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Totally regular property of alpha product of two fuzzy graphs

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Abstract
In this paper, the total degree of a vertex in Alpha product of two fuzzy graphs are obtained in terms of the degree and the total degree of vertices in the two given fuzzy graphs in some particular cases. In general, the Alpha product of two totally regular fuzzy graphs need not be a totally regular fuzzy graph. The necessary and sufficient conditions for the Alpha product of two fuzzy graphs to be totally regular under some restrictions are also obtained.

Keywords: Total degree of a vertex, Regular fuzzy graph, Complement of fuzzy graph, complete graph, Alpha product of two fuzzy graphs

1. Introduction
A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(uv) \le \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of G: (σ, μ) is denoted by G^*: (V, E) where $E \subseteq V \times V$. Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson JN and Peng CS. introduced the concept of operations on fuzzy graphs.
The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Mordeson JN and Peng CS [2]. The Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations union, join, Cartesian product and composition was discussed by Nagoorgani A and Radha K. The degree of a vertex in Alpha, Beta, Gamma Product of two fuzzy graphs was discussed by Nagoorgani A and Fathima Kani. B. In this paper we study about the total degree of a vertex in Alpha product of two fuzzy graphs and totally Regular property of Alpha product of two fuzzy graphs.
First we go through some preliminaries which can be found in [1-5].

2. Basic Definitions
Throughout this paper we assume that μ is reflexive and need not consider loops. Also, the underlying set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graph in all the examples and all these properties are satisfied for all fuzzy graphs except null graphs.

Definition 2.1 [1]**:** If $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all u,v $\in$ V, then G is called a *complete fuzzy graph*. The *Complement* $\overline{G^*}$ *of a graph* G^* also has V (G) as its point set, but two points are adjacent in G^* if and only if they are not adjacent in G^*. The *degree* $d_G^*(v)$ of a vertex v in G* is the number of edges incident with v.

Definition 2.2 [5]**:** Let G: (σ, μ) be a fuzzy graph. The *degree of a vertex u* in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$$

Definition 2.3 [7]**:** Let G: (σ, μ) be a fuzzy graph on G^*. The *total degree of a vertex* $u \in V$ is defined by

$$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u).$$

If each vertex of G has the same total degree k, then G is said to be a *totally regular fuzzy graph of total degree k or a k-totally regular fuzzy graph.*

Notations
The relation $\mu_1 \le \mu_2$ means that $\mu_1(e) \le \mu_2(e), \forall e \in E_1$ and $\forall e \in E_2$ where μ_1 is a fuzzy subset of E_1 and μ_2 is a fuzzy subset of E_2.

Lemma 2.4 [4]**:** If $G_1 : (\sigma_1, \mu_1)$ *and* $G_2 : (\sigma_2, \mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$. The relation $\sigma_1 \geq \sigma_2$ means that $\sigma_1(u) \geq \sigma_2(v)$, for every $u \in V_1$ and for every $v \in V_2$, where σ_i is a fuzzy subset of V_i, $i = 1, 2$.

Definition 2.5 [9]**:** The $\alpha-$*product of two fuzzy graphs* G_1 and G_2 is defined as a fuzzy graph $G_1 \times_\alpha G_2 = ((\sigma_1 \times_\alpha \sigma_2), (\mu_1 \times_\alpha \mu_2))$ *on* $G^*: (V, E)$ *where* $V = V_1 \times_\alpha V_2$ *and*
$E = \{((u_1, u_2), (v_1, v_2)) / u_1 = v_1, u_2v_2 \in E_2 \ (or) \ u_2 = v_2, u_1v_1 \in E_1 \ (or) u_1v_1 \in E_1, u_2v_2 \notin E_2 (or) u_1v_1 \notin E_1\, u_2v_2 \in E_2\}$
With $\sigma_1 \times_\alpha \sigma_2 = \sigma_1(u_1) \wedge \sigma_2(u_2) \quad \forall (u_1, u_2) \in V_1 \times_\alpha V_2$

$$(\mu_1 \times_\alpha \mu_2)(((u_1, u_2), (v_1, v_2))) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2), & if\ u_1 = v_1, u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1v_1), & if\ u_2 = v_2, u_1v_1 \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & if\ u_1v_1 \in E_1, u_2v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2), & if\ u_1v_1 \notin E_1\, u_2v_2 \in E_2 \end{cases}$$

