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# On supra regular generalized star star b- closed contra continuous functions and contra – irresolute functions and Supra $rg^{**}b$ separation axioms in supra topological spaces

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**Abstract**

In this paper we introduce the concept of contra $rg^{**}b^{\mu}$- continuous functions, almost contra $rg^{**}b^{\mu}$- continuous functions and contra $rg^{**}b^{\mu}$-irresolute. We obtain the basic properties and their relationship with other forms of contra supra continuous functions in supra topological spaces.

**Keywords:** contra $rg^{**}b^{\mu}$- continuous function, contra $rg^{**}b^{\mu}$-irresolute, almost contra $rg^{**}b^{\mu}$- continuous function and perfect contra $rg^{**}b^{\mu}$-irresolute

## 1. Introduction

In 1983, Mashhour et al., [8] introduced Supra topological spaces and studied S- continuous maps and S*- continuous maps. In the year 1989, [9] Noiri, introduced, on almost continuous functions.

During the year 1999, Ganster and Rielly, [4] came up with the idea of, on almost S- Continuity. In the year 1996, Andrijevic [1], introduced b- open sets. In 1996, Dontchev [3] presented a new notation of continuous function called contra- continuity in topological spaces. In 2008, Devi, Sampathkumar and Caldas [2], defined supra $\alpha$ open sets and S$\alpha$- continuous maps. In 2012, Jamal M. Mustafa, Hamzeh A. Qoqazeh [5] introduced a new class of supra D-sets and associated separation axioms in supra topological space.

During the year, 2015, Krishnaveni and Vigneshwaran [6] introduced Contra $bT^{\mu}$ - continuous function in supra topological spaces. In 2016 [7], Ludi Jancy Jenifer and Indirani introduced Supra regular generalized star star b- closed set in Supra topological spaces.

The purpose of this paper is to introduce the concept of contra $rg^{**}b^{\mu}$- continuous functions and contra $rg^{**}b^{\mu}$-irresolute and to study some of its basic properties. Also we define almost contra $rg^{**}b^{\mu}$- Continuous function and perfect contra $rg^{**}b^{\mu}$-irresolute function and investigate their relationship with other functions in supra topological spaces.

## 2. Preliminaries

**Definition 2.1** [8, 10]

Let X be a non-empty set. The subfamily $\mu \subseteq \wp(X)$ where $\wp(X)$ is the power set of X is said to be a supra topology on X if $X \in \mu$, $\emptyset \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ). Complements of supra open sets are called supra closed sets.

**Definition 2.2** [8]

Let A be a subset of (X, μ). Then the supra closure of A is denoted by $cl^{\mu}(A) = \cap \{ B / B$ is a supra closed set and $A \subseteq B\}$.

**Definition 2.3** [8]

Let A be a subset of (X, μ). Then the supra interior of A is denoted by $int^{\mu}(A) = \cup\{ B / B$ is a supra open set and $A \supseteq B\}$.

**Definition 2.4** [8]

Let (X, μ) be a topological space and μ be a supra topology on X. μ is supra topology associated with τ if $\tau \subseteq \mu$.

**Definition 2.5** [7]

A function f: $(X, \mu) \rightarrow (Y, \sigma)$ is called contra $rg^{**}b^{\mu}$-continuous function if $f^{-1}$(V) is $rg^{**}b^{\mu}$-closed set of X for every supra open set V of Y.

**Definition 2.6** [7]

A subset A of a supra topological space$(X,\mu)$ is called supra regular generalized star star b-closed set (briefly $rg^{**}b^{\mu}$-closed set) if $rg^{*}bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.

**Definition 2.7** [7]

A function f: $(X,\mu) \rightarrow (Y,\sigma)$ is called $rg^{**}b^{\mu}$-continuous if $f^{-1}$(V) is $rg^{**}b^{\mu}$closed set of $(X,\mu)$ for every closed set V of $(Y,\sigma)$.

**Definition 2.8** [7]

A function f: $(X,\mu) \rightarrow (Y,\sigma)$ is called $rg^{**}b^{\mu}$-irresolute if $f^{-1}$(V) is $rg^{**}b^{\mu}$closed ($rg^{**}b^{\mu}$-open)in X for every $rg^{**}b^{\mu}$-closed ($rg^{**}b^{\mu}$-open) set V of Y.

**Definition 2.9** [3]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called contra continuous if $f^{-1}$ (V) is supra closed in(X, τ) for every supra open set V of (Y, σ).

**Definition 2.10** [7]
A supra topological space (X,$\mu$) is called $T^{\mu}_{rg^{**}b}$-space, if every $rg^{**}b^{\mu}$-closed set is supra closed set.

## 3. Contra- $rg^{**}b^{\mu}$-continuous function

**Definition 3.1**
A function f: (X, τ) → (Y, σ) is called contra $rg^{**}b^{\mu}$ -continuous functions if $f^{-1}(V)$ is $rg^{**}b^{\mu}$-closed in (X, τ) for every supra open set V of (Y, σ).

**Theorem 3.2**
Every contra- supra continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.

**Proof:** Let f:(X,$\mu$)→(Y,$\sigma$) be contra supra continuous and V be a supra open set in (Y,$\sigma$).Since f is contra supra continuous, $f^{-1}(V)$ is supra closed in (X,$\mu$).Also every supra closed set is $rg^{**}b^{\mu}$ —closed set,$f^{-1}(V)$ is $rg^{**}b^{\mu}$ —closed set in (X,$\mu$).Therefore f is contra $rg^{**}b^{\mu}$continuous map.

**Example 3.3**
Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ , {b},{d},{b,d},{a,c,d},{a,b,c},{b,c,d}} and $\mu_2$ ={Y, ∅ , {d},{a,c},{a,d},{c,d},{a,c,d}}.Let f:(X, $\mu_1$) → (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d.
Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra supra continuous. Since for a supra open set {a,d} in Y, $f^{-1}$ ({a,d})={a,d} is not supra -closed in (X,$\mu_1$).

**Theorem 3.4**
i) Every contra-$s^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
ii) Every contra-$\alpha^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely. (iii)Every contra-supra- pre-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iii) Every contra-$r^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.

