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Special pairs of Pythagorean triangles and narcissistic number

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Abstract
We search for infinitely many pairs of Pythagorean triangles such that in each pair, the difference between their perimeters is 4 times the narcissistic number. Also the total number of pairs of primitive and non-primitive Pythagorean triangles are exhibited.

Keywords: Pairs of Pythagorean triangles, Narcissistic number, primitive and non-primitive

1. Introduction

In Number Theory Pythagorean triangles have been a very big interest to various Mathematicians, since it is a very big treasure house to hunt for. For various types of problems and ideas on Pythagorean triangles and special numbers one may refer [1-11]. In this communication we search for pairs of Pythagorean triangles so that, in each pair, the difference between the perimeters is 4 times the Narcissistic number.

2. Basic Definitions

Definition 2.1

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x,y, z are natural numbers. The above equation is also referred to as Pythagorean triangle.

Definition 2.2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$ where $m > n > 0$.
This solution is called primitive, if m, n are of opposite parity and gcd (m,n) = 1.

Definition 2.3: Narcissistic Numbers

An n-digit number which is the sum of n^{th} power of its digits is called an n-narcissistic number. It is also known as Armstrong number.

3. Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q (m > q > 0), and p, q (p > q > 0) respectively. Let P_1, P_2 be the perimeters of PT_1, PT_2 such that

$P_1 - P_2 = 4$ times the narcissistic number 153.

The above relation leads to

$$(2m+q)^2 - (2p+q)^2 = 1224 \qquad (1)$$

Which simplifies to

$$(m-p)(m+p+q) = 306 \qquad (2)$$

After performing numerical calculations, the values of m, q and p satisfying (2) are presented in the following Table-1.

Table 1: Numerical Illustrations

S. No.	m	q	p	P_1	P_2	P_1-P_2/4
1	153	1	152	47124	46512	153
2	152	3	151	47120	46508	153
3	151	5	150	47112	46500	153
4	150	7	149	47100	46488	153
5	149	9	148	47084	46472	153
6	148	11	147	47064	46452	153
7	147	13	146	47040	46428	153
8	146	15	145	47012	46400	153
9	145	17	144	46980	46368	153
10	144	19	143	46944	46332	153
11	143	21	142	46904	46292	153
12	142	23	141	46860	46248	153
13	141	25	140	46812	46200	153
14	140	27	139	46760	46148	153
15	139	29	138	46704	46092	153
16	138	31	137	46644	46032	153
17	137	33	136	46580	45968	153
18	136	35	135	46512	45900	153
19	135	37	134	46440	45828	153
20	134	39	133	46364	45752	153

21	133	41	132	46284	45672	153
22	132	43	131	46200	45588	153
23	131	45	130	46112	45500	153
24	130	47	129	46020	45408	153
25	129	49	128	45924	45312	153
26	128	51	127	45824	45212	153
27	127	53	126	45720	45108	153
28	126	55	125	45612	45000	153
29	125	57	124	45500	44888	153
30	124	59	123	45384	44772	153
31	123	61	122	45264	44652	153
32	122	63	121	45140	44528	153
33	121	65	120	45012	44400	153
34	120	67	119	44880	44268	153
35	119	69	118	44744	44132	153
36	118	71	117	44604	43992	153
37	117	73	116	44460	43848	153
38	116	75	115	44312	43700	153
39	115	77	114	44160	43548	153
40	114	79	113	44004	43392	153
41	113	81	112	43844	43232	153
42	112	83	111	43680	43068	153
43	111	85	110	43512	42900	153
44	110	87	109	43340	42728	153
45	109	89	108	43164	42552	153
46	108	91	107	42984	42372	153
47	107	93	106	42800	42188	153
48	106	95	105	42612	42000	153
49	105	97	104	42420	41808	153
50	104	99	103	42224	41612	153
51	103	101	102	42024	41412	153
52	53	49	51	10812	10200	153
53	54	47	52	10908	10296	153
54	55	45	53	11000	10388	153
55	56	43	54	11088	10476	153
56	57	41	55	11172	10560	153
57	58	39	56	11252	10640	153
58	59	37	57	11328	10716	153
59	60	35	58	11400	10788	153
60	61	33	59	11468	10856	153
61	62	31	60	11532	10920	153
62	63	29	61	11592	10980	153
63	64	27	62	11648	11036	153
64	65	25	63	11700	11088	153
65	66	23	64	11748	11136	153
66	67	21	65	11792	11180	153
67	68	19	66	11832	11220	153
68	69	17	67	11868	11256	153
69	70	15	68	11900	11288	153
70	71	13	69	11928	11316	153
71	72	11	70	11952	11340	153
72	73	9	71	11972	11360	153
73	74	7	72	11988	11376	153
74	75	5	73	12000	11388	153
75	76	3	74	12008	11396	153
76	77	1	75	12012	11400	153
77	52	1	49	5512	4900	153
78	51	3	48	5508	4896	153
79	50	5	47	5500	4888	153
80	49	7	46	5488	4876	153
81	48	9	45	5472	4860	153
82	47	11	44	5452	4840	153
83	46	13	43	5428	4816	153
84	45	15	42	5400	4788	153
85	44	17	41	5368	4756	153

86	43	19	40	5332	4720	153
87	42	21	39	5292	4680	153
88	41	23	38	5248	4636	153
89	40	25	37	5200	4588	153
90	39	27	36	5148	4536	153
91	38	29	35	5092	4480	153
92	37	31	34	5032	4420	153
93	28	1	22	1624	1012	153
94	27	3	21	1620	1008	153
95	26	5	20	1612	1000	153
96	25	7	19	1600	988	153
97	24	9	18	1584	972	153
98	23	11	17	1564	952	153
99	22	13	16	1540	928	153
100	21	1	12	924	312	153
101	20	3	11	920	308	153
102	19	5	10	912	300	153
103	18	7	9	900	288	153

Hence, there are in total 103 pairs of Pythagorean triangles, out of which there are 11 primitive pairs of Pythagorean triangle, 31 pairs of non-primitive Pythagorean triangles and in the remaining 61 pairs, one is primitive and the other is non-primitive

Observation

For every m, q, p the expression $3(P_1-P_2)/34$ represents the nasty number.

A similar observation regarding a 4-digit (8208) and 5-digit (54748) narcissistic numbers are presented below in Table: 2.

For simplicity and understanding, the total number of pairs of Pythagorean triangles satisfying the relation,

$$m + p + q = \text{Narcissistic number}$$

Is exhibited in Table-2.

Table 2: Examples

Narcissistic number	Pairs of Pythagorean triangles	Pairs of non-primitive Pythagorean triangles	Pairs of primitive Pythagorean triangles	Pair of one primitive and other non- primitive Pythagorean triangles	Observation representing Nasty Number
8208	1367	696	496	175	$\frac{2(P_1-P_2)}{19}$
54748	9124	4635	2321	2168	$82122(P_1-P_2)$

Conclusion

In this communication, pairs of Pythagorean triangles such that, in each pair, the difference between their perimeters representing 4 times the Narcissistic numbers are presented. It is observed that there are only finitely many Pythagorean triangles satisfying the property under consideration. The total numbers of pairs of primitive and non-primitive Pythagorean triangles are also given.

To conclude, one may attempt to determine pairs of Pythagorean triangles, where in each pair, the difference between their areas is represented by Narcissistic number or by other special number patterns namely, Sphenic, Harshad and so on.

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