^3)^2$$

Comparing (2) and (4), we acquire that

$$p=\frac{m(m^3+n^3)+3}{6}$$

Equating (3) and (5), we notice that

$$q=\frac{n(m^3+n^3)-2}{4}$$

Since the ranks of the numbers under consideration are integers, we examine that p and q are integers for the following choices of m and n

$$m=6k+3, n=4k+6$$

Thus, we have

$$p=(280k^4+752k^3+900k^2+540k+122)$$

$$q=(280k^4+1032k^3+1512k^2+1134k+364)$$

Hence,

$$a(k)=[6(f_k)-3]^3 \tag{6}$$

$$b(k)=[4(g_k)+2]^3 \tag{7}$$

Where

$$f_k=(280k^4+752k^3+900k^2+540k+122) \tag{8}$$

$$g_k=(280k^4+1032k^3+1512k^2+1134k+364) \tag{9}$$

Let $c(k)$ be any non-zero integer such that

$$a(k)+c(k)=\beta^2 \tag{10}$$

$$b(k)+c(k)=\gamma^2 \tag{11}$$

Subtracting (7) from (6), we get

$$\beta^2 - \gamma^2 = a(k) - b(k) \tag{12}$$

The choices $\beta = A+1, \gamma = A$ lead (12) to

$$\gamma = A = \frac{[6(f_k)-3]^3 - [4(g_k)+2]^3 - 1}{2} \tag{13}$$

Using (7) and (13) in (11), the values of c are represented by

$$c(k) = \frac{\left[[6(f_k)-3]^3 - [4(g_k)+2]^3 - 1\right]^2 - 4[4(g_k)+2]^3}{4} \tag{14}$$

Where

f_k and g_k are given by (8) and (9).

Hence,

$\left[[6(f_k)-3]^3, [4(g_k)+2]^3, \frac{\left[[6(f_k)-3]^3 - [4(g_k)+2]^3 - 1\right]^2 - 4[4(g_k)+2]^3}{4}\right]$ is a triple in which the sum of any two of them is a perfect square.

Some numerical examples are illustrated below:

k	$a(k)$	$b(k)$	$c(k)$	$a(k)+b(k)$	$a(k)+c(k)$	$b(k)+c(k)$
0	387420489	3099363912	$1.838659281\times 10^{18}$	$(59049)^2$	$(1355971711)^2$	$(1355971712)^2$
1	3768014003481	5168743489000	$4.905107729\times 10^{23}$	$(2989441)^2$	$(700364742759)^2$	$(700364742760)^2$
2	77324146816162	62867395218829	$5.224941668\times 10^{27}$	$(37442161)^2$	$(72283757986665)^2$	$(72283757986664)^2$

Remark
Similarly if we choose

$$a(p) = 1296Th_p + 108Gno_p + 324 \text{ and}$$
$$b(q) = 96HP_q + 14Gno_q + 15$$

and applying the procedure as explained above , we evaluate another triple

$$\left([6(f_k)+6]^3, [4(g_k)+1]^3, \frac{\left[[6(f_k)+6]^3 - [4(g_k)+1]^3 - 1\right]^2 - 4[4(g_k)+1]^3}{4}\right)$$

Where

$$f_k = \left(432k^4 + 468k^3 + 198k^2 + 39k + 2\right)$$
$$g_k = \left(648k^4 + 594k^3 + 216k^2 + 36k + 2\right)$$

such that sum of any two of them is a perfect square.

Some numerical examples are illustrated below

k	$a(k)$	$b(k)$	$c(k)$	$a(k)+b(k)$	$a(k)+c(k)$	$b(k)+c(k)$
0	5832	729	6506872	$(81)^2$	$(2552)^2$	$(2551)^2$
1	320013504000	214384046625	2.7894×10^{21}	$(731025)^2$	$(52814728688)^2$	$(52814728687)^2$
2	3.31001×10^{14}	2.65018×10^{14}	1.08844×10^{27}	$(24413481)^2$	$(32991487131344)^2$	$(32991487131343)^2$

Conclusion

In this communication, we discover the triple involving some figurate numbers in such a way that the sum of any two of them is a perfect square. In this manner one may search for other triples concerning various special numbers satisfying some other properties.

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