**Proof:** The proof is similar to theorem 3.2

**Example 3.5**
i) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ , {a},{c},{a,c},{a,c,d},{a,b,d},{b,c,d}} and $\mu_2$ ={Y, ∅ , {b},{a,b},{b,d},{c,d},{a,b,d},{b,c,d}}.Let f:(X, $\mu_1$)→(Y, $\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c, f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra $s^{\mu}$ continuous. Since for a supra open set {a,b} in Y,$f^{-1}$({a,b})={a,b} is not supra –semi -closed in (X,$\mu_1$).
ii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ , {a},{b},{a,b},{a,c,d},{a,b,d},{b,c,d}} and $\mu_2$ ={Y, ∅ , {b},{b,c},{b,d},{c,d},{b,c,d}}.Let f:(X, $\mu_1$)→(Y, $\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-$\alpha^{\mu}$ continuous. Since for a supra open set {b,c} in Y,$f^{-1}$({b,c})={b,c} is not supra –$\alpha$- closed in (X,$\mu_1$).
iii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ , {a},{d},{a,d},{a,c,d},{a,b,c},{b,c,d}} and $\mu_2$ ={Y, ∅ ,{d},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}}.Let f:(X, $\mu_1$ ) → (Y, $\mu_2$ ) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-supra-pre continuous. Since for a supra open set {a,b,d} in Y,$f^{-1}$({a,b,d})={a,b,d} is not supra pre-closed in (X,$\mu_1$).
iv) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ ,{b},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}, {a,c,d},{a,b,d},{b,c,d}} and $\mu_2$ ={Y, ∅ ,{c},{a,c},{a,d},{b,c},{a,c,d},{a,b,c}}.Let f:(X, $\mu_1$)→(Y, $\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.
Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-$r^{\mu}$ continuous. Since for a supra open set {c} in Y,$f^{-1}$({c})={c} is not supra r-closed in (X,$\mu_1$).

**Theorem 3.6**
i) Every contra-$b^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
ii) Every contra-$g^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iii) Every contra-$sg^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$ -continuous but not conversely.
iv) Every contra-$gs^{\mu}$-continuous function is contra-$rg^{**}b^{\mu}$-continuous but not conversely

**Proof:** The proof is similar to theorem 3.2

**Example 3.7**
i) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ , {a},{d},{a,d},{a,b,d},{b,c,d}} and $\mu_2$ ={Y, ∅ ,{a},{b}, {a,b},{a,c,d}, {a,b,c},{b,c,d}}.Let f:(X,$\mu_1$)→ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-$b^{\mu}$ continuous. Since for a supra open set {a,c,d} in Y,$f^{-1}$({a,c,d})={a,c,d} is not supra b-closed in (X,$\mu_1$).
ii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ ,{b},{c},{b,c},{a,b,c},{a,b,d}, {a,c,d}} and $\mu_2$={Y, ∅ , {d},{a,b},{b,d}, {c,d},{b,c,d},{a,b,d}}.Let f:(X,$\mu_1$)→ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-$g^{\mu}$ continuous. Since for a supra open set {a,b} in Y,$f^{-1}$({a,b})={a,b} is not supra g-closed in (X,$\mu_1$).
iii) Let X=Y={a,b,c} with $\mu_1$={X,∅,{a},{a,b},{b,c}} and $\mu_2$ ={Y,∅,{b},{a,c},{b,c}}.Let f:(X,$\mu_1$)→ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c.Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra-$sg^{\mu}$ continuous. Since for a supra open set {b} in Y, $f^{-1}$({b})={b} is not supra sgclosed in (X,$\mu_1$).
iv) Let X=Y={a,b,c,d} with $\mu_1$ ={X, ∅ ,{c},{d},{c,d},{a,b,c}{a,c,d},{a,b,d}} and $\mu_2$ ={Y, ∅ , {a},{c},{a,c},{a,b,d},{b,c,d},{a,b,c}}.Let f:(X, $\mu_1$)→ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d.Hence f is

contra- $rg^{**}b^{\mu}$ -continuous but not contra- $gs^{\mu}$ continuous. Since for a supra open set {a,c} in Y, $f^{-1}$ ({a,c})={a,c} is not supra gs-closed in (X,$\mu_1$).

**Theorem 3.8**

i) Every contra $g\alpha^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
ii) Every contra $\alpha g^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iii) Every contra $gp^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iv) Every contra $gr^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely

**Proof:** The proof is similar to theorem 3.2

**Example 3.9**

i) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ ,{b},{d},{b,d},{a,b,c}{a,c,d},{a,b,d}} and $\mu_2$ ={Y, $\emptyset$ ,{c},{a,d},{b,c},{c,d},{a,c,d},{b,c,d}}.Let f:(X, $\mu_1$) $\rightarrow$ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d.Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $g\alpha^{\mu}$ continuous. Since for a supra open set {a,d} in Y, $f^{-1}$({a,d})={a,d} is not supra g$\alpha$-closed in (X,$\mu_1$).
ii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {a},{c},{a,c},{a,b,c}{a,c,d}, {a,b,d}, {b,c,d}} and $\mu_2$ ={Y,$\emptyset$,{d},{a,d},{b,c},{b,d},{a,b,d},{b,c,d}}.Let f:(X,$\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.Hence f is contra- $rg^{**}b^{\mu}$ -continuous but not contra- $\alpha g^{\mu}$ continuous. Since for a supra open set {b,c} in Y, $f^{-1}$ ({b,c})={b,c} is not supra $\alpha$g-closed in (X,$\mu_1$).
iii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ ,{a},{a,c},{a,d}{c,d},{a,c,d}} and $\mu_2$ ={Y, $\emptyset$ ,{a},{a,b},{b,c},{a,b,c}}.Let f:(X, $\mu_1$) $\rightarrow$ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $gp^{\mu}$ continuous. Since for a supra open set {a} in Y, $f^{-1}$({a})={a} is not supra $gp$-closed in (X,$\mu_1$).
iv) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {a},{d},{a,d},{a,b,c},{a,b,d},{b,c,d}} and $\mu_2$ ={Y, $\emptyset$ , {b,c},{c,d},{b,d},{b,c,d}}.Let f:(X, $\mu_1$) $\rightarrow$ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $gr^{\mu}$ continuous. Since for a supra open set {b,c} in Y, $f^{-1}$({b,c})={b,c} is not supra gr-closed in (X,$\mu_1$).

**Theorem 3.10**

i) Every contra- $g^{*\mu}$-continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
ii) Every contra- $g^{*}s^{\mu}$ -continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iii) Every contra- $g^{\#\mu}$-continuous function is contra- $rg^{**}b^{\mu}$ -continuous but not conversely.
iv) Every contra- $g^{\#}s^{\mu}$ -continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.

