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Connection between special Pythagorean triangles and Jarasandha numbers

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Abstract
We present Special Pythagorean Triangles in connection with the Jarasandha numbers. Also we present the number of primitive and non-primitive Pythagorean triangles & some special cases are also discussed. A few interesting relations between the numbers and some special number patterns are presented.

Keywords: Pythagorean triangles, Jarasandha numbers, Primitive and Non-primitive Pythagorean triangles

1. Introduction
Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of Mathematics. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In Number theory, Pythagorean triangles have been a matter of interest to various mathematicians. For an extensive variety of fascinating problems one may refer [1-5]. Apart from the polygonal numbers we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [6-9].
In [10-11], special Pythagorean triangles connected with Polygonal numbers and Nasty numbers are obtained. Recently in [12], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are obtained.
In this communication, we present Special Pythagorean Triangles in connection with the Jarasandha numbers. Also we present the number of primitive and non-primitive Pythagorean triangles & some special cases are also discussed. A few interesting relations between the numbers and some special number patterns are presented.

2. Basic Definitions
Definition 1
The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y and z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$
Also, in Pythagorean triangle $T(x, y, z)$: $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2
Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

3. Notations
$T_{m,n}$ = Polygonal number of rank 'n' with sides 'm'.

CS_n = Centered Square number of rank 'n'.

O_n = Octahedral number of rank 'n'.

SO_n = Stella octangula number of rank 'n'.

$C_{m,n}$ = Centered m-gonal number of rank 'n'.

Gno_n = Gnomonic number of rank 'n'.

$Star_n$ = Star number of rank 'n'.

4. Jarasandha Numbers
In our Indian epic Mahabharatha, we come across a Person named 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha.

Consider a number of the form XC. This may split as two numbers X and C and if these numbers are added and squared we get the same number XC.

(i.e) $XC = (X+C)^2 = XC$

Note: If C is an n-digit number, then $(X+C)^2 = (10^n)(X) + C$

5. Method of Analysis

Special Pythagorean Triangles

Case 1:

When m, n are of Jarasandha number, we get six Pythagorean Triangles.

Table 1: Pythagorean Triangles with m and n of Jarasandha Number

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
2025	81	4094064	328050	4107186	16761360040000	107616802500	16868976840000
3025	81	9144064	490050	9157186	83613906440000	240149002500	83854055440000
9801	81	96053040	1587762	96066162	9226186493000000	2520988169000	9228707481000000
3025	2025	5050000	12251250	13251250	25502500000000	15009312660000	175595626600000
9801	2025	91958976	39694050	100160226	8456453267000000	1575617605000000	10032070870000000
9801	3025	86908976	59296050	105210226	7553170109000000	3516021546000000	11069191660000000

Thus it is seen that there are 6 Pythagorean triangles. Out of these 6 Pythagorean triangles, All the triangles are non-primitive triangles.

Case 2:

1. When x = 81(2-digit Jarasandha number), then $x = m^2 - n^2 = 81$

Table 2: Pythagorean Triangles with $x = 81$ (2-digit Jarasandha Number)

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
15	12	81	360	369	6561	129600	136161
41	40	81	3280	3281	6561	10758400	10764961

Thus it is seen that there are 2 Pythagorean triangles. Of these 2 Pythagorean triangles, 1 is a Primitive triangle and other is non-primitive triangle.

2. When x = 2025(4-digit Jarasandha number), then $x = m^2 - n^2 = 2025$

Table 3: Pythagorean Triangles with $x = 2025$ (4-digit Jarasandha Number)

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
51	24	2025	2448	3177	4100625	5992704	10093329
53	28	2025	2968	3593	4100625	8809024	12909649
75	60	2025	9000	9225	4100625	81000000	85100625
117	108	2025	25272	25353	4100625	638673984	642774609
205	200	2025	82000	82025	4100625	6724000000	6728100625
339	336	2025	227808	227817	4100625	51896484860	51900585490
1013	1012	2025	2050312	2050313	4100625	4203779297000	4203783398000

Thus it is seen that there are 7 Pythagorean triangles. Of these 7 Pythagorean triangles, 2 triangles are Primitive and remaining 5 triangles are non-primitive triangles.