Note: Throughout this paper $G_1 : (\sigma_1, \mu_1)$ *and* $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^*: (V_1, E_1)$ and $G_2^*: (V_2, E_2)$ respectively. Let the number of vertices in V_1 and V_2 be p_1 and p_2 respectively.

3. Total degree of a vertex in Alpha Product of fuzzy Graphs

For any $(u_1, u_2) \in V_1 \times V_2$,

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = \sum_{(u_1,u_2)(v_1,v_2) \in E} ((\mu_1 \times_\alpha \mu_2)(u_1, u_2)(v_1, v_2)) + (\sigma_1 \times_\alpha \sigma_2)(u_1, u_2)$$

$$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2v_2)$$

$$+ \sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1) + \sum_{u_1v_1 \in E_1, u_2v_2 \notin E_2} \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1)$$

$$+ \sum_{u_1v_1 \notin E_1\, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2) + (\sigma_1 \wedge \sigma_2)(u_1, u_2) \dashrightarrow (3.1)$$

In general, the total degree of a vertex in alpha product of two fuzzy graphs need not be expressed in terms of total degree or degree of vertices in the two given fuzzy graphs. In the following theorems, we obtain the total degree of a vertex in $G_1 \times_\alpha G_2$ in terms of degree and total degree of vertices in G_1 and G_2 in some particular cases.

Theorem 3.1: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then
$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$,
$\forall u_1 \in V_1$ and $u_2 \in V_2$

Proof: We have $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.
From (3.1), for any $(u_1, u_2) \in V_1 \times V_2$

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = \sum_{u_1 = v_1, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \mu_1(u_1v_1)$$

$$+ \sum_{u_1v_1 \in E_1, u_2v_2 \notin E_2} \mu_1(u_1v_1) + \sum_{u_1v_1 \notin E_1\, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$

$$\therefore\ td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$

Corollary 3.2: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_1}(u_1) + d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2).$$

Proof: Since both G_1^* and G_2^* are complete graphs, $d_{\overline{G_1^*}}(u_1) = 0$ and $d_{\overline{G_2^*}}(u_2) = 0$.
Therefore Theorem 3.1 becomes,

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_1}(u_1) + d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$

Theorem 3.3: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = td_{G_1}(u_1) + td_{G_2}(u_2) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) - \sigma_1(u_1) \vee \sigma_2(u_2),$$
$$\forall u_1 \in V_1 \text{ and } u_2 \in V_2.$$

Proof: From Theorem 3.1,

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$
$$= d_{G_1}(u_1) + \sigma_1(u_1) + d_{G_2}(u_2) + \sigma_2(u_2) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) - \sigma_1(u_1) - \sigma_2(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$
$$= td_{G_2}(u_2) + td_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) - \sigma_1(u_1) \vee \sigma_2(u_2),$$

Since $\sigma_1(u_1) \vee \sigma_2(u_2) = \sigma_1(u_1) + \sigma_2(u_2) - \sigma_1(u_1) \wedge \sigma_2(u_2)$.

Corollary 3.4: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = td_{G_2}(u_2) + td_{G_1}(u_1) - \sigma_1(u_1) \vee \sigma_2(u_2)$$

Proof: The proof follows by substituting $d_{\overline{G_1^*}}(u_1) = 0$ and $d_{\overline{G_2^*}}(u_2) = 0$ in Theorem 3.3.