**Proof:** The proof is similar to theorem 3.2

**Example 3.11**

i) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {b},{c},{b,c},{a,c,d},{a,b,d}, {b,c,d}} and $\mu_2$ ={Y, $\emptyset$ , {a},{a,c},{a,d},{b,d},{a,c,d},{a,b,d}}.Let f:(X,$\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$ -continuous but not contra- $g^{*\mu}$ continuous. Since for a supra open set {a,d} in Y, $f^{-1}$ ({a,d})={a,d} is not supra $g^{*}$-closed in (X,$\mu_1$).
ii) Let X=Y={a,b,c,d} with $\mu_1$={X,$\emptyset$,{b},{a,b},{a,c},{a,d},{a,b,c}{a,c,d},{a,b,d}} and $\mu_2$ ={Y, $\emptyset$ , {c},{b,c},{b,d},{c,d},{b,c,d}}.Let f:(X, $\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$ -continuous but not contra- $g^{*}s^{\mu}$ continuous. Since for a supra open set {b,c,d} in Y, $f^{-1}$ ({b,c,d})={b,c,d} is not supra $g^{*}$s-closed in (X,$\mu_1$).
iii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {b},{c},{b,c},{b,c,d},{a,b,d}} and $\mu_2$={Y, $\emptyset$ ,{c},{a,c},{a,d},{c,d},{a,c,d}}.Let f:(X,$\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $g^{\#\mu}$ –continuous. Since for a supra open set {a,c} in Y, $f^{-1}$({a,c})={a,c} is not supra $g^{\#}$-closed in (X,$\mu_1$).
iv) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {c},{d},{c,d},{b,c,d},{a,b,d}} and $\mu_2$={Y, $\emptyset$ ,{a},{a,c},{a,d},{b,c},{a,c,d},{a,b,c}}.Let f:(X, $\mu_1$) $\rightarrow$ (Y, $\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $g^{\#}s^{\mu}$ –continuous. Since for a supra open set {a,c} in Y, $f^{-1}$({a,c})={a,c} is not supra $g^{\#}s$-closed in (X,$\mu_1$).

**Theorem 3.12**

i) Every contra $rg^{*}b^{\mu}$ -continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
ii) Every contra $g^{*}b^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$ -continuous but not conversely.
iii) Every contra $g\alpha b^{\mu}$ -continuous function is contra- $rg^{**}b^{\mu}$-continuous but not conversely.
iv) Every contra $sgb^{\mu}$-continuous function is contra- $rg^{**}b^{\mu}$ -continuous but not conversely.

**Proof:** The proof is similar to theorem 3.2

**Example 3.13**

i) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {b},{d},{b,d},{a,b,c},{a,b,d}} and $\mu_2$={Y, $\emptyset$ ,{d},{a,c},{a,d},{b,c},{b,c,d},{a,c,d},{a,b,c}}.Let f:(X,$\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d.Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $rg^{*}b^{\mu}$ –continuous. Since for a supra open set {b,c,d} in Y, $f^{-1}$ ({b,c,d})={b,c,d} is not supra $rg^{*}b$ –closed in (X,$\mu_1$).
ii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ ,{a},{d},{a,d},{a,b,c},{b,c,d}} and $\mu_2$ ={Y, $\emptyset$ ,{b},{c},{b,c},{a,b,c},{a,c,d}}.Let f:(X,$\mu_1$) $\rightarrow$ (Y,$\mu_2$) be defined as

f(a)=a,f(b)=b,f(c)=d,f(d)=c. Hence f is contra-$rg^{**}b^{\mu}$-continuous but notcontra- supra-$g^*b$-continuous.Since for a supra open set {a,c,d} in Y, $f^{-1}$({a,c,d})={a,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$g^*b$-closed in (X,$\mu_1$).

iii) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ ,{c},{d},{c,d},{a,c,d},{a,b,d}} and $\mu_2$ ={Y, $\emptyset$ ,{a},{a,b},{b,c},{b,d},{b,c,d},{a,b,d},{a,b,c}}.Let f:(X, $\mu_1$ ) $\rightarrow$ (Y, $\mu_2$ ) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d.Hence f is contra- $rg^{**}b^{\mu}$-continuous but not contra- $g\alpha b^{\mu}$ –continuous. Since for a supra open set {b.d} in Y,$f^{-1}$({b,d})={b,d} is not supra g$\alpha b$-closed in (X,$\mu_1$).

iv) Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ ,{a},{c},{a,c},{a,c,d},{a,b,d}} and $\mu_2$={Y, $\emptyset$,{c},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}}.Let f:(X,$\mu_1$)$\rightarrow$ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$ -continuous but not contra- $sgb^{\mu}$ – continuous. Since for a supra open set {a,c} in Y,$f^{-1}$({a,c})={a,c} is not supra $sgb$-closed in (X,$\mu_1$).

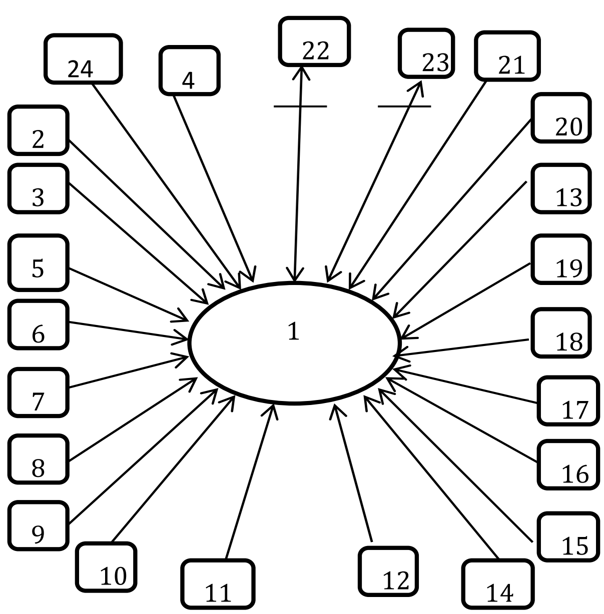
**Theorem 3.14**
Contra-$rg^{**}b^{\mu}$-continuity is independent from contra- r$g^{\mu}$-continuity and contra-$gpr^{\mu}$-continuity.

**Proof:** The proof is similar to theorem 3.2

**Example 3.15**
Let X=Y={a,b,c,d,e} with $\mu_1$ ={X, $\emptyset$ , {e},{d},{b,c},{c,d},{e,d},{b,c,e},{c,d,e}, {b,c,d},{a,b,c,e}, {a,c,d,e},{b,c,d,e}} and $\mu_2$ ={Y, $\emptyset$ , {c},{d},{c,d},{a,b},{b,c},{a,b,c}, {a,b,d},{c,b,d},{a,b,c,d}, {a,b,d,e},{c,b,d,e,}}.Let f:(X, $\mu_1$) $\rightarrow$(Y, $\mu_2$) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d,f(e)=e.
For a supra open set {b,c,d} and {c,b,d,e} in Y, $f^{-1}$ ({b,c,d})={b,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$gpr$-closed in (X,$\mu_1$) and $f^{-1}$({c,b,d,e})={c,b,d,e} is supra gpr-closed but not $rg^{**}b^{\mu}$-closed in (X,$\mu_1$).
Let X=Y={a,b,c,d,e} with $\mu_1$ ={X, $\emptyset$ , {e},{d},{e,d},{c,d},{c,e},{b,c,d},{b,c,e},{c,d,e},{a,b,c,d},{a, b,c,e},{b,c,d,e}} and $\mu_2$ ={Y, $\emptyset$ , {a,e},{b,e}, {a,d,e},{b,d,e},{a,b,e},{a,c,d,e}, {c,b,d,e},{a,b,d,e}} Let f:(X, $\mu_1$ ) $\rightarrow$ (Y, $\mu_2$ ) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d,f(e)=e.For a supra open set {a,d,e} and {b,d,e} in Y,$f^{-1}$({a,d,e})={a,d,e} is $rg^{**}b^{\mu}$ closed but not supra-$rg$ -closed in (X,$\mu_1$) and $f^{-1}$({b,d,e})={b,d,e} is supra rg-closed but not $rg^{**}b^{\mu}$-closed in (X,$\mu_1$).

