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An application of generalized two dimensional fractional cosine transform

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Abstract
An integral transform is useful if it allows one to turn a complicated problem into a simpler one. In this paper an application of fractional cosine transform to differential equation is presented.

Keywords: fractional Fourier transform, fractional Cosine transform, fractional Sine transform.

1. Introduction

Integral transforms play wide and important role in mathematical physics, theoretical physics. : Fourier and Mellin transform has many applications such as signal processing, algorithm, watermarking, pattern recognition, correlators, navigation, vowel recognition, cryptographic scheme, quantum calculus, radar system and have applications in agriculture, medical stream. Integral transforms provide a way to solve otherwise intractable physical problems. They work by expressing the equations of a physical system in a new form that can be solved with simple computation. An integral transform maps some function onto another one and as a consequence a function space into or onto another one. Operations in the original space are converted in general into operations in the image space. Integral transforms are therefore used in the first place if handling with the operations in the image space is easier to do or is better known as in the original space. As an example: the Laplace transforms converts differentiation in the space of definition into a simple algebraic operation in the image space. It will be clear however that having obtained results in the image space, these results have to be put back into the original space in order to have the possibility to interpret them in relation with the problem one started with. In applications the nature of the original space is more or less determined by the special properties of the functions under consideration. Operations in this space are often suggested by the mathematical description of the problems to be solved. In justifying the appeal of integral transforms to physicists, this opening sortie introduces a pair of problems amenable to solution by their use. Of course, they can't be solved properly ahead of introducing the transforms themselves; the intent is to explain the problems in sufficient depth to see the manner in which transforms are useful.
In our previous work we defined the following definitions.

1.1. Generalized two dimensional fractional Cosine transform

Two dimensional fractional Cosine transform with parameter α f(x, y) denoted by $F_C^\alpha(x,y)$ perform a linear operation given by the integral transform.

$$F_C^\alpha\{f(x,y)\}(u,v) = \int_0^\infty \int_0^\infty f(x,y) K_\alpha(x,y,u,v)\, dx\, dy \quad \ldots\ldots (1.1)$$

Where the kernel,

$$K_c^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\,.\cos(cosec\alpha.vy) \quad \ldots\ldots (1.2)$$

1.2. The test function space E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to $E(R^n)$ if for each copactset$I \subset S_{a,b}$, where,

$S_{a,b} = \{x,y : x,y \in R^n, |x| \le a, |y| \le b, a > 0, b > 0\}, I \in R^n$

$\gamma_{E_{p,q}}(\emptyset) = \sup_{x,y} \left| D_{x,y}^{p,q} \emptyset(x,y) \right| < \infty$ Where, p, q =1, 2, 3….

Thus $E(R^n)$ will denote the space of all$\emptyset \in E(R^n)$ with support contained in $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member ofE^*, the dual space of E.

In the present work, differential equation is solved by using two dimensional fractional cosine transform.

2. Distributional two-dimensional fractional Cosine transform

The two dimensional distributional fractional Cosine transform of $f(x,y) \in E^*(R^n)$ defined by

$$F_c^\alpha\{f(x,y)\} = F^\alpha(u,v) = \langle f(x,y), K_\alpha(x,y,u,v) \rangle \quad \ldots\ldots (2.1)$$

$$K_c^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\,.\cos(cosec\alpha.vy) \quad \ldots\ldots (2.2)$$

Where, RHS of equation (2.1) has a meaning as the application of $f \in E^{*}$to$K_{\alpha}(x,y,u,v) \in E$

3. Kernel to testing function space for two-dimensional fractional Cosine transform

To prove:-

For each $u, v \in R^{n}$ and $0 < \alpha \le \frac{\pi}{2}$, the function $k_{\alpha}(x,y,u,v)$ as in (equation -1) belongs to $E(R^{n})$ as a function of x, y

Solution: $\gamma_{E,p,q}\big(k_{\alpha}(x,y,u,v)\big) = \sup_{x \epsilon k, y \epsilon I}\left|D_x^p D_y^q k_{\alpha}(x,y,u,v)\right|$

$$\gamma_{E,p,q}\big(k_{\alpha}(x,y,u,v)\big) = \sup_{\substack{x \in k \\ y \in I}}\left|D_x^p D_y^q \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cot\alpha} cos(cosec\alpha.ux)cos(cosec\alpha.vy)\right|$$

$$\gamma_{E,p,q}\big(k_{\alpha}(x,y,u,v)\big) = \sup_{\substack{x \in k \\ y \in I}}\left|\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(u^2+v^2)cot\alpha} D_x^p e^{\frac{i}{2}(x^2)cot\alpha} cos(cosec\alpha.ux) D_y^q e^{\frac{i}{2}(y^2)cot\alpha} cos(cosec\alpha.vy)\right|$$