Theorem 3.5: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$. Then

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = c_1 \left[1 + d_{G_2^*}(u_2) + d_{G_2^*}(u_2) d_{\overline{G_1^*}}(u_1)\right] + [1 + d_{\overline{G_2^*}}(u_2)] d_{G_1}(u_1)$$

Where c_1 is the constant value of σ_1.

Proof: We have $\sigma_1 \leq \mu_2$. Hence $\sigma_2 \geq \mu_1$ and $\sigma_1 \leq \sigma_2$.
From (3.1)

$$td_{G_1 \times_\alpha G_2}(u_1, u_2)$$
$$= \sum_{u_1 = v_1, u_2 v_2 \in E_2} \sigma_1(u_1) + \sum_{u_2 = v_2, u_1 v_1 \in E_1} \mu_1(u_1 v_1) + \sum_{u_1 v_1 \in E_1, u_2 v_2 \notin E_2} \mu_1(u_1 v_1) + \sum_{u_1 v_1 \notin E_1, u_2 v_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(v_1)$$
$$+ \sigma_1(u_1)$$
$$= \sigma_1(u_1) d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + c_1\, d_{\overline{G_1^*}}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1)$$
$$= c_1 \left[1 + d_{G_2^*}(u_2) + d_{G_2^*}(u_2) d_{\overline{G_1^*}}(u_1)\right] + [1 + d_{\overline{G_2^*}}(u_2)] d_{G_1}(u_1) \quad \text{--------- (3.2)}$$

Theorem 3.6: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$. Then

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = c_1 \, [d_{G_2^*}(u_2) + d_{\overline{G_1^*}}(u_1) d_{G_2^*}(u_2) - d_{\overline{G_2^*}}(u_2)] + [1 + d_{\overline{G_2^*}}(u_2)] td_{G_1}(u_1)$$

Where c_1 is the constant value of σ_1.

Proof: From (3.2),

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = c_1 \left[1 + d_{G_2^*}(u_2) + d_{G_2^*}(u_2) d_{\overline{G_1^*}}(u_1)\right] + [1 + d_{\overline{G_2^*}}(u_2)]\, [td_{G_1}(u_1) - \sigma_1(u_1)]$$
$$= c_1 \left[1 + d_{G_2^*}(u_2) + d_{G_2^*}(u_2) d_{\overline{G_1^*}}(u_1)\right] + [1 + d_{\overline{G_2^*}}(u_2)]\, [td_{G_1}(u_1) - c_1]$$
$$= c_1 \, [d_{G_2^*}(u_2) + d_{\overline{G_1^*}}(u_1) d_{G_2^*}(u_2) - d_{\overline{G_2^*}}(u_2)] + [1 + d_{\overline{G_2^*}}(u_2)] td_{G_1}(u_1)$$

4. Totally Regular Property of Alpha product of two Fuzzy graphs

Theorem 4.1: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs on regular graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \wedge \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.

Proof: Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$

Suppose that G_1 and G_2 are regular fuzzy graphs of degree k_1 and k_2 respectively.

By Theorem 3.1, for any $(u_1, u_2) \in V_1 \times_\alpha V_2$.

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\, d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$

We know that $d_{\overline{G^*}}(v) = p - 1 - d_{G^*}(v)$.

Then
$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_2}(u_2) + d_{G_1}(u_1) + [p_2 - 1 - d_{G_2^*}(u_2)]\, d_{G_1}(u_1)$$
$$+ [p_1 - 1 - d_{G_1^*}(u_1)] d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$$
$$\Rightarrow td_{G_1 \times_\alpha G_2}(u_1, u_2) = [p_2 - d_{G_2^*}(u_2)] d_{G_1}(u_1) + [p_1 - d_{G_1^*}(u_1)]\, d_{G_2}(u_2)$$
$$+ \sigma_1(u_1) \wedge \sigma_2(u_2) \qquad ----\rightarrow (4.1)$$
$$\therefore\ td_{G_1 \times_\alpha G_2}(u_1, u_2) = [p_2 - r_2]\, k_1 + [p_1 - r_1] k_2 + c$$

Hence $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.