**From the above discussions we have the following implications**

| | | | |
|---|---|---|---|
| 1. contra -$rg^{**}b^{\mu}$-continuous | 2.contra-$s^{\mu}$-continuous | 3.contra-$\alpha^{\mu}$-continuous | 4. contra-supra pre-continuous |
| 5.contra-$r^{\mu}$-continuous | 6.contra-$b^{\mu}$-continuous | 7.contra-$g^{\mu}$ -continuous | 8.contra-$sg^{\mu}$ -continuous |
| 9.contra-$gs^{\mu}$-continuous | 10. contra-g$\alpha^{\mu}$ -continuous | 11.contra-$\alpha g^{\mu}$ -continuous | 12.contra-$gp^{\mu}$ -continuous |
| 13.contra-$gr^{\mu}$-continuous | 14.contra-$g^{*\mu}$ –continuous | 15.contra-$g^*s^{\mu}$ -continuous | 16.contra-$g^{\#\mu}$ -continuous |
| 17.contra-$g^{\#}s^{\mu}$-continuous | 18. contra-r$g^*b^{\mu}$-continuous | 19.contra-$g^*b^{\mu}$-continuous | 20.contra-$g\alpha b^{\mu}$-continuous |
| 21.contra-$sgb^{\mu}$-continuous | 22.contra-$rg^{\mu}$-continuous | 23.contra-$gpr^{\mu}$-continuous | 24. contra -supra continuous |



**Theorem 3.16**
The composition of two contra $rg^{**}b^{\mu}$- continuous map need not be contra $rg^{**}b^{\mu}$-continuous. Let us prove the remark by the following example.

**Example 3.17**
Let X=Y=Z={a,b,c,d} with supra topologies $\theta$ ={X, $\emptyset$ , {a,c},{a,d}, {b,c}, {a,c,d}, {a,b,c}}, $\sigma$ ={Y, $\emptyset$ , {a,c},{a,d},{c,d},{a,c,d}}, $\eta$ ={Z, $\emptyset$ , {a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}} Let f:X ⟶ Y be defined by f(a)=d,f(b)=c,f(c)=b,f(d)=a. Define a map g:Y⟶Z be the identity map. Let g∘f:X⟶Z be th identity map. Both f and g are contra $rg^{**}b^{\mu}$ continuous but g∘f:X⟶Z is not a contra $rg^{**}b^{\mu}$-continuous. Since for open set {a,b,c} in Z, $(g \circ f)^{-1}$ ({a,b,c})={a,b,c} which is not contra $rg^{**}b^{\mu}$ continuous.

**Theorem 3.18**
If f:(X, τ ) → (Y, σ) is contra $rg^{**}b^{\mu}$- continuous function and g: (Y, σ) → (Z, $\theta$) is supra continuous function then composition gof is contra $rg^{**}b^{\mu}$ - continuous function.

**Proof:** obvious.

**Theorem 3.19**
If f:(X, τ ) → (Y, σ) is $rg^{**}b^{\mu}$- irresolute function and g: (Y, σ) → (Z, $\theta$) is contra $rg^{**}b^{\mu}$- continuous function then composition gof is contra $rg^{**}b^{\mu}$- continuous function.

**Proof:** obvious

**Remark 3.20**
The concept of $rg^{**}b^{\mu}$- continuity and contra $rg^{**}b^{\mu}$-continuity are independent as shown in the following example.

**Example 3.21**
Let X=Y={a,b,c} with $\mu_1$={X,$\emptyset$,{a,b},{a,c}} and $\mu_2$={Y,$\emptyset$, {c,b}, {a,c}}.Let f:(X,$\mu_1$)⟶ (Y,$\mu_2$) be an identity map..Hence f is $rg^{**}b^{\mu}$-continuous but not contra $rg^{**}b^{\mu}$- continuous. Since V={b,c} is supra open set in Y but $f^{-1}$ ({b,c}) = {b,c} is not $rg^{**}b^{\mu}$- closed set in X.
Let X=Y={a,b,c,d} with $\mu_1$ ={X, $\emptyset$ , {b},{d},{b,d},{a,c,d},{a,b,c},{b,c,d}} and $\mu_2$ ={Y, $\emptyset$ ,{d}, {a,c},{a,d},{c,d},{a,c,d}}.Let f:(X,$\mu_1$)⟶ (Y,$\mu_2$) be defined as f(a)=a,f(b)=b, f(c)=c,f(d)=d. Hence f is contra- $rg^{**}b^{\mu}$ - continuous,but not $rg^{**}b^{\mu}$ - continuous function, since V={b,d} is supra closed set in Y but $f^{-1}$ ({b,d}) ={b,d} is not $rg^{**}b^{\mu}$- closed set in X.

**Theorem 3.22**
If f:(X, τ ) → (Y, σ) is contra $rg^{**}b^{\mu}$- continuous function and X is $T^{\mu}_{rg^{**}b}$-space, then f is contra supra continuous.

**Proof:** Let V be supra open set in Y. Since f is contra $rg^{**}b^{\mu}$ - continuous function, then $f^{-1}$(V) is $rg^{**}b^{\mu}$- closed in X. Since X is $T^{\mu}_{rg^{**}b}$-space, we have every $rg^{**}b^{\mu}$- closed set is supra closed in X, then $f^{-1}$ (V) is supra closed in X. Therefore f is contra supra continuous function.

## 4. Almost contra $rg^{**}b^{\mu}$- continuous function

**Definition 4.1**
A map f:(X, τ ) → (Y, σ) is called almost contra $rg^{**}b^{\mu}$- continuous function if $f^{-1}$(V) is $rg^{**}b^{\mu}$- closed in (X, τ ) for every supra regular open set V in (Y, σ).

**Theorem 4.2**
Every contra supra continuous function is almost contra $rg^{**}b^{\mu}$- continuous function.

**Proof:** Obvious.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.3**
Let X=Y={a, b,c,d} and τ = {X, υ, {b},{c},{b,c},{a,c,d},{b,c,d}},σ={Y, υ, {b},{d},{b,d},{b,c,d},{a,c,d}}.f:(X, τ ) → (Y, σ) be the identity function. Here f is almost contra $rg^{**}b^{\mu}$- continuous, but it is not contra supra continuous, Since V={b,c,d} is supra open in Y but $f^{-1}$({b,c,d}) = {b,c,d} is not supra closed in X.