3. When x = 3025(4-digit Jarasandha number), then $x = m^2 - n^2 = 3025$

Table 4: Pythagorean Triangles with $x = 3025$ (4-digit Jarasandha Number)

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
73	48	3025	7008	7633	9150625	49112064	58262689
143	132	3025	37752	37873	9150625	1425213504	1434364129
305	300	3025	183000	183025	9150625	33489000000	33498150630
1513	1512	3025	4575312	4575313	9150625	20933479900000	20933489050000

Thus it is seen that there are 4 Pythagorean triangles. Of these 4 Pythagorean triangles, 2 triangles are Primitive and remaining two are non-primitive triangles.

4. When x = 9801(4-digit Jarasandha number), then $x = m^2 - n^2 = 9801$

Table 5: Pythagorean Triangles with $x = 9801$ (4-digit Jarasandha Number)

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
101	20	9801	4040	10601	96059601	16321600	112381201
165	132	9801	43560	44649	96059601	1897473600	1993533201
195	168	9801	65520	66249	96059601	4292870400	4388930001
451	440	9801	396880	397001	96059601	157513734400	157609794000
549	540	9801	592920	593001	96059601	351554126400	351650186000
1635	1632	9801	5336640	5336649	96059601	28479726490000	28479822550000
4901	4900	9801	48029800	48029801	96059601	2306861688000000	2306861784000000

Thus it is seen that there are 7 Pythagorean triangles. Of these 7 Pythagorean triangles, 2 triangles are Primitive and remaining 5 are non-primitive triangles.

Case 3:

When y = Jarasandha number, then $y = 2mn$.

Since we had taken only the Jarasandha numbers 81, 2025, 3025 & 9801. All these numbers are odd, so for y = Jarasandha number we get no Pythagorean triangles for these numbers.

Case 4:

When z = Jarasandha number, then we get Pythagorean triangles only for the Jarasandha numbers 2025 & 3025.

I. $z = m^2 + n^2 = 2025$, we get one Pythagorean triangle.

II. $z = m^2 + n^2 = 3025$, we get one Pythagorean triangle.

Table 6: Pythagorean Triangles with $z = 2025$ **&** 3025 (4-digit Jarasandha Number)

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
36	27	567	1944	2025	321489	3779136	4100625
44	33	847	2904	3025	717409	8433216	9150625

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangles are non-primitive.

Case 5:

Hypotenuse and one leg are consecutive and the other leg equals Jarasandha number.

When hypotenuse and one leg are consecutive, then either $z = x+1$ **or** $z = y+1$.

1. If $z = x+1$ then we get,

$$m^2 + n^2 = m^2 - n^2 + 1$$

$$\Rightarrow 2n^2 = 1$$

Which gives n as irrational number, which is not possible.

2. If $z = y+1$ then we get,

$$m^2 + n^2 = 2mn + 1$$

$$\Rightarrow (m-n)^2 = 1$$

$$\Rightarrow m = n+1$$

$$\therefore \quad x = 2n+1, \quad y = 2n^2 + 2n, \quad z = 2n^2 + 2n + 1$$

Taking x = Jarasandha number, and y, z are consecutive we have the following Table7.

Table 7: Pythagorean Triangles with x = Jarasandha number, and $z = y+1$

m	n	x	y	z	x^2	y^2	$x^2+y^2=z^2$
41	40	81	3280	3281	6561	10758400	10764961
1013	1012	2025	2050312	2050313	4100625	4203779297000	4203783398000
1513	1512	3025	4575312	4575313	9150625	20933479900000	20933489050000
4901	4900	9801	48029800	48029801	96059601	2306861688000000	2306861784000000

Thus it is seen that there are 4 Pythagorean triangles. Of these, All the triangles are primitive.

6. Observations

The following observations are made:

1. $4x+3y+5z$ is a Perfect square.
2. $12(x+z)$ represents a Nasty number.
3. $y(x-z+1)+2m\,So_n=0$.
4. $x+2y+z-4T_{3,m}-2m\ Gno_n=0$.
5. $n(z-x)+y-3O_n-n\,Gno_m=0$.
6. $y+m(x-z)+4C_{m,n}\equiv 0 \pmod 4$.
7. $y-m(z-x)+mCS_n\equiv 0 \pmod m$.
8. $x+y+z-T_{6,m}-m\,Gno_n\equiv 0 \pmod{2m}$.
9. $3(x+z)-Star_m-3\,Gno_m\equiv 0 \pmod 2$.
10. $2x+4y-3z+T_{12,n}-4n\,Gno_m\equiv 0 \pmod{m^2}$.

7. Conclusion

To conclude, one may search for the connections between the Pythagorean triangles and other Jarasandha numbers of higher order and other number patterns.

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