$$\gamma_{E,p,q}\big(k_{\alpha}(x,y,u,v)\big) = \sup_{\substack{x \in k \\ y \in I}}\left|\begin{array}{c}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(u^2+v^2)cot\alpha} \sum_{n=0}^{p}\sum_{r=0}^{k}\binom{p}{n}\frac{n!}{(k-2r)!\,r!}(icot\alpha)^{k-r}(2x)^{(k-2r)}(csc\alpha.ux)^{p-n} \\ \cos\left(csc\alpha.ux + \frac{(p-n)\pi}{2}\right) e^{\frac{i}{2}(x^2)cot\alpha}\sum_{m=0}^{q}\sum_{s=0}^{l}\binom{q}{m}\frac{m!}{(l-2s)!\,s!}(icot\alpha)^{l-s}(2y)^{(l-2s)}(csc\alpha.vy)^{q-m} \\ \cos\left(csc\alpha.vy + \frac{(q-m)\pi}{2}\right) e^{\frac{i}{2}(y^2)cot\alpha}\end{array}\right|$$

$$\gamma_{E,p,q}\big(k_{\alpha}(x,y,u,v)\big) = \sup_{\substack{x \in k \\ y \in I}}\left|\begin{array}{c}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cot\alpha}\sum_{m=0}^{q}\sum_{s=0}^{l}\sum_{n=0}^{p}\sum_{r=0}^{k}\binom{p}{n}\binom{q}{m}\frac{n!}{(k-2r)!r!} \\ \frac{m!}{(l-2s)!s!}(icot\alpha)^{k+l-r-s}(2x)^{(k-2r)}(2y)^{(l-2s)}(csc\alpha.ux)^{p-n}(csc\alpha.vy)^{q-m} \\ \cos\left(csc\alpha.ux+\frac{(p-n)\pi}{2}\right)\cos\left(csc\alpha.vy+\frac{(q-m)\pi}{2}\right)\end{array}\right| < \infty$$

4. Solution of differential equation for two-dimensional fractional Cosine transform

Consider the differential equation $P(D)w = f(x,y)$………………………………………………….. (1)

Where $f \in E'$ and $P(D) = \sum_{|\beta| \le m} a_{\beta} D^{\beta}$ is linear differential operator of order m with constant coefficient.

Suppose that the equation (1) possesses a solution w. applying the fractional cosine transform to (1) and using

$$D_x^p D_y^q k_{\alpha}(x,y,u,v) = \left[\begin{array}{c}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cot\alpha}\sum_{m=0}^{q}\sum_{s=0}^{l}\sum_{n=0}^{p}\sum_{r=0}^{k}\binom{p}{n}\binom{q}{m}\frac{n!}{(k-2r)!r!} \\ \frac{m!}{(l-2s)!s!}(icot\alpha)^{k+l-r-s}(2x)^{(k-2r)}(2y)^{(l-2s)}((csc\alpha)^2.uxvy) \\ \cos\left(csc\alpha.ux+\frac{(p-n)\pi}{2}\right)\cos\left(csc\alpha.vy+\frac{(q-m)\pi}{2}\right)\end{array}\right] \text{………. (2)}$$

$$F_{\alpha}^{c}\{P(D)w\} = F_{\alpha}^{c} f = f^{\wedge}$$

For different values of n, p in (2) we can reform them to the fractional cosine transform and hence we get,

$P(x,y,u,v).w^{\wedge} = f^{\wedge}$

Where $P(x,y,u,v)$ is polynomial in x, y, u, v

$w^{\wedge} = F_{\alpha}^{c}\{w\}$

Under the assumption that the polynomial p is such that

$$P(D_x^p D_y^q k_{\alpha}(x,y,u,v)) > \xi > 0$$

For $\xi = \xi_1, \xi_2, \xi_3 \ldots\ldots\ldots. \xi_n \epsilon R^n$……………………………………………………………..……..… (4)

Equation (3) gives

$w^{\wedge} = P\left[D_x^p D_y^q k_{\alpha}(x,y,u,v)\right]^{-1} f^{\wedge}$……………………………………………….........……...….. (5)

Applying inversion of fractional cosine transform to (5)

We get,

$w = [F_{\alpha}^{c}]^{-1}\left[\frac{f^{\wedge}}{P(D_x^p D_y^q k_{\alpha}(x,y,u,v))}\right]$……………………………………………………..………….…….. (6)

Next we show that if $f \epsilon E'$ then P satisfies (4) then equation (6) defines a tempered distribution which is the solution of equation (1) indeed since $f \epsilon E'$ then $0 < \alpha \leq \frac{\pi}{2}$, $F_\alpha^c(f) \epsilon E'$and hence by assumption (4) and the definition that if $\theta \epsilon \theta_M$ and $f \epsilon E'$ then the product θ_f is defined by $\langle \theta_f, \emptyset \rangle = \langle f, \theta\emptyset \rangle \; \forall \; \emptyset \in E'$

We have

$\frac{[F_\alpha^c(f)]}{\left[P(D_x^p D_y^q k_\alpha(x,y,u,v))\right]} \in E'$ And so $w \epsilon E'$

To show that w satisfies (1) we apply F_α^c to both sides of (6) and (5). Since tempered distribution admits multiplication by polynomials, we hence obtain equation (3). Finally applying $[F_\alpha^c]^{-1}$ to (3),
We get (1).

Conclusion: In the present work, we have solved differential equation for generalized two dimensional cosine transform.

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