**Theorem 4.4**
Every contra $rg^{**}b^{\mu}$- continuous function is almost contra $rg^{**}b^{\mu}$- continuous function.

**Proof:** Obvious.

**Remark 4.5**
The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.6**
Let X=Y={a, b, c} and τ = {X, υ, {a},{a,b},{b,c},{a,c}}, σ = {Y, υ, {a},{a,b},{b,c}}.f:(X, τ ) → (Y, σ) be the identity function. Here f is almost contra $rg^{**}b^{\mu}$- continuous, but it is not contra $rg^{**}b^{\mu}$- continuous, Since V={a,b} is supra open in Y but $f^{-1}$ ({a,b}) = {a,b} is not $rg^{**}b^{\mu}$- closed in X.

**Theorem 4.7**
If a map f: X⟶Y from supra topological space X into a supra topological space Y. The following statement are equivalent.
(a) f is almost contra $rg^{**}b^{\mu}$- continuous.
(b) For every supra regular closed set F of Y, $f^{-1}$(F) is $rg^{**}b^{\mu}$- open in X.

**Proof:**
(a) ⟶(b) :Let F be a supra regular closed set in Y, then Y-F is a supra regular open set in Y. By (a) $f^{-1}$(Y-F) = X – $f^{-1}$(F) is $rg^{**}b^{\mu}$- closed set in X. This implies $f^{-1}$(F) is $rg^{**}b^{\mu}$ - open set in X. Therefore (b) holds.
(b) ⟶(a):Let G be a supra regular open set of Y. The Y-G is supra regular closed set of Y. By (b) $f^{-1}$(Y-G) is $rg^{**}b^{\mu}$- open in X. This implies X- $f^{-1}$(G) is $rg^{**}b^{\mu}$ - open in X, which implies $f^{-1}$(G) is $rg^{**}b^{\mu}$-closed set in X. Therefore (a) holds.

**Definition 4.8**
A space (X, τ ) is $rg^{**}b^{\mu}$- locally indiscrete if every $rg^{**}b^{\mu}$ - open ($rg^{**}b^{\mu}$- closed) set is supra closed(supra open) in (X, τ ).

**Theorem 4.9**
If f:(X, τ ) → (Y, σ) is $rg^{**}b^{\mu}$- continuous function and X is $rg^{**}b^{\mu}$ - locally indiscrete, then f is contra $rg^{**}b^{\mu}$ - continuous.

**Proof:** Let V be supra open set in Y. Since f is $rg^{**}b^{\mu}$- continuous function, then $f^{-1}(V)$ is $rg^{**}b^{\mu}$- open in X. Since X is $rg^{**}b^{\mu}$- locally indiscrete, then $f^{-1}(V)$ is supra closed set in X. We know that every supra closed set is $rg^{**}b^{\mu}$- closed set. Therefore $f^{-1}(V)$ is $rg^{**}b^{\mu}$- closed set in X. Hence f is contra $rg^{**}b^{\mu}$-continuous function.

**Theorem 4.10**
If f:(X, τ ) → (Y, σ) is a surjective $rg^{**}b^{\mu}$- irresolute and g: (Y, σ) → (Z, $\gamma$) be any function such that gof: (X, τ ) → (Z, $\gamma$ ) is contra $rg^{**}b^{\mu}$- -continuous function, iff g is contra $rg^{**}b^{\mu}$- continuous.

**Proof:** Suppose gof is contra $rg^{**}b^{\mu}$-continuous, Let V be a supra closed set in Z, then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $rg^{**}b^{\mu}$- open in (X, τ ). Since f is subjective and $rg^{**}b^{\mu}$- irresolute, then $f(gof)^{-1}(V)=f( f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is supra N-open in (Y, σ).Hence g is contra $rg^{**}b^{\mu}$- continuous function.
Conversely, suppose g is contra $rg^{**}b^{\mu}$- continuous, Let V be supra closed in Z, then $g^{-1}(V)$ is $rg^{**}b^{\mu}$ -open in Y. Since f is surjective and $rg^{**}b^{\mu}$ - irresolute, then $f^{-1}(g^{-1}(V))$ is $rg^{**}b^{\mu}$- open in X. Hence gof is contra $rg^{**}b^{\mu}$- continuous function.

**Theorem 4.11**
If f:(X, τ ) → (Y, σ) is a $rg^{**}b^{\mu}$- continuous and g: (Y, σ) → (Z, $\gamma$) is contra $rg^{**}b^{\mu}$- continuous function and (Y, σ) is $T^{\mu}_{rg^{**}b}$-space, then gof: (X, τ ) → (Z, $\gamma$) is contra $rg^{**}b^{\mu}$- continuous function.

**Proof:** Let V be any supra open set in Z, then $g^{-1}(V)$ is $rg^{**}b^{\mu}$ - closed set in Y. since Y is $T^{\mu}_{rg^{**}b}$space, $g^{-1}(V)$ is supra closed set in Y. Since f is $rg^{**}b^{\mu}$- continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $rg^{**}b^{\mu}$- closed set in X. Hence gof is contra $rg^{**}b^{\mu}$- continuous.

## 5. *Contra - irresolute function*

**Definition 5.1**
A function f:(X, τ ) → (Y, σ) is called contra $rg^{**}b^{\mu}$ - irresolute function if $f^{-1}(V)$ is $rg^{**}b^{\mu}$- closed in (X, τ) for every $rg^{**}b^{\mu}$- open set V in (Y, σ).

**Definition 5.2**
A function f: (X, τ ) → (Y, σ) is called perfectly contra $rg^{**}b^{\mu}$ - irresolute function if $f^{-1}(V)$ is $rg^{**}b^{\mu}$- closed and $rg^{**}b^{\mu}$- open in (X,$\tau$) for every $rg^{**}b^{\mu}$- open set V in (Y,$\sigma$).

**Theorem 5.3**
Every contra $rg^{**}b^{\mu}$- irresolute function is contra $rg^{**}b^{\mu}$- continuous.

**Proof:** obvious.

**Remark 5.4**
The converse of the above theorem need not be true. It is shown by the following example.

**Example 5.5**
Let X=Y= {a, b, c, d}, $\tau$ = {X, ∅ , {b},{d},{b,d},{b,c,d},{a,c,d}}, $\sigma$ ={ Y, ∅ ,{b}, {c},{b,c},{a,c,d},{a,b,c}}. A function f: (X, τ ) → (Y, σ) is defined by f(a) = a, f(b) = b, f(c) = c,f(d)=d.Here f is contra $rg^{**}b^{\mu}$- continuous but not contra $rg^{**}b^{\mu}$- irresolute. Since V = {b,c,d} is $rg^{**}b^{\mu}$ - open set in (Y, $\sigma$ ) and $f^{-1}$({b,c,d})={b,c,d} is not in $rg^{**}b^{\mu}$- closed set in (X,$\tau$).