Conversely, assume that $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.

Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times_\alpha V_2$.

$$td_{G_1 \times_\alpha G_2}(u_1, u_2) = td_{G_1 \times_\alpha G_2}(v_1, v_2)$$
$$\text{From (4.1)}, [p_2 - d_{G_2^*}(u_2)] d_{G_1}(u_1) + [p_1 - d_{G_1^*}(u_1)]\, d_{G_2}(u_2) + c$$
$$= [p_2 - d_{G_2^*}(v_2)] d_{G_1}(v_1) + [p_1 - d_{G_1^*}(v_1)]\, d_{G_2}(v_2) + c$$
$$\Rightarrow [p_2 - d_{G_2^*}(u_2)] d_{G_1}(u_1) + [p_1 - d_{G_1^*}(u_1)]\, d_{G_2}(u_2)$$
$$= [p_2 - d_{G_2^*}(v_2)] d_{G_1}(v_1) + [p_1 - d_{G_1^*}(v_1)]\, d_{G_2}(v_2) \qquad ----\rightarrow (4.2)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times_\alpha V_2$ where $u_2, v_2 \in V_2$ are arbitrary.

From (4.2),

$$[p_2 - r_2] d_{G_1}(u) + [p_1 - d_{G_1^*}(u)]\, d_{G_2}(u_2) = [p_2 - r_2] d_{G_1}(u) + [p_1 - d_{G_1^*}(u)]\, d_{G_2}(v_2)$$
$$\Rightarrow [p_1 - d_{G_1^*}(u)]\, d_{G_2}(u_2) = [p_1 - d_{G_1^*}(u)]\, d_{G_2}(v_2)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$.

Thus G_2 is a regular fuzzy graph.

Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times_\alpha V_2$ where $u_1, v_1 \in V_1$ are arbitrary.

From (4.2),

$$[p_2 - d_{G_2^*}(v)] d_{G_1}(u_1) + [p_1 - r_1]\, d_{G_2}(v) = [p_2 - d_{G_2^*}(v)] d_{G_1}(v_1) + [p_1 - r_1]\, d_{G_2}(v)$$
$$\Rightarrow [p_2 - d_{G_2^*}(v)] d_{G_1}(u_1) = [p_2 - d_{G_2^*}(v)] d_{G_1}(v_1)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$.

Thus G_1 is a regular fuzzy graph.

Theorem 4.2: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two regular fuzzy graphs on crisp graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \wedge \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1^* and G_2^* are regular fuzzy graphs.

Proof: The proof of the theorem is similar to the proof of Theorem 4.1.

Theorem 4.3: Let $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ be two fuzzy graphs on regular graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \vee \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are totally regular fuzzy graphs.

Proof: This theorem can be proved in the same way theorem 4.1 is proved, by using the formula obtained in Theorem 3.3.

Theorem 4.4: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two totally regular fuzzy graphs on crisp graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \vee \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1^* and G_2^* are regular fuzzy graphs.
Proof: This theorem can be proved in the same way theorem 4.1 is proved, by using the formula obtained in Theorem 3.3.

Theorem 4.5: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \wedge \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if $G_1$and G_2 are regular fuzzy graphs.
Proof: Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$
Suppose that G_1 and G_2 are regular fuzzy graphs of degree k_1 and k_2 respectively.
By corollary 3.2, for any $(u_1, u_2) \in V_1 \times_\alpha V_2$.

$$td_{G_1 \times_\alpha G_2}(u_1,u_2) = d_{G_1}(u_1) + d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \quad \text{----}\rightarrow (4.3)$$
$$td_{G_1 \times_\alpha G_2}(u_1,u_2) = k_1 + k_2 + c$$