**Theorem 5.6**
If f: (X, τ) → (Y, σ) is a $rg^{**}b^{\mu}$- irresolute and g: (Y,$\sigma$) →( Z, $\gamma$) is contra $rg^{**}b^{\mu}$- irresolute function, then gof : (X,$\tau$) →(Z, $\gamma$) is contra $rg^{**}b^{\mu}$- irresolute function.

**Proof:** obvious.

**Theorem 5.7**
If f: (X, τ) → (Y, σ) is a contra $rg^{**}b^{\mu}$- irresolute and g: (Y, $\sigma$) →(Z,$\gamma$) is $rg^{**}b^{\mu}$- irresolute function, then gof: (X,$\tau$) → (Z, $\gamma$) is contra $rg^{**}b^{\mu}$- irresolute function.

**Proof:** Obvious

**Theorem 5.8**
Every perfectly contra $rg^{**}b^{\mu}$- irresolute is contra $rg^{**}b^{\mu}$- irresolute function.

**Proof:** obvious.

**Remark**
The converse of the above theorem need not be true. It is shown by the following example.

**Example 5.9**
Let X=Y= {a,b,c,d} and $\tau$ = {X, ∅ , {c},{d},{c,d},{a,b,c},{a,b,d}}, $\sigma$ = {Y, ∅ ,{b}, {c},{b,c},{a,b,c},{a,b,d}} f: (X, τ ) → (Y, σ) be a function defined by f(a) = a, f(b) = b, f(c)=c, f(d)=d. Here f is contra $rg^{**}b^{\mu}$- irresolute function but not perfectly contra $rg^{**}b^{\mu}$- irresolute function. Since V = {a,b} is $rg^{**}b^{\mu}$- open set in (Y, $\sigma$) and $f^{-1}$({a,b}) = {a,b} is not $rg^{**}b^{\mu}$- open set in (X,$\tau$).

**Theorem 5.10**
Every perfectly contra $rg^{**}b^{\mu}$ - irresolute is $rg^{**}b^{\mu}$ - irresolute function.

**Proof:** Obvious.

**Remark 5.11**
The converse of the above theorem need not be true. It is shown by the following example.

**Example 5.12**

Let X=Y= {a,b,c} and $\tau$ = {X,∅,{a},{a,b},{b,c}},$\sigma$= {Y,∅ ,{a},{a,b},{b,c},{c,a}}.f: (X, τ ) → (Y, σ) be a identity function. Here f is $rg^{**}b^{\mu}$- irresolute function but not perfectly contra $rg^{**}b^{\mu}$- irresolute function.Since V = {a,b} is $rg^{**}b^{\mu}$- open set in (Y,$\sigma$) and $f^{1}$({a,b}) = {a,b} is not $rg^{**}b^{\mu}$- closed in (X,$\tau$).

## 6. Applications

### 6.1 Supra $rg^{**}b$ separation axioms

**Definition 6.1.1**

Let (X, μ) be a supra topological space, then:

i) X is **S-$rg^{**}b$- $T_0$** if for every two distinct points x and y in X there exists a supra $rg^{**}b$ open set U that contains only one of the points x and y.
ii) X is **S -$rg^{**}b$ -$T_1$** if for every two distinct points x and y in X there exists two supra $rg^{**}b$ open sets U and V such that x ∈ U, y ∉ U and y ∈ V, x ∉V.
iii) X is **S-$rg^{**}b$ -$T_2$** if for every two distinct points x and y in X there exists two disjoint supra $rg^{**}b$ open sets U and V such that x ∈ U and y ∈ V.

**Remark 6.1.2**

Every S-$rg^{**}b$- Ti space is S-$rg^{**}b$ -$T_{i-1}$ for each i = 1, 2 but the converse need not be true.

**Example 6.1.2**

Let X = {a, b, c, d}, μ = {X,∅,{a},{d},{a,d},{a,c,d},{b,c,d}} $rg,^{**}b$ O(X, $\mu_1$)={X, ∅, {a}, {c},{d},{a,c},{a,d}, {b,d}, {c,d}, {a,b,d},{a,c,d},{b,c,d}}.Then (X, μ) is an S-$rg^{**}b$-$T_0$ space but not S-$rg^{**}b$-$T_1$. Since for the open subset {b,d} of X, there are no two supra $rg^{**}b$ open sets U and V such that b ∈ U, d ∉ U and d ∈ V, b ∉V.

**Example 6.1.3**

Let X = {a, b, c}, μ = {X,∅,{a}, {a, b}, {a, c}, {b, c}},$rg^{**}b$ O(X,$\mu_1$)={X,∅,{a},{a,b},{b,c},{c,a}}.Then (X, μ) is an S-$rg^{**}b$-$T_1$ space but not S-$rg^{**}b$-$T_2$.

**Theorem 6.1.4**

A supra topological space (X, μ) is S-$rg^{**}b$ -$T_0$ if and only if for each pair of distinct points x, y in X, $rg^{**}b$cl$^{\mu}$({x}) ≠ $rg^{**}b$cl$^{\mu}$({y}).

**Proof:⇒)** Let (X, μ) be an S-$rg^{**}b$ -$T_0$ space and x, y be any two distinct points in X. There exists a supra $rg^{**}b$ open set U containing x or y, say x but not y. Then X − U is a supra $rg^{**}b$ closed set containing y but not x. Now, $rg^{**}b$cl$^{\mu}$ ({y}) ⊆ X − U, and therefore x ∉ $rg^{**}b$cl$^{\mu}$({y}). Hence $rg^{**}b$cl$^{\mu}$ ({x}) ≠ $rg^{**}b$cl$^{\mu}$ ({y}).

**⇐)** Suppose that x, y ∈ X, x≠ y and $rg^{**}b$cl$^{\mu}$ ({x})≠ $rg^{**}b$ cl$^{\mu}$ ({y}). Then there exists a point z ∈ X such that z belongs only to one of the sets $rg^{**}b$cl$^{\mu}$ ({x}) and $rg^{**}b$cl$^{\mu}$ ({y}), say z ∈ $rg^{**}b$cl$^{\mu}$ ({x}) but z ∉ $rg^{**}b$cl$^{\mu}$ ({y}). We claim that x ∉ $rg^{**}b$cl$^{\mu}$ ({y}). For, if x ∈ $rg^{**}b$cl$^{\mu}$ ({y}) then $rg^{**}b$cl$^{\mu}$ ({x}) ⊆ $rg^{**}b$cl$^{\mu}$ ({y}). This contradicts the fact that z ∉ $rg^{**}b$cl$^{\mu}$ ({y}). Consequently x belongs to the supra open set X − $rg^{**}b$cl$^{\mu}$ ({y}) to which y does not belong.

**Theorem 6.1.5**

A supra topological space (X, μ) is S-$rg^{**}b$ -$T_1$ if and only if the singletons are supra $rg^{**}b$ closed sets.

**Proof:** Let (X, μ) be S-$rg^{**}b$ -$T_1$ and let x be any point in X. Suppose y ∈ X\{x}, then x≠y, and so there exists a supra rg**b-open set U such that y ∈ U and x □ U. Consequently y ∈ U⊆ X\{x}, that is X\{x} is supra rg**b open.

Conversely, suppose {p} is supra rg**b-closed for every p ∈ X. Let x,y ∈ X with x≠y, Now x≠y implies y ∈ X\{x}.Hence X\{x} is a supra rg**b-open set contains y but not x. Similarly X\{y} is a supra rg**b-open set contains x but not y. Accordingly (X, μ) is a S-rg**b-$T_1$ space.

**Definition 6.1.6**

A supra topological space (X, μ) is called a supra $rg^{**}b$-symmetric space if for x and y in X, x ∈ $rg^{**}b$cl$^{\mu}$({y}) implies y ∈ $rg^{**}b$cl$^{\mu}$ ({x}).

**Theorem 6.1.7**

Let (X, μ) be a supra $rg^{**}b$- symmetric space. Then the following are equivalent:

1) (X, μ) is S-$rg^{**}b$- $T_0$;
2) (X, μ) is S-$rg^{**}b$- $T_1$.

**Proof:** It is enough to show that (1) ⇒ (2). Let x ≠ y. Since (X, μ) is S-$rg^{**}b$- $T_0$, we may assume that x ∈ U ⊆ X−{y} for some supra $rg^{**}b$ open set U. Then x ∉ $rg^{**}b$cl$^{\mu}$({y}) and hence y ∉ $rg^{**}b$cl$^{\mu}$ ({x}).Therefore there exists a supra $rg^{**}b$ open set V, such that y ∈ V ⊆ X − {x} and (X, μ) is an S-$rg^{**}b$- $T_1$ space.

**Theorem 6.1.8**

The following properties are equivalent:

1) X is S-$rg^{**}b$- $T_2$.
2) Let x ∈ X. For each y≠ x, there exists a supra $rg^{**}b$ open set U such that x ∈ U and y∉ $rg^{**}b$cl$^{\mu}$ (U).
3) For each x ∈ X, ∩{$rg^{**}b$cl$^{\mu}$ (U) : U is a supra $rg^{**}b$ open set with x ∈ U} = {x}.

**Proof:**

**(1) ⇒(2)** Let x ∈ X and y≠ x. Then there are disjoint supra $rg^{**}b$ open sets U and V such that x ∈ U and y ∈ V. Now X − V is supra $rg^{**}b$ closed with $rg^{**}b$cl$^{\mu}$ (U) ⊆ X − V and y ∉X − V and therefore y∉ $rg^{**}b$cl$^{\mu}$ (U).

**(2) ⇒(3)** If y∉{x}, then there exists a supra $rg^{**}b$ open set U such that x ∈ U and y∉ $rg^{**}b$cl$^{\mu}$ (U). So y∉ ∩{$rg^{**}b$cl$^{\mu}$ (U) : U is a supra $rg^{**}b$ open set with x ∈ U}.

**(3) ⇒(1)** If y≠x. By assumption we have ∩{$rg^{**}b$cl$^{\mu}$ (U) : U is a supra $rg^{**}b$ open set with x ∈ U} = {x}, then there exists a supra $rg^{**}b$ open set U such that x ∈ U, y∉ $rg^{**}b$cl$^{\mu}$ (U). Let V = X-$rg^{**}b$cl$^{\mu}$ (U), then V is a supra $rg^{**}b$ open set with y ∈ V and U ∩ V = ∅.

**Definition 6.1.9**

A subset A of a supra topological space (X, μ) is called a supra D−set if there are two supra open sets U and V such that U≠X and A = U − V.

Observe that every supra open set U different from X is a supra D−set if A = U and V=∅.

**Definition 6.1.10**
A supra topological space (X, μ) is called:
1) **S-$rg^{**}b$-D$_0$** if for any distinct pair of points x and y of X there exists a supra $rg^{**}b$ D-set in X containing x but not y or a supra $rg^{**}b$ D-set in X containing y but not x.
2) **S-$rg^{**}b$-D$_1$** if for any distinct pair of points x and y in X there exists a supra $rg^{**}b$ D-set in X containing x but not y and a supra $rg^{**}b$ D−set in X containing y but not x.
3) **S-$rg^{**}b$-D$_2$** if for any distinct pair of points x and y in X there exist disjoint supra $rg^{**}b$ Dsets G and E in X containing x and y, respectively.

**Remark 6.1.11**
Every proper supra $rg^{**}b$ -open set is a supra $rg^{**}b$ –D set. But the converse is not true in general as shown in the example below.

**Example 6.1.12**
Consider X={a,b,c,d} with μ={X, ∅ ,{a},{d},{a,d},{a,c,d},{b,c,d}}, $rg^{**}b$ O(X, $\mu_1$ )={X, ∅ , {a},{c},{d},{a,c},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}.Then U={b,c,d} ≠ X and V={a,c,d} are supra $rg^{**}b$ open sets in X and A=U\V={b},then we have A={b} is a supra $rg^{**}b$ –D set but it is not supra $rg^{**}b$ open.

**Remark 6.1.13**
1) If (X, μ) is S-$rg^{**}b$- T$_i$, then (X, μ) is S-$rg^{**}b$ -D$_i$, i = 0, 1, 2.
2) If (X, μ) is S-$rg^{**}b$ -D$_i$, then it is S-$rg^{**}b$- D$_{i-1}$, i = 1, 2.
However, S-$rg^{**}b$ –D$_1$ ↛ S-$rg^{**}b$ –T$_1$ and S-$rg^{**}b$ –D$_2$ ↛ S-$rg^{**}b$ –T$_2$ and shown in the examples below.

**Example 6.1.14**
Let X={a,b,c,d} with $\mu$={X,∅,{a},{d},{a,d},{a,c,d},{b,c,d}} then $rg^{**}b^{\mu}$ O(X, $\mu$ )={X, ∅ ,{a},{c},{d},{a,c},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Therefore the supra topological space (X,$\mu$) is S-$rg^{**}b$ –D$_1$ but not S-$rg^{**}b$ –T$_1$.
Let X={a,b,c} with $\mu$={X,∅,{a,b},{b,c}} then$rg^{**}b^{\mu}$ O(X, $\mu$ )={X, ∅ ,{b},{a,b},{a,c}, {b,c}}.Therefore the supra topological space (X,$\mu$) is S-$rg^{**}b$ –D$_2$ but not S-$rg^{**}b$ –T$_2$.