Hence $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.
Conversely, assume that $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.
Then for any two points (u_1,u_2) and (v_1,v_2) in $V_1 \times_\alpha V_2$.

$$td_{G_1 \times_\alpha G_2}(u_1,u_2) = td_{G_1 \times_\alpha G_2}(v_1,v_2)$$
$$d_{G_1}(u_1) + d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) = d_{G_1}(v_1) + d_{G_2}(v_2) + \sigma_1(v_1) \wedge \sigma_2(v_2) \text{---}\rightarrow (4.4)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times_\alpha V_2$ where $u_2, v_2 \in V_2$ are arbitrary.
From (4.4), $d_{G_1}(u) + d_{G_2}(u_2) + c = d_{G_1}(u) + d_{G_2}(v_2) + c$

$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$.
Thus G_2 is a regular fuzzy graph.
Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times_\alpha V_2$ where $u_1, v_1 \in V_1$ are arbitrary.
From (4.4), $d_{G_1}(u_1) + d_{G_2}(v) + c = d_{G_1}(v_1) + d_{G_2}(v) + c$

$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$.
Thus G_1 is a regular fuzzy graph.

Theorem 4.6: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. Let $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\sigma_1 \vee \sigma_2$ be a constant function. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if $G_1$and G_2 are totally regular fuzzy graphs.
Proof: This theorem can be proved in the same way theorem 4.5 is proved, by using the formula obtained in Corollary 3.4.

Theorem 4.7: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs on regular crisp graphs G_1^* and G_2^* respectively such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph.
Proof: Suppose that G_1^* and G_2^* are regular fuzzy graphs of degree k_1 and k_2 respectively and c_1 be the constant value of σ_1.
Suppose that G_1 is a regular fuzzy graph of degree r_1.
By Theorem 3.5, for any $(u_1, u_2) \in V_1 \times_\alpha V_2$.

$$td_{G_1 \times_\alpha G_2}(u_1,u_2) = \left[1 + d_{G_2^*}(u_2) + d_{G_2^*}(u_2) d_{\overline{G_1^*}}(u_1)\right] + [1 + d_{\overline{G_2^*}}(u_2)] d_{G_1}(u_1)$$
$$= c_1[1 + k_2 + k_2(p_1 - 1 - k_1)] + [1 + (p_2 - 1 - k_2)r_1$$

Hence $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.
Conversely, assume that $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph.
Then for any two points (u_1,u_2) and (v_1,v_2) in $V_1 \times_\alpha V_2$,

$td_{G_1 \times_\alpha G_2}(u_1, u_2) = td_{G_1 \times_\alpha G_2}(v_1, v_2)$ gives

$c_1[1 + k_2 + k_2(p_1 - 1 - k_1)] + [1 + (p_2 - 1 - k_2)d_{G_1}(u_1)$

$= c_1[1 + k_2 + k_2(p_1 - 1 - k_1)] + [1 + (p_2 - 1 - k_2)d_{G_1}(v_1)$

Which implies $d_{G_1}(u_1) = d_{G_1}(v_1)$ for any u_1, v_1 in V_1. Hence G_1 is a regular fuzzy graph.

Theorem 4.8: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on regular crisp graphs G_1^* and G_2^* respectively such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$. Then $G_1 \times_\alpha G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph.

Proof: This theorem can be proved in the same way theorem 4.7 is proved, by using the formula obtained in Theorem 3.6.

5. Conclusion

In this paper, we have obtained the total degree of a vertex in $G_1 \times_\alpha G_2$ in terms of degree and total degree of vertices in G_1 and G_2 in some particular cases. They will be helpful in studying many properties of alpha product of two fuzzy graphs. And we have showed that the Alpha product of two totally regular fuzzy graphs need not be a totally regular fuzzy graph. We have obtained necessary and sufficient condition for the Alpha product of two fuzzy graphs to be totally regular in some particular cases.

6. References

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