**Theorem 6.1.15**
A supra topological space (X, μ) is S-$rg^{**}b$ –D$_0$ if and only if it is S-$rg^{**}b$ –T$_0$.

**Proof:** Let (X, μ) be S-$rg^{**}b$ –D$_0$. Then for each distinct pair x, y in X, at least one of x, y, say x, belongs to a supra $rg^{**}b$ –D set U where y ∉ U. Let U = U$_1$ − U$_2$ such that U$_1$≠X and U$_1$ and U$_2$ are supra $rg^{**}b$ open sets in X. Then x ∈ U$_1$. For y ≠ U we have two cases:
(a) y U$_1$
(b) y ∈ U$_1$ and y ∈ U$_2$. In case (a), x ∈ U$_1$ but y∉ U$_1$; In case (b), y ∈ U$_2$ but x∉U$_2$.
Hence X is S$rg^{**}b$ –T$_0$.

The converse part of the theorem holds true by Remark 6.1.13.

**Theorem 6.1.16**
A supra topological space (X, μ) is S-$rg^{**}b$ –D$_1$ if and only if it is S-$rg^{**}b$ –D$_2$.

**Proof:** Suppose that X is S-$rg^{**}b$ –D$_1$. Then for each distinct pair x, y ∈ X, we have supra-$rg^{**}b$D−sets G$_1$, G$_2$ such that x ∈ G$_1$, y ∉ G$_1$; y ∈ G$_2$, x∉ G$_2$. Let G$_1$ = U$_1$ −U$_2$, G$_2$ = U$_3$ −U$_4$ where U$_1$, U$_2$, U$_3$ and U$_4$ are supra $rg^{**}b$ open sets such that U≠X and U$_3$ ≠ X. Since x ∉ G$_2$, it follows that either x∉ U$_3$ or x ∈ U$_3$ and x ∈ U$_4$. Now we consider two cases: **(a)**x ∉U$_3$.
Since y∉ G$_1$ we have two subcases:
**(a1)** y ∉U$_1$. Since x ∈ U$_1$ − U$_2$, it follows that x ∈ U$_1$ − (U$_2$ ∪ U$_3$) and y ∈ U$_3$−U$_4$ we have y ∈ U$_3$−(U$_1$∪U$_4$). Hence (U$_1$−(U$_2$ ∪U$_3$))∩(U$_3$−(U$_1$∪U$_4$)) = ∅.
**(a2)** y ∈ U$_1$ and y ∈ U$_2$. We have x ∈ U$_1$−U$_2$, y ∈ U$_2$ and (U$_1$−U$_2$)∩U$_2$ =∅. **(b)** x ∈ U$_3$ and x ∈ U$_4$. We have y ∈ U$_3$−U$_4$, x ∈ U$_4$ and (U$_3$−U$_4$)∩U4 =∅.
Therefore X is S-$rg^{**}b$- D$_2$.
The converse part of the theorem holds true by Remark 6.1.13.

**Theorem 6.1.17**
Let (X, $\mu$) and (Y, $\delta$) be two topological spaces and $\gamma$,$\theta$ be associated supra topologies with $\mu$ and $\delta$ respectively. Let f : (X, $\mu$ ) → (Y, $\delta$ ) be a supra $rg^{**}b$-irresolute surjective function and G be a supra-$rg^{**}b$ –D set in Y, then f$^{-1}$(G) is a supra-$rg^{**}b$ D-set in X.

**Proof:** Let G be a supra-$rg^{**}b$-D set in Y. Then there are supra $rg^{**}b$- open sets U$_1$ and U$_2$ in Y such that G = U$_1$ − U$_2$ and U$_1$ ≠Y. By the supra $rg^{**}b$ irresoluteness of f, f$^{-1}$(U$_1$) and f$^{-1}$(U$_2$) are supra $rg^{**}b$ open in X. Since U$_1$ ≠ Y, we have f$^{-1}$(U$_1$) ≠ X. Hence f$^{-1}$(G) = f$^{-1}$(U$_1$) − f$^{-1}$(U$_2$) is a supra $rg^{**}b$ D−set.

**Theorem 6.1.18**
Let (X, $\mu$) and (Y, $\delta$) be two topological spaces and $\gamma$,$\theta$ be associated supra topologies with $\mu$ and $\delta$ respectively. Let f : (X, $\mu$ ) → (Y, $\delta$ ) be a supra $rg^{**}b$-irresolute bijective function. If (Y, $\theta$) is S-$rg^{**}b$- D$_1$ then (X,$\gamma$ ) is also S-$rg^{**}b$ - D$_1$.

**Proof:** Suppose that Y is a S-$rg^{**}b$- D$_1$ space. Let x and y be any pair of distinct points in X. Since f is injective and Y is S-$rg^{**}b$-D$_1$, there exist supra $rg^{**}b$-D sets G$_x$ and G$_y$ of Y containing f(x) and f(y) respectively, such that f(y) ∉ G$_x$ and f(x) ∉ Gy. By Theorem 6.1.17, f$^{-1}$(G$_x$) and f$^{-1}$(G$_y$) are supra $rg^{**}b$-D sets in X containing x and y, respectively, such that y ∉ f$^{-1}$(G$_x$) and x∉ f$^{-1}$(G$_y$).
This implies that X is a S-$rg^{**}b$ -D$_1$ space.

**Theorem 6.1.19**
Let (X, $\mu$) and (Y, $\delta$) be two topological spaces and $\gamma$,$\theta$ be associated supra topologies with $\mu$ and $\delta$ respectively. Then (X, μ) is S-$rg^{**}b$ -D$_1$ if and only if for each pair of distinct points x, y ∈ X, there exists a supra $rg^{**}b$ irresolute subjective function f:(X,$\mu$ )→(Y,$\delta$), where (Y,$\theta$) is a S-$rg^{**}b$ -D$_1$ space such that f(x) and f(y) are distinct.

**Proof:** For every pair of distinct points of X, it suffices to take the identity function on X.

Conversely, Let x and y be any pair of distinct points in X. By assumption there exists a supra $rg^{**}b$ irresolute, surjective function f of a space (X, μ) onto a S-$rg^{**}b$-$D_1$ space (Y,$\theta$) such that f(x) $\neq$f(y). Therefore, there exist two supra $rg^{**}b$-D-sets $G_x$ and $G_y$ of Y such that f(x) $\in G_x$ and f(y) $\in G_y$ but f(y) $\notin G_x$ and f(x) $\notin G_y$. Since f is supra $rg^{**}b$- irresolute and surjective, by Theorem 6.1.17, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are distinct supra $rg^{**}b$ D–sets in X containing x and y, respectively, such that y$\notin f^{-1}(G_x)$ and x $\notin f^{-1}(G_y)$.This implies that X is a S-$rg^{**}b$-$D_1$ space.

## 7. References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